Simple Harmonic Motion

Revised AH Physics 2013

7. A "saucer" swing consists of a bowl shaped seat of mass 1·2 kg suspended by four ropes of negligible mass as shown in Figure 7A.

Figure 7A

When the empty seat is pulled back slightly from its rest position and released its motion approximates to simple harmonic motion.

(*a*) Define the term *simple harmonic motion*. **1**

(*b*) The acceleration-time graph for the seat with no energy loss is shown in Figure 7B.

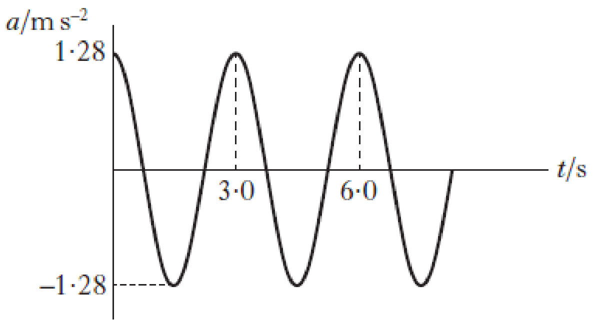

Figure 7B

(i) Show that the amplitude of the motion is 0·29 m. **3**

(ii) Calculate the velocity of the seat when its displacement is 0·10 m. **2**

(*c*) Calculate the displacement of the seat when the kinetic energy and potential energy are equal. **3**

(9)

8. Car engines use the ignition of fuel to release energy which moves the pistons up and down, causing the crankshaft to rotate.

The vertical motion of the piston approximates to simple harmonic motion.

Figure 8 shows different positions of a piston in a car engine.

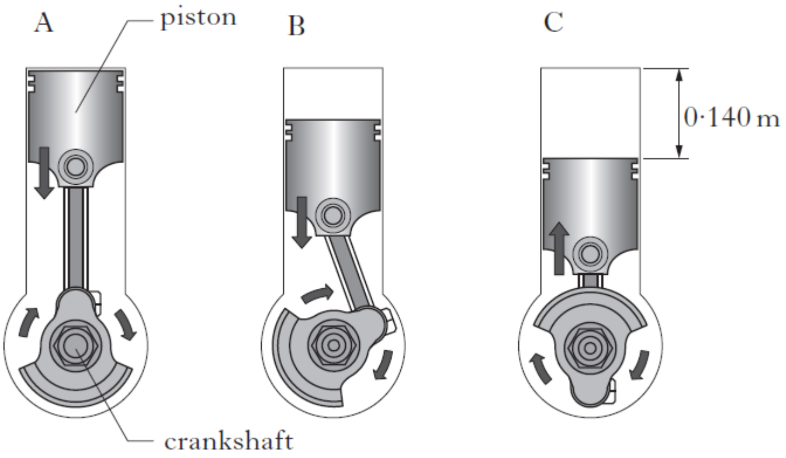

Figure 8

(*a*) Define *simple harmonic motion*. **1**

(*b*) Determine the amplitude of the motion. **1**

(*c*) In this engine the crankshaft rotates at 1500 revolutions per minute and the piston has a total mass of 1·40 kg.

(i) Calculate the maximum acceleration of the piston. **3**

(ii) Calculate the maximum kinetic energy of the piston. **2**

(7)

14. A group of students were evaluating an experiment to investigate the relationship between the mass on a spring and its period of oscillation. Figure 14 shows some of the apparatus used.

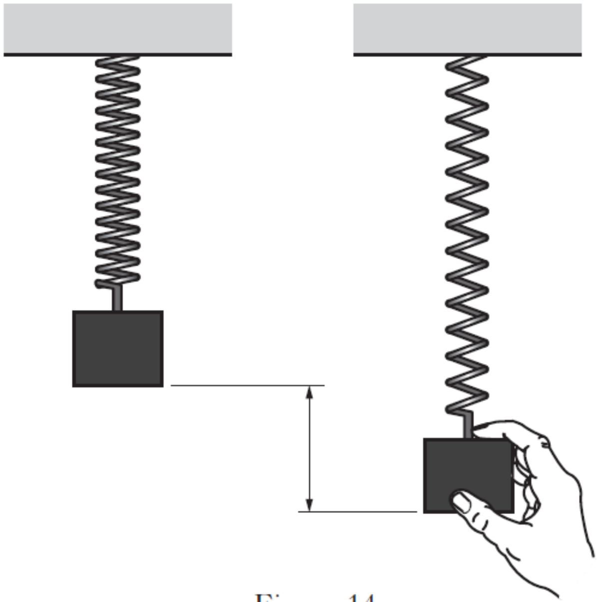

Figure 14

Student A stated "*I think we should use a balance that reads to 0·001 g instead of 0·1 g. This will give us a more accurate answer.*"

