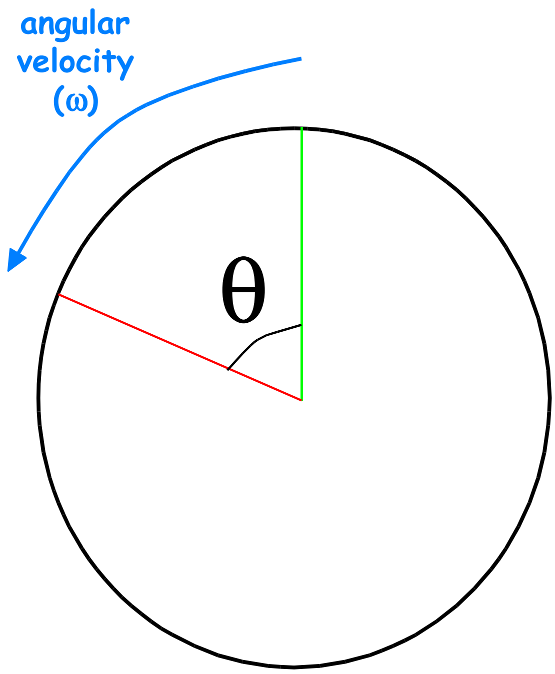
For an object travelling in a circular path:

- **State that angular velocity (ω) is the rate of change of angular displacement (θ).**

$$\omega = \frac{d\theta}{dt}$$

- **State that angular acceleration (α) is the rate of change of angular velocity (ω).**

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$



Comparing Linear Motion and Circular Motion

Linear Motion	Circular Motion
s is the	θ is the
u is the	ω_o is the
v is the	ω is the
a is the	α is the
t is the	t is the
$v =$	$\omega =$
$v^2 =$	$\omega^2 =$
$s =$	$\theta =$

---

1) **(a)** State the relationship between **radians** and **degrees**:

(b) Convert the following from **degrees** to **radians**:

30°	**45°**	**60°**	**90°**
180°	**270°**	**360°**	**720°**

(c) Convert the following from **radians** to **degrees**:

1 rad	**10 rad**	**0.1 rad**	**π rad**
2π rad	**$\pi/2$ rad**	**$\pi/4$ rad**	**$\pi/6$ rad**

2) Engineers usually quote the value for the **angular velocity** of a rotating machine shaft in units of **revolutions per minute** (**rpm**)**.**

(a) How do we convert units of **rpm** to units of **rad s^{-1}** ?

(b) Convert the following from **rpm** to **rad s^{-1}**:

10 rpm	**33 rpm**	**45 rpm**	**78 rpm**
100 rpm	**500 rpm**	**1 000 rpm**	**2 500 rpm**

3) Using **calculus notation**, write down an expression for:

(a) angular velocity in terms of **angular displacement**.

(b) angular acceleration in terms of **angular velocity**.

(c) angular acceleration in terms of **angular displacement**.

4) State the **3 equations** which can be applied to any object moving with constant (uniform) **angular acceleration**. (State the meaning of each symbol used).

- **Carry out calculations involving constant angular accelerations.**

5) A wheel accelerates uniformly from rest to 3.0 rad $s^{-2.}$ This takes 5.0 s.

(a) Calculate the **angular velocity** of the wheel after the 5.0 s.

(b) Determine the **angular displacement** of the wheel after the 5.0 s.

6) A disc is rotating with a constant angular velocity of 200 rad s^{-1}.
The disc decelerates uniformly at 5.0 rad s^{-2} for 4.0 s.

(a) Calculate the disc's **angular velocity** at the end of the 4.0 s.

(b) Determine the disc's **angular displacement** in this time.

7) The angular velocity of an engine shaft is increased uniformly from 800 rpm to 3 000 rpm in 8.0 s.

(a) Convert these angular velocity values to **rad s^{-1}**.

(b) Calculate the uniform **angular acceleration** of the engine shaft.

(c) Determine the total **angular displacement** of the motor shaft in this time.

(d) How many **revolutions** does the engine shaft make during these 8.0 s?

8) The blades of an electric fan are rotating with an angular velocity of 80 rad s^{-1}.
The fan is now switched off. The fan blades take 12 s to come to rest.

Calculate the **angular deceleration** of the fan blades.

9) A rotating lawn mower shaft accelerates from rest to 8 rad s^{-1}. Its angular displacement is 8 rad.
Determine the **acceleration** of the shaft.

10) A rotating toy is moving with constant angular velocity. It then accelerates uniformly at 3.0 rad s^{-2} for 3 s, achieving a final angular velocity of 20 rad s^{-1}.

Calculate the **initial angular velocity** of the rotating toy.

11) A spinning wheel is travelling at a constant angular velocity of 1.5 rad s^{-1}. It then accelerates at 2.5 rad s^{-1} for a time of 6.0 s.

Calculate the **angular displacement** of the spinning wheel during the 6.0 s.

12) A rotating machine decelerates uniformly from an angular velocity of 30 rad s^{-1}. During the 50 s this takes, its angular displacement is 1 000 rad.

Determine the **deceleration** of the rotating machine during these 50 s.

13) What time will it take a turning rod to decelerate uniformly at 60 rad s^{-2} from an angular velocity of 340 rad s^{-1} to one of 100 rad s^{-1}.

14) An engine cog, rotating at 12.5 rad s^{-1}, comes to rest when the engine is turned off. The cog decelerates uniformly at 1.25 rad s^{-2}.

Calculate the **angular displacement** during this stopping period

15) A rotating machine part accelerates from rest to 5 rad s^{-1}. If the angular acceleration is 0.25 rad s^{-2}, calculate the **angular displacement** of the machine part.

16) A reel of thread on a sewing machine is rotating at 4.0 rad s^{-1}. It accelerates with a constant angular acceleration of 2.0 rad s^{-2} for 6.0 s.

Calculate the reel's **angular velocity** at the end of the 6.0 s.

17) A rotating drill accelerates at 2.5 rad s^{-2} for 2.0 s. During this time, its angular displacement is 12 rad.

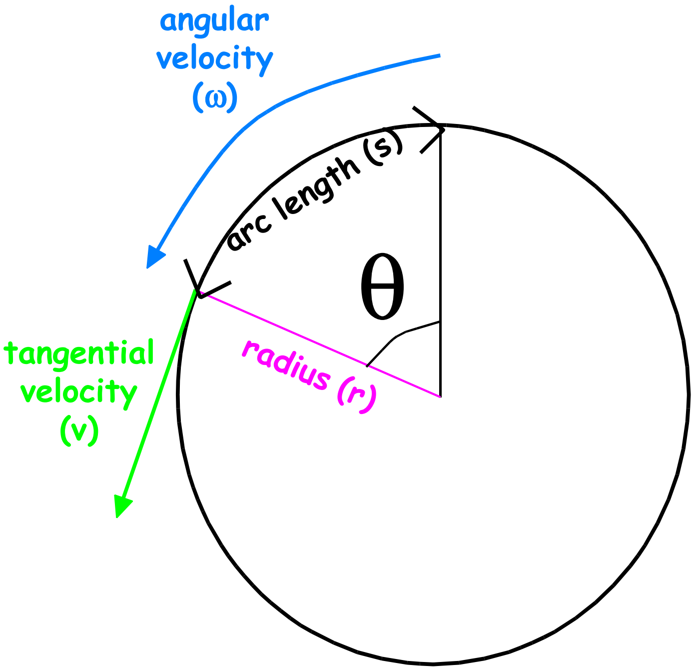
Calculate the drill's **initial angular velocity**.

18) Determine the **angular acceleration** of a lathe shaft, originally rotating at 2.0 rad s^{-1}, which accelerates constantly for 3.0 s. Its angular displacement during this time is 15 rad.

19) A metal bar is initially rotating at 3.0 rad s^{-1}. Over the next 4.0 s, the bar decelerates uniformly at 0.25 rad s^{-2}.

Calculate the **angular displacement** of the bar during the 4.0 s.

- **Derive the expression v = rω for a particle in circular motion.**



● Carry out calculations involving the relationship between tangential velocity (v), radius (r) and angular velocity (ω).

20) In each case, calculate the **tangential velocity** of the particle in uniform circular motion:		
● angular velocity = 8.0 rad s^{-1} **● radius of rotation = 0.15 m**	**● angular velocity = 18.0 rad s^{-1}** **● radius of rotation = 1.20 m**	**● angular velocity = 4.0 rad s^{-1}** **● radius of rotation = 0.75 m**

21) In each case, calculate the **radius of rotation** of the particle in uniform circular motion:		
● angular velocity = 6.0 rad s^{-1} **● tangential velocity = 15 m s^{-1}**	**● angular velocity = 0.25 rad s^{-1}** **● tangential velocity = 7.5 m s^{-1}**	**● angular velocity = 0.50 rad s^{-1}** **● tangential velocity = 0.25 m s^{-1}**

22) In each case, calculate the **angular velocity** of the particle in uniform circular motion:		
● tangential velocity = 12.5 m s^{-1} **● radius of rotation = 0.5 m**	**● tangential velocity = 9.0 m s^{-1}** **● radius of rotation = 0.15 m**	**● tangential velocity = 11.7 m s^{-1}** **● radius of rotation = 1.8 m**

23) A computer storage disc is revolving with a constant angular velocity of 1.5 rad s^{-1}.