Student B stated "*I think we should repeat the time measurement and calculate a mean value.*"

Student C stated "*I think we should time the pendulum for 10 oscillations and divide this value by 10 to get the time for one complete oscillation. This will give us a more precise answer.*"

Student D stated "*I think it would be good to check the mass on another balance.*"

Using your knowledge of experimental analysis, comment on these statements. **(3)**

6. (*a*) (i) State what is meant by *simple harmonic motion*. 1

(ii) The displacement of an oscillating mass can be described by the expression

$$y = A\sin\omega t$$

where the symbols have their usual meanings.

Show that this mass exhibits simple harmonic motion (SHM). 2

(iii) The displacement of an object exhibiting SHM can also be written as

$$y = A\cos\omega t$$

Identify the initial condition for which this equation would be used. 1

(*b*) A mass attached to a spring is displaced from its equilibrium position and allowed to oscillate vertically. A motion sensor, connected to a computer, is placed below the mass as shown in Figure 6A.

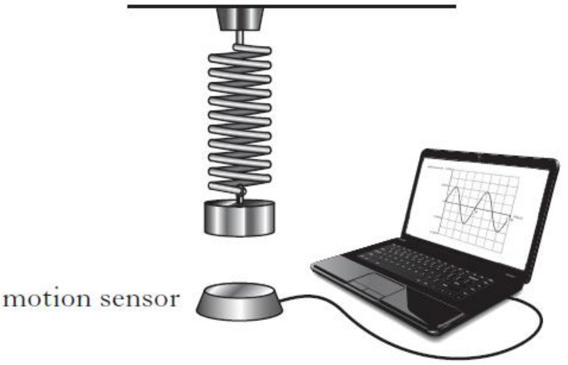

Figure 6A

Figure 6B shows the graph of the displacement from equilibrium position against time for the mass.

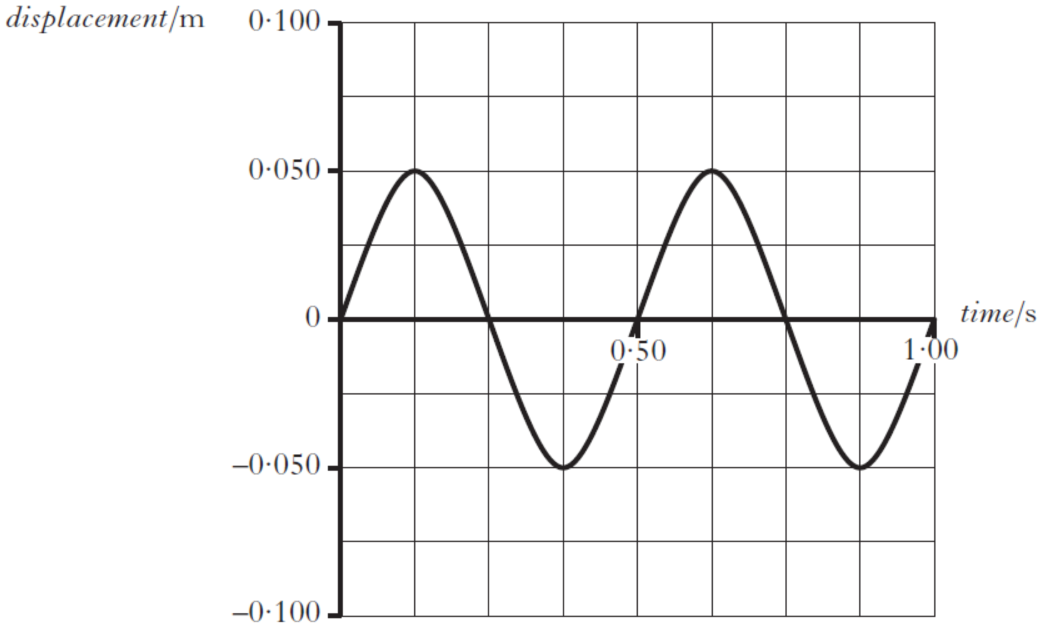

Figure 6B

(i) Using data from the graph, determine the velocity of the mass at 0·50 s. **3**

(ii) Calculate the maximum acceleration of the mass. **2**

(*c*) The system is modified by attaching a rigid card of negligible mass as shown in Figure 6C.