(a) Calculate the **tangential velocity** of a spot of dust which is stuck on the disc surface at the following distances from the centre of rotation:

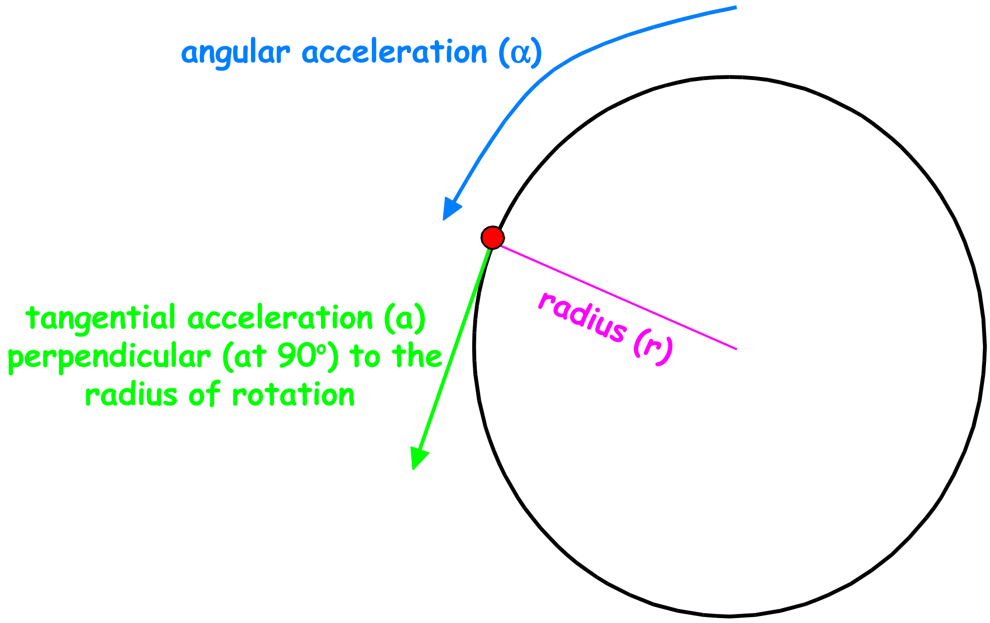
0.10 m	**0.20 m**	**0.25 m**

(b) If the angular velocity remains constant, what happens to the **tangential velocity** of a particle in uniform circular motion as its distance from the centre of rotation increases?

● Carry out calculations involving the relationship between tangential acceleration (a), radius (r) and angular acceleration (α).

Consider a particle in circular motion.

When the angular acceleration (α) of the particle changes, the tangential acceleration (a) will also change.



24) In each case, calculate the **tangential acceleration** of the particle in circular motion:

● **angular acceleration = 4.0 rad s^{-2}** ● **radius of rotation = 0.15 m**	● **angular acceleration = 8.0 rad s^{-2}** ● **radius of rotation = 1.2 m**	● **angular acceleration = 12 rad s^{-2}** ● **radius of rotation = 0.75 m**

25) In each case, calculate the **radius of rotation** of the particle in circular motion:

● **angular acceleration = 0.50 rad s^{-2}** ● **tangential acceleration = 2.5 m s^{-2}**	● **angular acceleration = 0.25 rad s^{-2}** ● **tangential acceleration = 2.5 m s^{-2}**	● **angular acceleration = 0.45 rad s^{-2}** ● **tangential acceleration = 0.90 m s^{-2}**

26) In each case, calculate the **angular acceleration** of the particle in circular motion:

● **tangential acceleration = 1.25 m s^{-2}** ● **radius of rotation = 3.75 m**	● **tangential acceleration = 0.90 m s^{-2}** ● **radius of rotation = 0.15 m**	● **tangential acceleration = 0.21 m s^{-2}** ● **radius of rotation = 0.70 m**

27) A particle in circular motion has a radius of rotation of 10 m.
Its angular velocity is increased from 1.1 rad s^{-1} to 1.9 rad s^{-1} in a time of 10 s.

(a) Using an equation of rotational motion, calculate the **angular acceleration** of the particle.

(b) Calculate the **tangential acceleration** of the particle.

(c) State the direction of the **tangential acceleration**.

28) Graham swings a conker on a 1.2 m long string around his head. The conker accelerates from rest. After 3.0 s, its angular displacement is 45 radians.

(a) Using an equation of rotational motion, calculate the **angular acceleration** of the conker.

(b) Calculate the **tangential acceleration** of the conker.

(c) State the direction of the **tangential acceleration**.

29) A particle following a circular path of radius 1.2 m decelerates from an angular velocity of 95 rad s^{-1} to 50 rad s^{-1} in 5.0 s.

(a) Using an equation of rotational motion, calculate the **angular deceleration** of the particle.

(b) Calculate the **tangential deceleration** of the particle.

(c) State the direction of the **tangential deceleration**.

30) A rotating drum of radius 3.50 m slows down from an angular velocity of 1.50 rad s^{-1} to 1.20 rad s^{-1}. The angular displacement of a particle on the circumference of the drum is 162 rad.

(a) Using an equation of rotational motion, calculate the **angular deceleration** of the particle on the drum's circumference.

(b) Calculate the **tangential deceleration** of the particle on the drum's circumference.

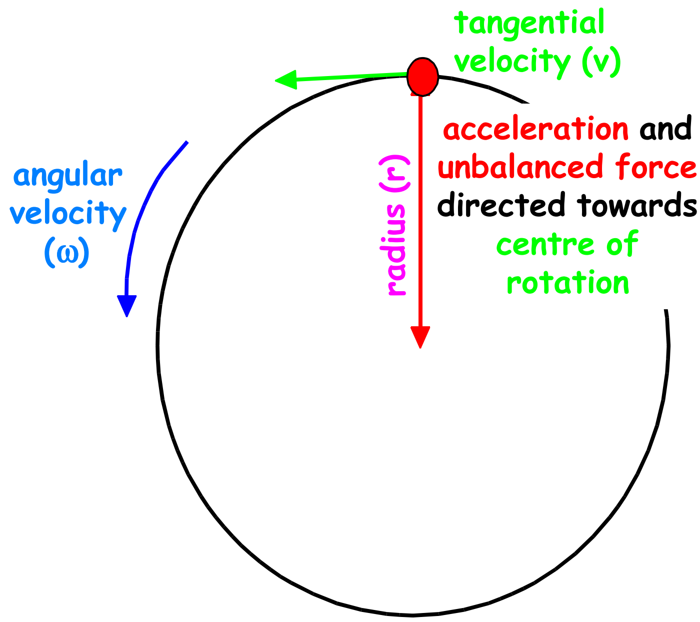
(c) State the direction of the **tangential deceleration**.

● **Explain that a central force is require to maintain circular motion.**

We now consider how Newton's laws of motion apply to uniform circular motion.

The direction of a particle in uniform circular motion keeps changing, so the velocity of the particle changes at a constant rate - This means the particle is **accelerating**.

To cause this **acceleration**, there must be an **unbalanced force** acting on the particle. The **acceleration** and **unbalanced force** are always directed towards the **centre of rotation** - The **unbalanced force** stops the particle "flying off" at a tangent to the circle.



We use the terms **centripetal** (or **radial**) **acceleration** and **centripetal force**.

- Centripetal means "centre seeking".
- Radial indicates that the acceleration and unbalanced force are directed along the radius of rotation, towards the centre of rotation.

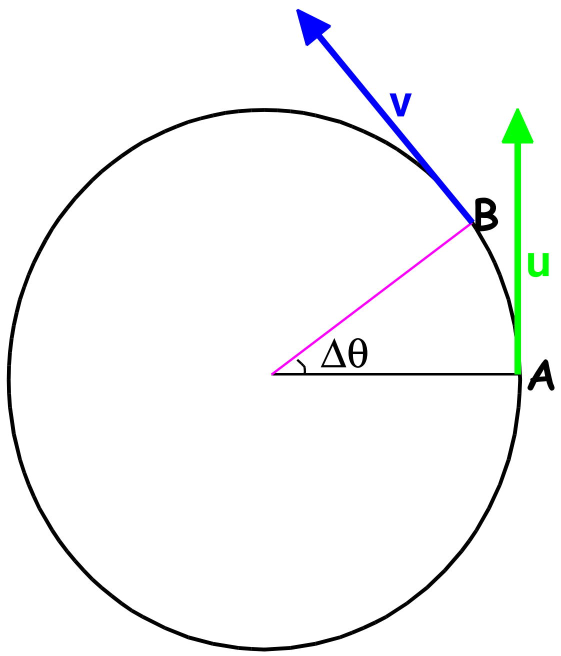
You must understand that **centripetal force** is not a "magic quantity" created as a result of uniform circular motion - **Centripetal force** is an applied force (or forces), the presence of which cause an object to undergo uniform circular motion. These applied forces include tension in a string, the force of friction between car tyres and a road as the car travels round a bend or the force of the rails on a train's wheels as the train rounds a curved section of track.

WARNING

Do not confuse **centripetal acceleration** with **tangential acceleration**.

- **Tangential acceleration** is always perpendicular to the radius of rotation, directed along the tangent to the circle. It only occurs when the **speed** of an object in circular motion changes.
- **Centripetal acceleration** is always directed towards the centre of rotation. Any object moving in a circular path has **centripetal acceleration**. The object does not need to speed up or slow down.

● **Derive the expressions $a_r = v^2/r$ and $a_r = r\omega^2$ for centripetal (radial) acceleration.**

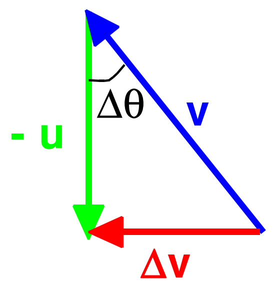


A particle in uniform circular motion travels from position A to position B through a small angle Δθ in a short time interval Δt.

- **u** is the **initial velocity vector**.
- **v** is the **final velocity vector**.

Change in velocity (**Δv**) = **v - u**.

Using **vector addition**:



Change in velocity (**Δv**) and therefore **acceleration** are directed towards the **centre of rotation**.

- **State that the central force require to maintain circular motion depends on mass, speed and radius of rotation**

Newton's second law of motion tells us that:

Unbalanced force = mass x acceleration

In the case of the **unbalanced force** **centripetal force** (F_r):

$$F_r = m\,a_r$$

$$= m\,\frac{v^2}{r}$$

$$F_r = m\,a_r$$

$$= m\,r\omega^2$$

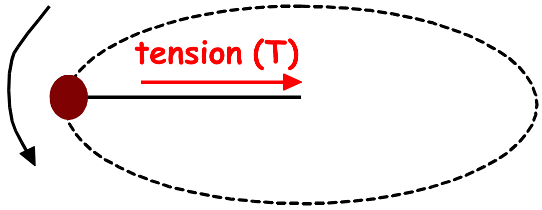
- **Carry out calculations involving central (centripetal) forces and radial accelerations (a_r).**

31) In each case, the data applies to an object in uniform circular motion.
Calculate the **centripetal force** (F_r) acting on each object:

• mass = 0.25 kg • tangential velocity = 1.2 m s^{-1} • radius of rotation = 0.15 m	• mass = 0.60 kg • tangential velocity = 1.5 m s^{-1} • radius of rotation = 0.50 m	• mass = 10 kg • tangential velocity = 0.80 m s^{-1} • radius of rotation = 1.6 m
• mass = 0.15 kg • radius of rotation = 0.30 m • angular velocity = 4.0 rad s^{-1}	• mass = 1.6 kg • radius of rotation = 0.60 m • angular velocity = 5.0 rad s^{-1}	• mass = 2.8 kg • radius of rotation = 2.5 m • angular velocity = 3.0 rad s^{-1}

Objects Moving in a Horizontal Circle

Consider a conker on a length of string being swung in a horizontal circle.



The only force acting in the horizontal direction is the **tension** in the string, so the **tension** must provide the **centripetal force**:

tension = centripetal force

$$T = m\frac{v^2}{r} = m r \omega^2$$

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32) A 0.0012 kg conker on a 0.90 m long piece of string is moving in a horizontal circle with a tangential velocity of 1.5 m s^{-1}.

(a) Sketch this situation. On your sketch, you should name an label any horizontal force(s) acting on the conker and show the force direction.

(b) Calculate the value of the **centripetal force** acting on the conker.

33) A swingball (mass 1.2 kg) is fixed on a 1.5 m long cord which is free to rotate. The swingball moves in a horizontal circle with an angular velocity of 6.0 rad s^{-1}.

(a) What produces the **centripetal force** acting on the swingball?

(b) Calculate the value of the **centripetal force** acting on the swingball.

34) The centripetal force acting on a 1.4 kg object rotating in a horizontal circle of diameter 1.00 m is 25 N.

(a) Determine the object's **angular velocity**.

(b) Determine the object's **tangential velocity**.

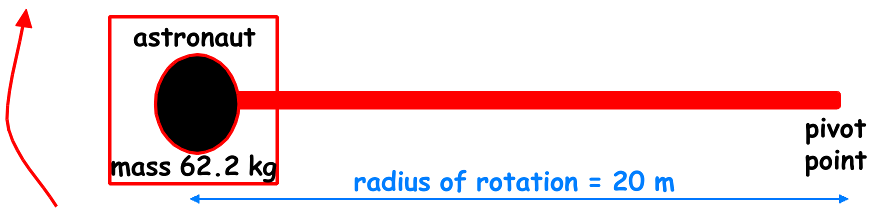
35) A length of twine will break when a force greater than 56 N is applied to it.

(a) If the twine is used to twirl a mass in a horizontal circle, what will be the maximum value of **centripetal force** it will withstand without snapping?

(b) If the twine is used to twirl a 0.15 kg mass in a horizontal circle of diameter 2.4 m, calculate the **maximum angular velocity** the mass can attain before the twine snaps.

(c) What will be the **maximum length of twine** which will allow the 0.15 kg mass to be rotated in a horizontal circle with an angular velocity of 85 rpm?

36) The diagram shows a plan view of a device used to train astronauts.
The device rotates about the pivot point with a constant angular velocity.



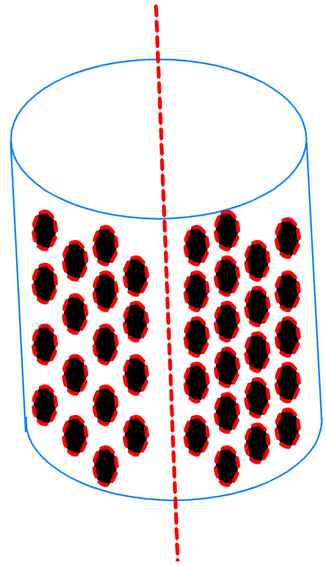
(a) The angular velocity of the astronaut is constant, yet she experiences a **horizontal force**.

(i) State the **direction** of the **horizontal force**.

(ii) Explain why the astronaut experiences this **horizontal force**.

(b) If the **horizontal force** acting on the astronaut has a magnitude of 2 800 N, determine her **angular velocity**.

37) Clothes can be dried in a spin dryer. This consists of a hollow drum which rotates about a central axis. There are a number of holes in the drum wall.



(a) Explain why, when wet clothes are spun in the drum, water passes from the clothes out of the drum through the holes.

(b) The drum rotates at 1 000 rpm.

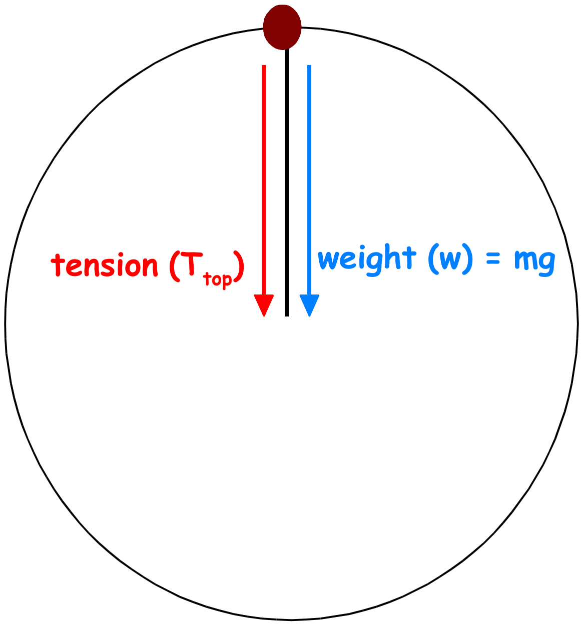
(i) Change this to an angular velocity expressed in **rad** $\mathbf{s^{-1}}$.

(ii) Calculate the **centripetal force** which would be exerted on Craig's wet jeans (mass 0.50 kg) when they are spun in the rotating drum which has a diameter of 0.80 m.

Objects Moving in a Vertical Circle

Consider a conker on a length of string being swung in a vertical circle.

When conker is at **top** of circle:

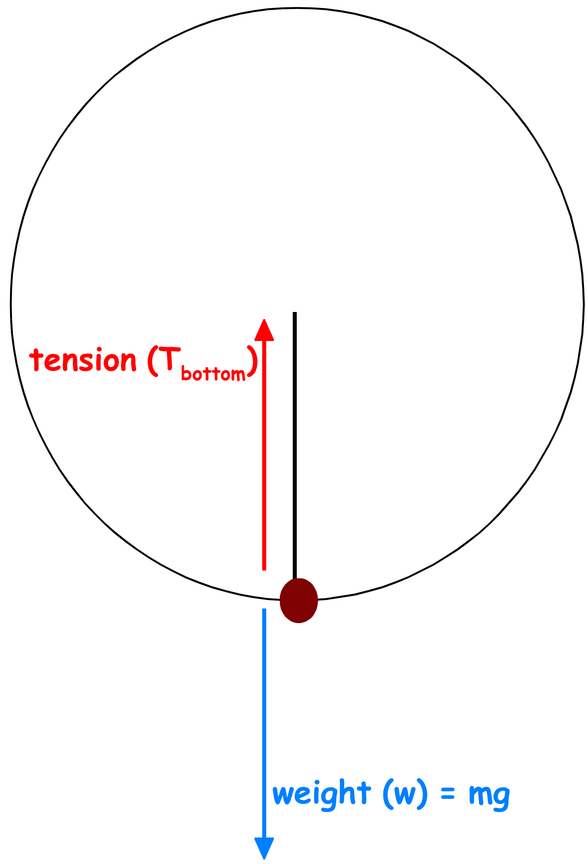


Both tension and weight act down towards centre of rotation.

The resultant of these 2 forces provides the centripetal force.

tension + weight = centripetal force

When conker is at **bottom** of circle:



Tension and weight act in opposite directions.

The resultant of these 2 forces provides the centripetal force.

tension − weight = centripetal force

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46) A 3.0 kg mass is whirled round in a vertical circle of radius 0.75 m with a tangential velocity of 8.0 m s^{-1}.

Calculate the **tension** in the string when the circling mass is:

(a) at the **top** of the circle;

(b) at the **bottom** of the circle.

(a) 227 N (b) 285 N

47) Helen the hammer thrower swings a 5.0 kg steel ball on a 1.2 m long chain in a **vertical** circle with an angular velocity of 9.0 rad s^{-1}.

(a) Sketch this situation, showing the forces acting on the chain when the steel ball is at its highest and lowest points.

(b) At what points in the circle will the tension in the chain have its maximum and minimum value?

(c) Calculate these maximum and minimum tension values.

(d) At what point of the rotation is the chain most likely to snap? Explain why.

(c) 535 N and 437 N

48) A hump-backed bridge takes the form of a circular arc, radius 35 m.
A car travelling over the bridge is at the top of the arc.

(a) Sketch this situation.

(b) Determine the greatest **tangential velocity** with which the car can cross the bridge without leaving the road at its highest point.