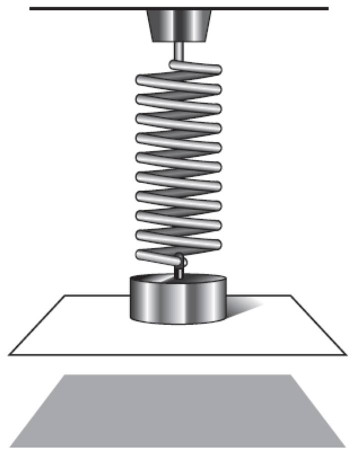

Figure 6C

The mass is displaced from its equilibrium position and allowed to oscillate vertically.

Sketch a displacement time graph of this motion. **1**

(10)

10. (a) (i) State what is meant by *simple harmonic motion*. **1**

(ii) The displacement of an oscillating object can be described by the expression

$$y = A\cos\omega t$$

where the symbols have their usual meaning.

Show that this expression is a solution to the equation

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

2

10. (continued)

(b) A mass of 1·5 kg is suspended from a spring of negligible mass as shown in Figure 10. The mass is displaced downwards 0·040 m from its equilibrium position.

The mass is then released from this position and begins to oscillate. The mass completes ten oscillations in a time of 12 s.

Frictional forces can be considered to be negligible.

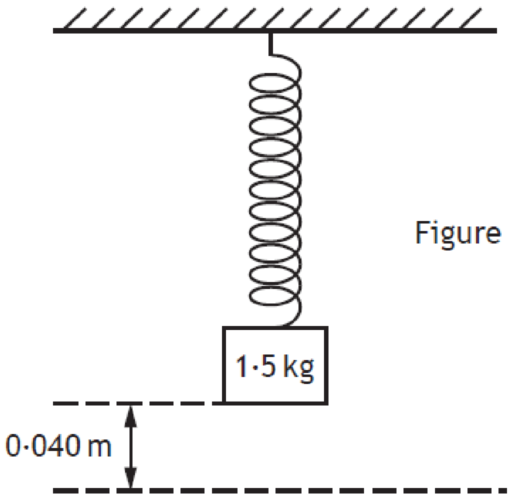

Figure 10

(i) Show that the angular frequency ω of the mass is $5 \cdot 2\ \text{rad s}^{-1}$. **3**

8. A student is investigating simple harmonic motion. An oscillating mass on a spring, and a motion sensor connected to a computer, are used in the investigation. This is shown in Figure 8A.

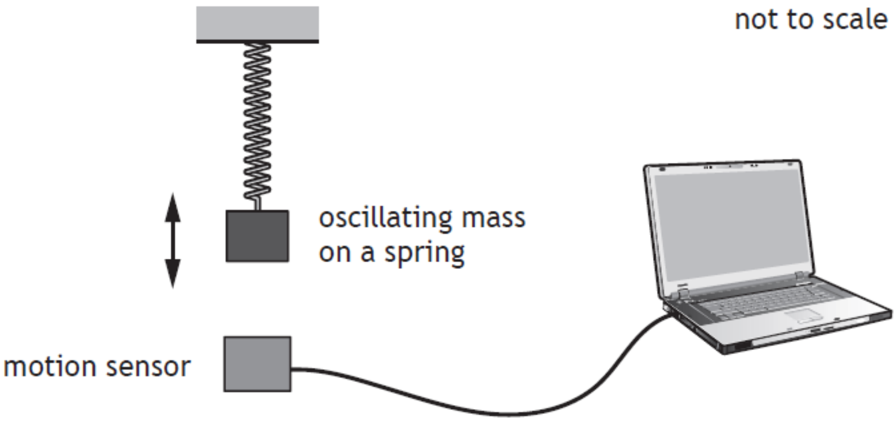

Figure 8A

The student raises the mass from its rest position and then releases it. The computer starts recording data when the mass is released.

(a) The student plans to model the displacement y of the mass from its rest position, using the expression

$$y = A\sin\omega t$$

where the symbols have their usual meaning.

Explain why the student is incorrect. **1**

8. (continued)

(b) (i) The unbalanced force acting on the mass is given by the expression

$$F = -m\omega^2 y$$

Hooke's Law is given by the expression

$$F = -ky$$

where k is the spring constant.

By comparing these expressions, show that the frequency of the oscillation can be described by the relationship

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

2

(ii) The student measures the mass to be 0·50 kg and the period of oscillation to be 0·80 s.

Determine a value for the spring constant k. **3**

8\. (b) (continued)

(iii) The student plans to repeat the experiment using the same mass and a second spring, which has a spring constant twice the value of the original.

Determine the expected period of oscillation of the mass. **2**

(c) The student obtains graphs showing the variation of displacement with time, velocity with time and acceleration with time.

The student forgets to label the y-axis for each graph.

Complete the labelling of the y-axis of each graph in Figure 8B. **2**