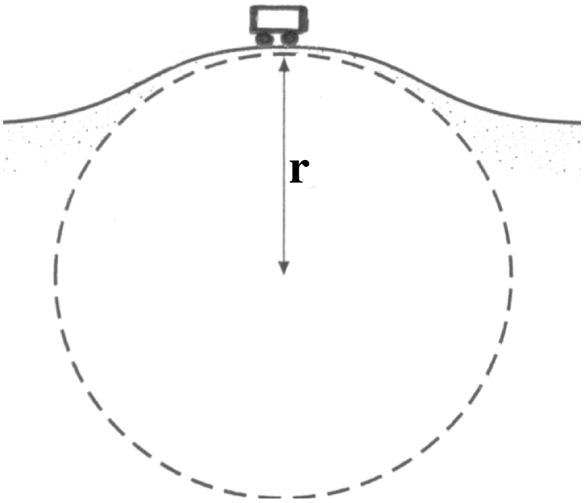
(b) 18.5 m s^{-1}

50) A vehicle is travelling with constant tangential speed across a small mound which has a profile approximating to the arc of a circle with radius **r**.

The vehicle has just reached the very top of the mound.

(a) On the diagram, show and label any forces which make up the **centripetal force** acting on the vehicle.

(b) In terms of **tangential velocity (v)**, state when the vehicle's wheels will be able to remain on the ground when it has reached the top of the mound.



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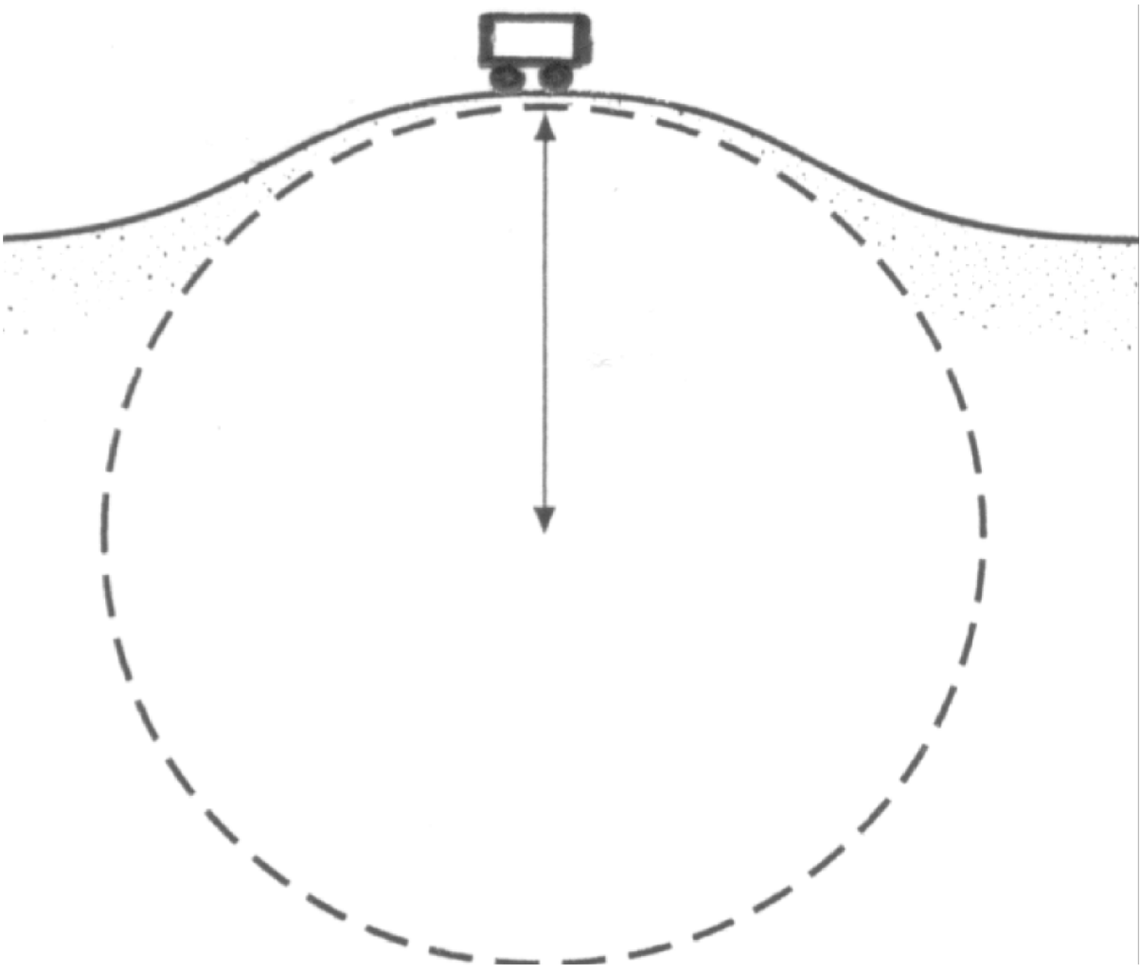
(c) For the values of radius given below, calculate the minimum value of **tangential velocity** the vehicle can reach at the top of the mound without its wheels leaving the ground.

(i) **r = 5.0 m**	(ii) **r = 10.0 m**
(iii) **r = 15.0 m**	(iv) **r = 20.0 m**

49) The moving vehicle shown has just reached the top of the humped-back bridge.

(a) On the diagram, show and name any force(s) which contribute to the centripetal force.

(b) In terms of **gravitational field strength** (**g**), explain why the vehicle will leave the road surface if its tangential velocity exceeds a certain limiting value.



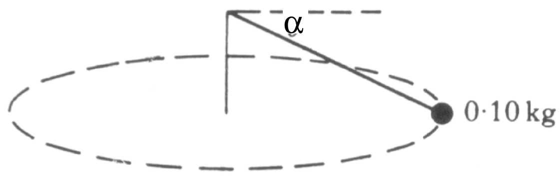
(a) In terms of centripetal acceleration, and hence force, explain why a conical pendulum follows a circular path.

(b) If the pendulum bob is given a greater horizontal speed, it moves outwards, away from the centre of rotation. Explain this in terms of the size of the centripetal force acting on the bob.

(c) If the bob of such a pendulum has a mass of 0.1 kg and takes 0.3 s to make 1 complete revolution, calculate its angular velocity.

(d) Calculate the angle θ the string makes with the vertical if the string is 0.1 m long.

An object of mass 0.10 kg moves in a horizontal circle at the end of a length of string 0.50 m long, as shown in the diagram. The angle shown, α, is the angle the string makes with the horizontal.



(a) On the diagram, show the forces acting on the 0.10 kg object.

(b) The speed of rotation is steadily increased.

(i) Explain what happens to the angle α.

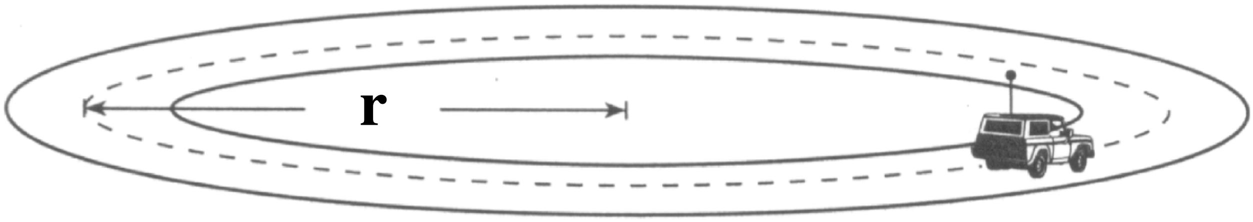
(ii) Calculate the centripetal force acting on the object when α = 10º.

(iii) If the string can withstand a force of 10 N, calculate the angle α at which the string would break.

Vehicle Rounding a Horizontal Curve

For a vehicle rounding a horizontal curve, the centripetal force is provided by the force of friction acting between the tyres and the road.

$F_{friction} =$ or $F_{friction} =$

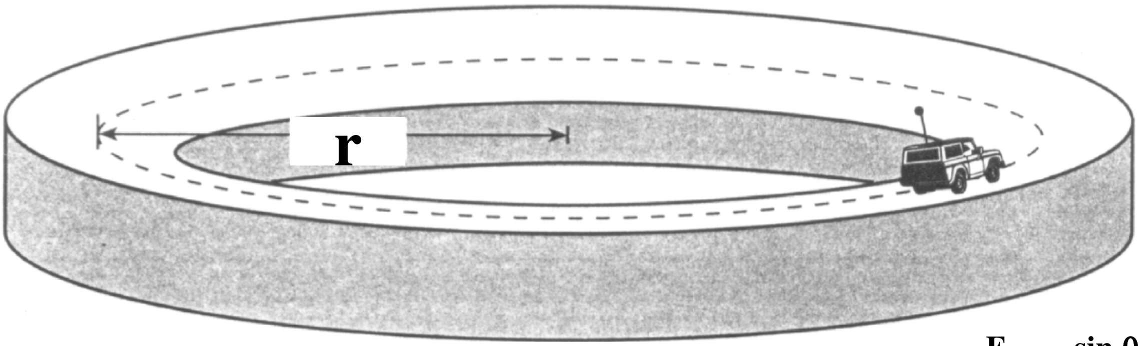


If the vehicle's speed increases to a certain value, the force of friction will no longer be large enough to maintain the circular path - so the vehicle will skid off at a tangent to the curve.

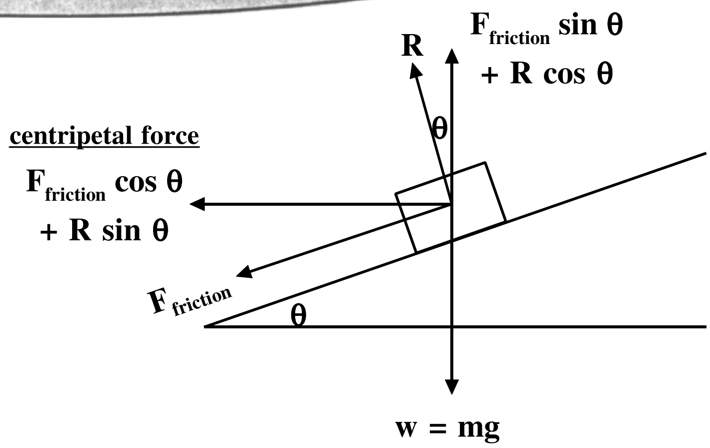
38) For each case below, calculate the smallest value of friction force between the vehicle and road that will allow the vehicle to follow the circular path without skidding off at a tangent.

m = 1 000 kg, v = 25 m s^{-1}, r = 30 m	m = 750 kg, v = 12 m s^{-1}, r = 12 m	m = 2 kg, v = 3 m s^{-1}, r = 1.5 m
m = 5 kg, v = 2.5 m s^{-1}, r = 0.25 m	m = 10 000 kg, v = 25 m s^{-1}, r = 50 m	m = 1 250 kg, v = 7.5 m s^{-1}, r = 18 m
m = 300 kg, v = 22.5 m s^{-1}, r = 17.5 m	m = 3 000 kg, v = 8 m s^{-1}, r = 45 m	m = 0.15 kg, v = 2.5 m s^{-1}, r = 2.2 m

Vehicle Rounding a Banked Curve



A vehicle is able to travel faster on a banked curve than on a horizontal curve before skidding off at a tangent because the centripetal force is greater on the banked curve - It is provided by a **component of the friction force ($F_{friction}$) between the vehicle tyres and road** + a **component of the normal reaction force (R) of the road on the vehicle**.



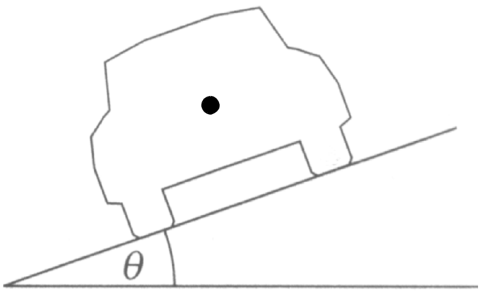
39) For a certain angle of banking (θ), the friction force (**$F_{friction}$**) between the vehicle tyres and road becomes zero, hence its components **$F_{friction}$ sin θ** and **$F_{friction}$ cos θ** become zero.

1) Show this situation on the diagram below, i.e., use arrows to represent the remaining forces which act on the vehicle.

2) Give an equation for the forces acting on the vehicle in the **vertical** direction.

3) Give equations (one involving **v** and one involving **ω**) for the forces acting on the vehicle in the **horizontal** direction.

4) Combine your equation from **2** and your **equation** involving **v** from **3** to form an equation involving **tan θ** which represents the motion of the vehicle rounding a banked curve when θ is such that the force of friction between the vehicle tyres and the road is zero.



40) A racing car moves around a banked circular track of radius 75 m with a constant tangential velocity of 25 m s^{-1}. The angle of the banking to the horizontal is such that the friction force between the tyres and track surface is zero. Calculate the **banking angle**.

40.4°

41) A supersonic aircraft banks in a circular path of diameter 30 km with a constant tangential velocity of 400 m s^{-1}. If the banking angle is such that the frictional forces are zero, determine the **angle above the horizontal** at which the aircraft banks.

47.5°

42) A circular cycle track is banked at 30° to the horizontal. When travelling at a constant tangential velocity of 15 m s^{-1} around the track, the frictional force between a bicycle's tyres and the track surface is 0 N. Determine the **track radius**.

39 m

43) A circular train track makes an angle of 15° above the horizontal. A train travels around the track at a tangential velocity of 12.5 m s^{-1} which eliminates frictional forces. Calculate the **diameter** of the train track.

118.8 m

44) A toy car travels around a circular track banked at 25° to the horizontal with a tangential velocity **v** such that the friction acting between the tyres and track top is zero. If the track has a radius of 3 m, calculate the value of **v** in m s^{-1}.

3.7 m s^{-1}

45) A marble rolls around a roulette wheel of diameter 0.5 m banked at 35° to the horizontal. If the tangential velocity of the marble is such that frictional forces are eliminated, determine its **tangential velocity**.

1.3 m s^{-1}

A student collects a radio-controlled toy car from her teacher.

Angular Momentum/Rotational Kinetic Energy Problems

With her arms outstretched, Claire the ice skater spins on the same spot with the constant angular velocity show on the diagram. For this arm position, her moment of inertia is 75.0 kg m^2.

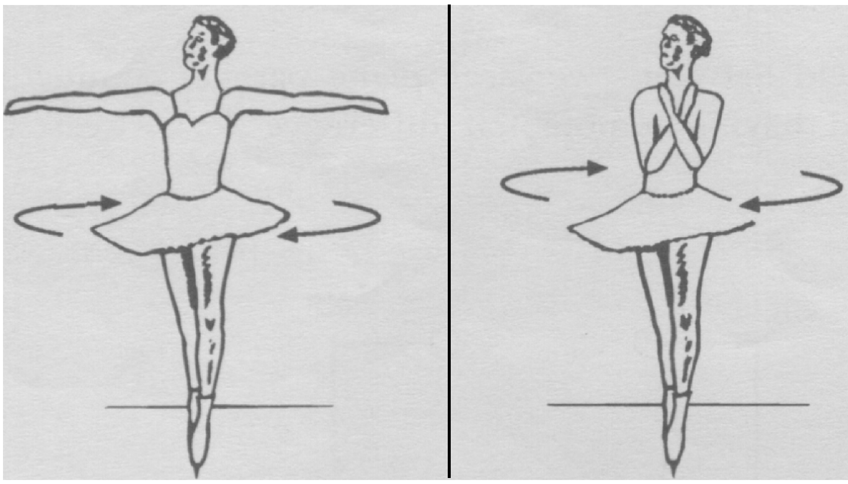
When Claire draws her arms in close to her body, her angular velocity increases, as shown on the diagram.

(a) (i) Calculate Claire's **moment of inertia** for the "close in" arms position;

(ii) What assumption have you made?

(b) Calculate Claire's **rotational kinetic energy** for both arm positions.

9.50 rad s^{-1}



27.5 rad s^{-1}

(a) (i) 25.9 kg m^2 (b) arms out = 3 384 J, arms in = 9 793 J

A children's roundabout has a mass of 500 kg and radius 4.0 m. The formula for its moment of inertia is 1/2 mr^2. John (mass 50 kg) is positioned on the edge of the roundabout and can be considered to be a point mass. The roundabout rotates with an angular velocity of 4.8 rad s^{-1}.

(a) Calculate the **total moment of inertia** for the rotating system.

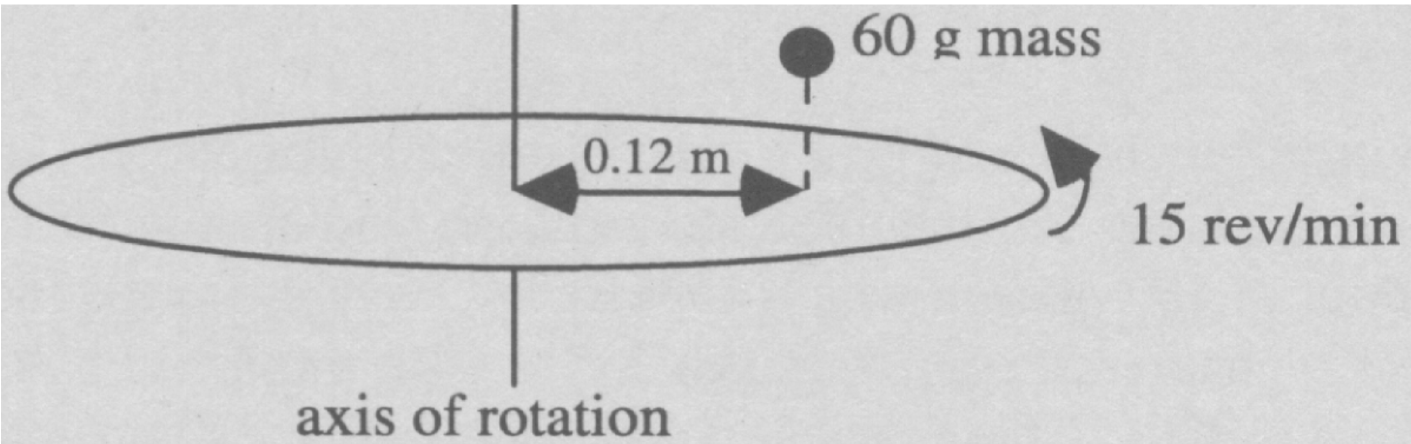
(b) Calculate the **angular momentum** of the rotating system.

John falls off the roundabout.

(c) (i) Calculate the new **angular velocity** of the roundabout. (ii) State any assumption you have made.

(d) Calculate the **rotational kinetic energy** of the roundabout before and after John falls off.

(a) 4 800 kg m^2 (b) 23 040 kg m^2 s^{-1} (c) (i) 5.8 rad s^{-1} (d) before = 55 296 J, after = 67 280 J



A CD rotates around a central axis at 15 revolutions per minute. The moment of inertia for the CD is 1.74×10^{-3} kg m^2.

(a) Calculate the **angular velocity** of the CD.

A 60 g mass is now dropped on the CD. It sticks to the CD surface at a distance of 0.12 m from the central axis. The mass should be regarded as a point mass.

(b) Calculate the new **moment of inertia** for the rotating system.

(c) (i) Determine the new **angular velocity** of the rotating system.

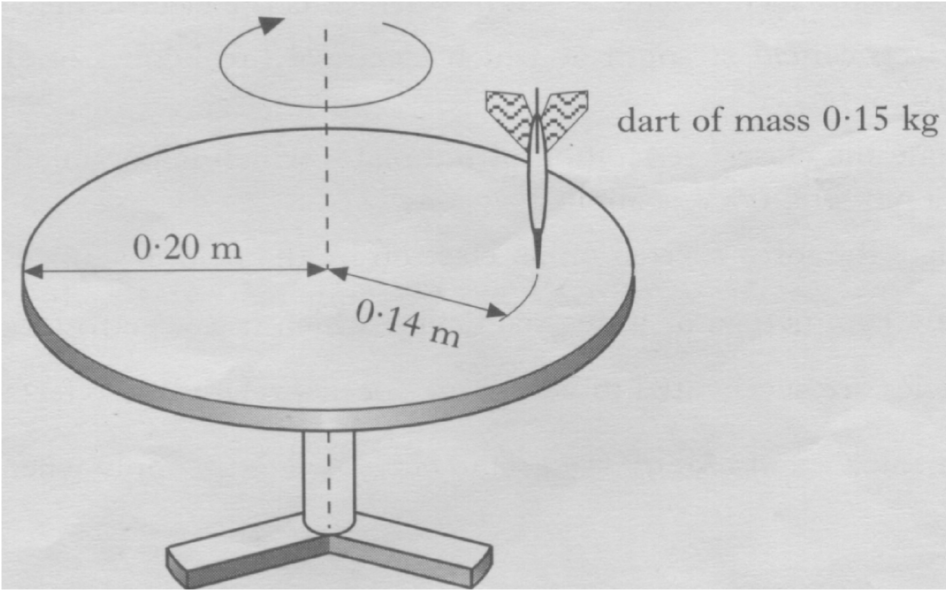
(ii) State any assumption you have made.

(d) Use values for rotational kinetic energy to show that the impact of the mass with the CD is **inelastic**.

(a) 1.6 rad s^{-1} (b) 2.60×10^{-3} kg m^2 (c) (i) 1.1 rad s^{-1} (d) 2.2×10^{-3} J before, 1.6×10^{-3} J after

A turntable is in the form of a uniform disc of mass 1.2 kg and radius 0.20 m. It rotates in the horizontal plane on frictionless bearings around its centre. Its initial angular velocity is 8.0 rad s^{-1}.
Moment of inertia for the turntable = 1/2 mr^2.

A small dart of mass 0.15 kg is dropped from rest just above the rotating turntable and sticks into it at a distance of 0.14 m from its centre, as shown in the diagram below.



(a) (i) Calculate the new **angular velocity** of the system. (ii) State any assumption(s) you have made.

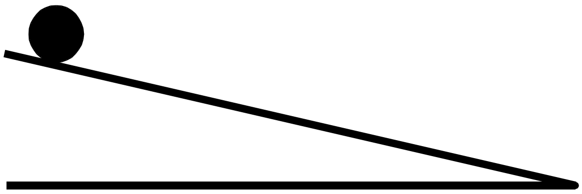
(b) (i) Calculate the **kinetic energy** of the turntable before the dart is dropped.

(ii) Calculate the **kinetic energy** of the turntable and dart after the dart has been dropped.

(iii) Account for any change in **kinetic energy**.

(a) (i) 7.2 rad s^{-1} (b) (i) 1.5 J, (ii) 0.7 J

Cylinder/Sphere on Slope Problems



When a cylinder or sphere is rolled down a slope from rest, the **gravitational potential energy** of the cylinder/sphere at the point of release (**mgh**) is converted to both **linear kinetic energy** (**1/2 mv^2**) and **rotational kinetic energy** (**1/2 Iω^2**).

Assuming no slipping takes place:

gravitational potential energy at point of release = linear kinetic energy at bottom of slope + rotational kinetic energy at bottom of slope

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

A solid cylinder of mass 1.5 kg and radius 0.12 m rolls down a long, shallow slope, starting from rest. At the instant the cylinder reaches the bottom of the slope, its tangential (linear) speed (**v**) is 1.6 ms^{-1}. For the solid cylinder, moment of inertia (**I**) = 1/2 mr^2.

(a) Calculate the numerical value for the cylinder's **moment of inertia**.

(b) Determine the **angular velocity** of the cylinder at the instant it reaches the bottom of the slope.

(c) Calculate the **gravitational potential energy** of the cylinder at its release height (**h**) on the slope.

(d) Determine the **release height** (**h**).

(a) 0.01 kg m^2 (b) 13.3 rad s^{-1} (c) 2.8 J (d) 0.2 m

For a solid sphere of mass '**m**' and radius '**r**': **I = 2/5 mr²**.

(a) Calculate the **moment of inertia** for a marble of mass 0.5 g and diameter 1.2 cm.

The marble is released from rest on a long, shallow ramp. At the instant it reaches the bottom of the ramp, its angular velocity is 360.0 rad s^{-1}.

(b) Determine the marble's **tangential** (**linear**) **speed** at the instant it reaches the bottom of the ramp.

(c) Calculate the **gravitational potential energy** of the marble at its starting point on the ramp.

(d) Determine the **height** on the ramp from which the marble was released.

(a) 7.2×10^{-9} kg m^2 (b) 2.2 ms^{-1} (c) 1.7×10^{-3} J (d) 0.3 m

A solid cylinder (**I = 1/2 mr^2**) has a mass of 20 kg and a radius of 0.2 m. It is released from rest at the top of a 2.5 m high slope.

(a) Calculate the cylinder's **moment of inertia**.

(b) Determine the cylinder's **tangential** (**linear**) **speed** at the instant it reaches the bottom of the slope.
[HINT: $\omega = v/r$, so substitute v/r for ω in a suitable equation].

(a) 0.4 kg m^2 (b) 5.7 m s^{-1}

A solid cylinder (mass 10 kg and radius 10 cm) is released from a height of 0.8 m on a wooden ramp. The cylinder rolls down the ramp, reaching the bottom with a tangential (linear) speed of 2.5 ms^{-1}.

Determine the cylinder's **moment of inertia**.

0.2 kg m^2

A solid cylinder has a mass of 3.0 kg and a radius of 5 cm. For such a cylinder, $\mathbf{I = 1/2\ mr^2}$. The cylinder rolls down a 30 cm long slope which makes an angle of 40º above the horizontal. Assuming the cylinder does not slip during this motion, calculate:

(a) The loss in gravitational potential energy as the cylinder rolls from the top to the bottom of the slope.

(b) The tangential (linear) speed of the cylinder at the instant it reaches the bottom of the slope.

(a) 5.7 J (b) 1.6 m s^{-1}

- **Explain that the moment of inertia of an object depends on the mass of the object and the distribution of the mass about a fixed axis.**

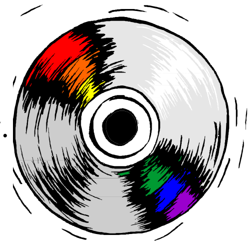
- **Carry out calculations involving moment of inertia.**

For a **point mass** undergoing rotational motion, $\mathbf{I = mr^2}$.

If a point mass has a moment of inertia of 4.5×10^{-5} kg m^2 and a radius of rotation of 0.15 m, calculate its **mass**.

For a **rotating disc**, $\mathbf{I = 1/2\ mr^2}$.

A CD of mass 1.0×10^{-4} kg has a moment of inertia of 2.0×10^{-7} kg m^2. Calculate the **diameter** of the CD.

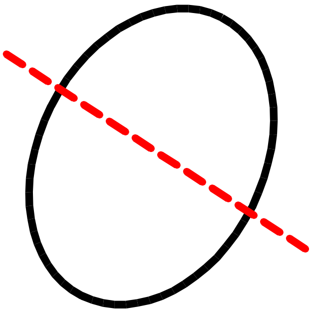


In each case, calculate the value for the **moment of inertia** of the object about the axis of rotation shown.

m = **r =** **l =**

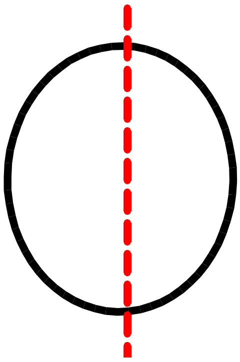
hoop: $I = mr^2$

m = 0.05 kg, r = 0.15 m



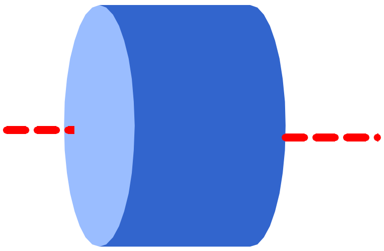
hoop: $I = 1/2\ mr^2$

m = 0.05 kg, r = 0.15 m



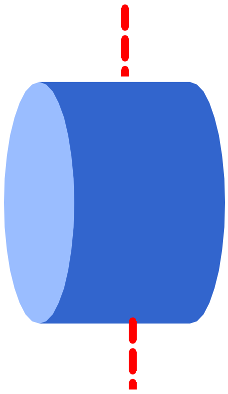
solid cylinder: $I = 1/2\ mr^2$

m = 1.5 kg, r = 0.20 m



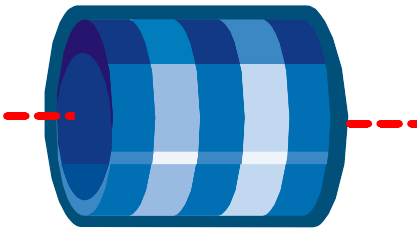
solid cylinder: $I = 1/4\ mr^2 + 1/12\ ml^2$

m = 1.5 kg, r = 0.20 m, l = 0.30 m



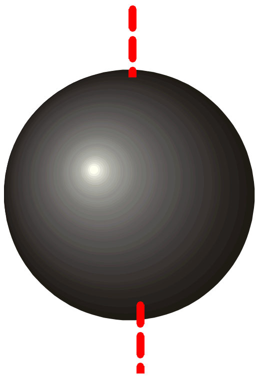
thick-walled cylinder: $I = 1/2\ m(r_1^2 + r_2^2)$

m = 0.3 kg, r_1 = 0.20 m, r_2 = 0.25 m



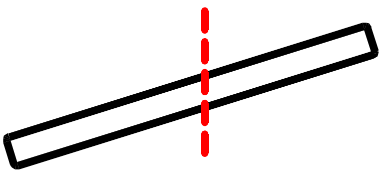
solid sphere: $I = 2/5\ mr^2$

m = 0.75 kg, r = 0.10 m



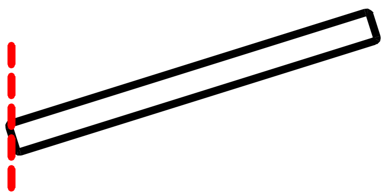
thin rod: $I = 1/12\ ml^2$

m = 0.01 kg, l = 0.15 m



thin rod: $I = 1/3\ ml^2$

m = 0.01 kg, l = 0.15 m

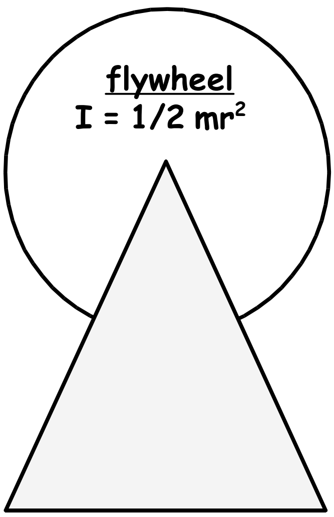


● **State what is meant by the moment of a force.**

● **State that an unbalanced torque produces an angular acceleration.**

● **State that the angular acceleration produced by an unbalanced torque depends on the moment of inertia of the object.**

● **Carry out calculations involving the relationships between torque, force, radius, moment of inertia and angular acceleration given the moment of inertia when required.**

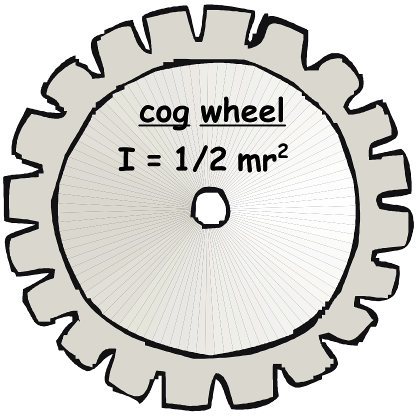


A flywheel of mass 30 kg an radius 0.20 m is mounted on friction-free bearings. A constant tangential force of 20 N is applied to the flywheel by an electric motor.

(a) Calculate the **moment of inertia** of the flywheel.

(b) Calculate the **torque** acting on the flywheel.

(c) Calculate the **angular acceleration** of the flywheel.



A cog wheel of mass 2.50 kg and radius 0.100 m is mounted on a friction-free shaft.
A uniform tangential force of 15.0 N is applied to the cog wheel.

(a) Calculate the cog wheel's **moment of inertia**.

(b) Calculate the **torque** acting on the cog wheel.

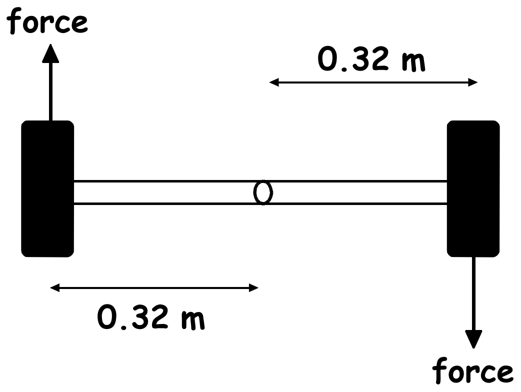
(c) Calculate the cog wheel's **angular acceleration**.

An artistic display in a shopping centre comprises a solid sphere of mass 12.0 kg and radius 0.50 m which rotates on a friction-free shaft positioned through its centre when a tangential force of 500 N is applied.

(a) Determine the solid sphere's **moment of inertia**.

(b) Determine the **torque** acting on the solid sphere.

(c) Determine the **angular acceleration** of the solid sphere.



A display firework consists of a 0.64 m long rod of negligible mass, pivoted about its centre. At both ends of the rod, a firework of mass 0.50 kg is attached. Both fireworks are lit at the same time and each provides a tangential force of 25 N perpendicular to the rod, causing the rod to rotate about its central pivot point.

(a) Calculate the **total moment of inertia** of the system.

(b) Calculate the **total torque** acting on the system.

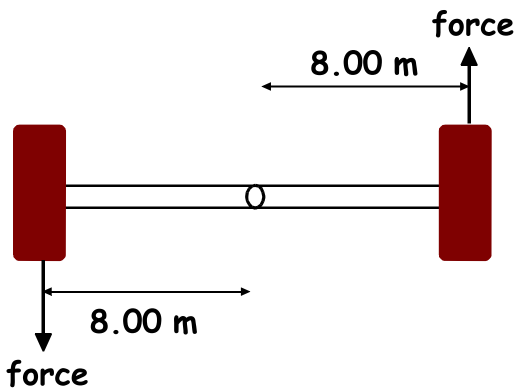
(c) Calculate the **angular acceleration** of the system.

Two donkeys (each of mass 250 kg) are positioned in a water pumping apparatus which has negligible mass. The apparatus is pivoted at its centre. Each donkey provides a tangential force of 5 000 N as they walk round in a circle, turning the apparatus.

(a) Calculate the **total moment of inertia** of the system.

(b) Calculate the **total torque** acting on the system.

(c) Calculate the **angular acceleration** of the system.



Newton's Universal Law of Gravitation

Every particle of matter in the universe attracts every other particle with **gravitational force** whose magnitude (**F**) is directly proportional to the product of the particle masses (**m_1** and **m_2**) and inversely proportional to the square of the straight line distance between them (**r^2**).

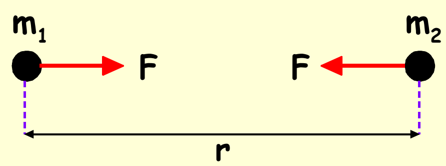
$$F \; \alpha \; \frac{m_1 m_2}{r^2}$$

Putting in a constant of proportionality (**G**) called the **gravitational constant**:

$$F = \frac{G m_1 m_2}{r^2}$$

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

This diagram shows the attractive forces acting on 2 particles. The attractive forces act along the straight line joining the particles. They form an "action-reaction pair", equal in magnitude but opposite in direction.

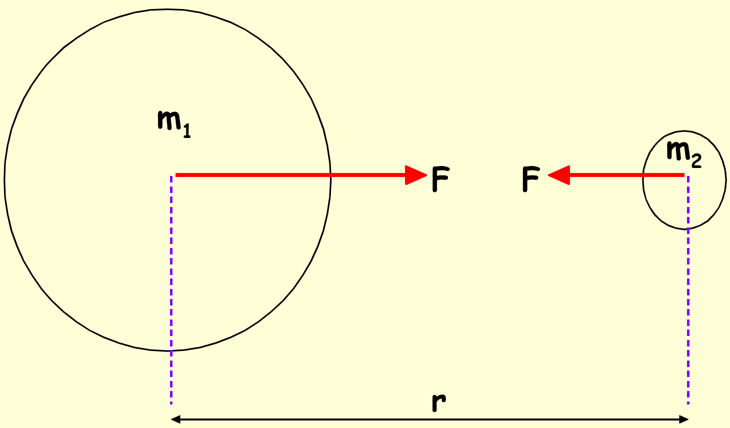


The **gravitational force** acting between everyday objects has such a small value that we do not notice its attractive effect - A pencil and eraser sitting on a desk do not accelerate towards each other because the tiny **gravitational force of attraction** acting between them is not large enough to overcome **frictional forces**.

The **gravitational force of attraction** only becomes significant for extremely large objects such as **planets** and their **moons**.

We can apply the above equation to objects such as planets and their moons if we assume that:

- the objects are perfectly **spherical**;
- the **mass** of each object is concentrated at the **centre** of the sphere;
- **r** is the distance between the **centre** of the spheres.



- **Carry out calculations involving Newton's universal law of gravitation.**

(a) Two point masses of value 1.50 kg an 2.00 kg are placed 1.25 m apart on a bench top. Calculate the **gravitational force of attraction** which exists between the two masses.

(b) What can you say about the **magnitude** (**size**) of the attractive force?

(c) Explain why the attractive force does not cause the two masses to accelerate towards each other.

Determine the magnitude of the **gravitational force of attraction** acting between two point masses of 0.25 kg and 0.75 kg which are separate by a distance of 1.5 m.

Two point masses (0.05 kg and 0.06 kg) experience a gravitational force of attraction of 2.55×10^{-14} N.

Calculate the **distance** between them.

Two point masses (0.50 kg and ? kg), separated by a distance of 15 m, experience a gravitational force of attraction of magnitude 1.19×10^{-13} N.

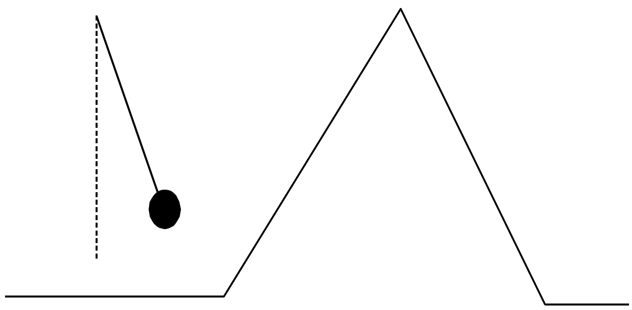
Calculate the **mass** of the unknown point mass.

Two point masses, of equal mass, are separated by a distance of 4.00 m. They experience a gravitational force of attraction of magnitude 2.61×10^{-13} N.

Calculate the **mass** of each point mass.

If we apply Newton's law of gravitation equation to objects which are not point masses, what **assumptions** must we make about the objects?

Calculate the **force** which:		
A molecule of mass 3.00×10^{-25} kg exerts on another molecule of the same mass when the molecules are 2.40×10^{-9} m apart.	The earth (mass 6.00×10^{24} kg) exerts on Craig (mass 65 kg) who is standing on its surface. (Radius of earth = 6.40×10^{6} m).	The sun (mass 2.00×10^{30} kg) exerts on the earth (mass 6.00×10^{24} kg). Assume the distance between centres to be 1.50×10^{11} m



A mountain of mass 4.00×10^{12} kg and a plumb bob of mass 2.00 kg interact. They are separate by a horizontal distance of 3 km between their centres.

(a) Determine the **force** the mountain exerts on the plumb bob.

(b) If the earth has a mass of 6.00×10^{24} kg and a diameter of 12.0×10^{6} m, determine the **force** the earth exerts on the plumb bob.

(c) Sketch a vector diagram showing these 2 forces acting on the plumb bob and use the diagram to determine the plumb bob's ***tiny*** **angle of deflection** from the vertical.

Gravitational Fields

A **gravitational field** exists around every object.

The **gravitational field** of an object is the region of space around the object in which the object will exert a **gravitational force** on any other object.

- **Sketch gravitational field lines for an isolated point mass and for two point masses.**

We can represent the **gravitational field** around an object by drawing **gravitational field lines**.

The **arrow** on each **gravitational field line** shows the direction of the **gravitational force** which would be exerted on an object positioned on the **field line**.

The closer together the **field lines** are, the greater the strength of the **gravitational field**.

- **isolated point mass**

- **isolated planet**

- **2 point masses**

- **planet-moon system**

Gravitational Field Strength

- **Define gravitational field strength.**

The **gravitational field strength** (**g**) at any point in a **gravitational field** is the **gravitational force** which would act on a **1 kg mass** placed at that point in the **gravitational field**.

$$g = \frac{F}{m}$$

A particle of mass 0.15 kg experiences a gravitational force of 0.75 N in a gravitational field. Calculate the **gravitational field strength** at that position in the gravitational field.	The gravitational field strength where a 0.40 kg particle is positioned has a value of 8.5 N kg^{-1} . Calculate the **gravitational force** which will act on the particle at that position.	The gravitational field strength where a particle experiences a gravitational force of 4.2 N is 8.4 N kg^{-1}. Calculate the **mass** of the particle.

An object (mass **M**) is attracted towards a more massive object (mass **m**) by a gravitational force (**F**). If **r** is the straight line distance between their centres:

$$F = \frac{G\,m\,M}{r^2}$$

Substituting into $g = \frac{F}{M}$

$$g = \frac{G\,m\,M/r^2}{M}$$

$$\therefore\ g = \frac{G\,m}{r^2}$$

Calculate the value for the **gravitational field strength** at the stated distance from the centre of each planet:

mercury	**venus**
● mass = 3.30×10^{23} kg ● distance from centre = 2.50×10^{6} m	● mass = 4.87×10^{24} kg ● distance from centre = 8.00×10^{6} m
earth ● mass = 5.98×10^{24} kg ● distance from centre = 7.50×10^{6} m	**mars** ● mass = 6.42×10^{23} kg ● distance from centre = 3.40×10^{6} m

Compare the values for the **gravitational field strength** on the surface of earth (mass 5.98×10^{24} kg and radius 6.38×10^{6} m) and the moon (mass 7.35×10^{22} kg and radius 1.74×10^{6} m):

The earth orbits the sun with a mean radius (centre to centre) of 1.50×10^{11} m. Calculate the **gravitational field strength** on earth due to the sun, given that the mass of the sun is 1.99×10^{30} kg.

The moon orbits the earth with a mean radius (centre to centre) of 3.85×10^{8} m. Calculate the **gravitational field strength** on the moon due to the earth, given that the mass of the earth is 5.98×10^{24} kg.

Assuming the earth to be a perfect sphere, determine its **density**.

- radius of earth = 6.37×10^6 m.
- volume of sphere = $4/3\ \pi\ r^3$.

Assuming the moon to be a perfect sphere, determine its **density**.

- radius of moon = 1 738 km.
- gravitational field strength on moon = 1.6 N kg^{-1}
- volume of sphere = $4/3\ \pi\ r^3$.

Gravitational Potential (V)

- **State that the gravitational potential at a point in a gravitational field is the work done by external forces in bringing unit mass from infinity to that point.**
- **State that the zero of gravitational potential energy is taken to be at infinity.**

The **gravitational potential** at a point in a gravitational field is the work done by external forces in bringing a 1 kg mass from infinity to that point.

The "**zero**" of **gravitational potential energy** is taken to be at **infinity**
- So values for **gravitational potential** are **negative**.

working definition	The **gravitational potential** at a point in a gravitational field is the **gravitational potential energy** of a 1 kg mass placed at that point.

$$V = -\frac{Gm}{r}$$

Calculate the **gravitational potential** (**V**) at the following distances (**r**) from the centre of the earth. (Mass of earth = 5.98×10^{24} kg).

• $r = 6.38 \times 10^6$ m (This is at the earth's surface)	• $r = 7.50 \times 10^6$ m	• $r = 9.00 \times 10^6$ m
• $r = 1.25 \times 10^7$ m	• $r = 2.50 \times 10^7$ m	• $r = 3.00 \times 10^7$ m

Gravitational Potential Energy

The **gravitational potential** (**V**) at a point in a gravitational field is the **gravitational potential energy** of a 1 kg mass placed at that point.

∴ To find the **gravitational potential energy** (**E_p**) of an object with mass **M** at a point in a gravitational field, we multiply the mass **M** of the object by the **gravitational potential** (**V**) at that point:

$$E_p = M \times V$$

$$\therefore E_p = M \times -\frac{G\,m}{r}$$

$$\therefore E_p = -\frac{G\,m\,M}{r}$$

- **Carry out calculations involving the gravitational potential energy of a mass in a gravitational field.**

Calculate the **gravitational potential energy** of the following objects in the earth's gravitational field. (Mass of earth = 5.98×10^{24} kg).		
• mass of object = 1.00 kg • distance from centre of earth = 6.38×10^6 m	• mass of object = 1.75 kg • distance from centre of earth = 7.25×10^6 m	• mass of object = 2.25 kg • distance from centre of earth = 7.75×10^6 m
• mass of object = 50.0 kg • distance from centre of earth = 8.00×10^6 m	• mass of object = 750 kg • distance from centre of earth =1.25×10^7 m	• mass of object = 225 kg • distance from centre of earth = 8.50×10^7 m

A spacecraft (mass 4.00×10^5 kg) starts at a distance of 3.00×10^6 m from the centre of the moon. The spacecraft fires its engine in order to move away from the moon to a new moon centre to spacecraft distance of 3.20×10^6 m. The moon has a mass of 7.35×10^{22} kg.

(a) Calculate the spacecraft's ***initial*** **gravitational potential energy**.

(b) Calculate the spacecraft's ***final*** **gravitational potential energy**.

(c) Calculate the spacecraft's ***change in*** **gravitational potential energy**.

(d) State the **work done** by the spacecraft's engine in moving the spacecraft.

A space capsule, mass 5.25×10^5 kg, is positioned 6 750 km from the centre of planet venus. The capsule uses its propulsion system to travel to a new position 7 250 km from the centre of venus. (Mass of venus = 4.87×10^{24} kg).

(a) Calculate the ***initial*** **gravitational potential energy** of the space capsule.

(b) Calculate the ***final*** **gravitational potential energy** of the space capsule.

(c) Calculate the ***change in*** **gravitational potential energy** of the space capsule.

(d) State the **work done** by the space capsule's engine in moving the space capsule.

(a) Calculate the **gravitational potential energy** of a rocket (mass 10 000 kg) which is sitting on the surface of the Earth. (Mass of earth = 5.98 x 10^{24} kg. Radius of earth = 6.38 x 10^{6} m).

(b) When the rocket's engine is fired, the rocket travels to a distance of 7.43 x 10^{6} m from the earth's centre. Calculate the **gravitational potential energy** of the rocket at this position in the earth's gravitational field.

(c) Calculate the ***change in*** **gravitational potential energy** of the rocket.

(d) Calculate the **work done** by the rocket engine in launching the rocket.

The planet mars has a mass of 6.42 x 10^{23} kg and a radius of 3 390 km. Determine the **work done** by the engine of a spacecraft which, on firing, causes the spacecraft to travel from the martian surface to a distance of 3 650 km from the planet centre.

- **State that a gravitational field is a conservative field.**
- **Explain what is meant by a conservative field.**

A **gravitational field** is a **conservative field** - The work done in moving a mass between two points in the field is independent of the path taken.

Escape Velocity

- **Explain the term 'escape velocity'.**
- **Derive the expression for the escape velocity.**

The **escape velocity** from a point in a **gravitational field** is the **minimum velocity** with which a small mass must be projected from the point in order to escape from the **gravitational field** and reach **infinity**.

- **State that the motion of photons is affected by gravitational fields.**
- **State that, within a certain distance from a sufficiently dense object, the escape velocity is greater than c, hence nothing can escape from such an object - a black hole.**

The motion of **photons** is affected by **gravitational fields**.

A **black hole** is formed when a star collapses on itself. The radius of the star decreases dramatically, so its density increases dramatically.
The **escape velocity** becomes greater than the **speed of light in a vacuum** (**c**), so no **photons** can escape.

Explain why we cannot see a black hole: ______________________________________
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____
____
____
____

Define the term **"escape velocity"**: ____

____

____

____

Determine the **escape velocity** for a spacecraft taking off from the moon's surface. ● mass of moon = 7.35×10^{22} kg ● radius of moon = 1.74×10^{6} m	Calculate the **escape velocity** for a space ship on the surface of mars. ● mass of mars = 6.42×10^{23} kg ● radius of mars = 3.40×10^{6} m
If a rocket is to escape from the surface of the earth to infinity, what **velocity** must it achieve at take off? ● mass of earth = 5.98×10^{24} kg ● radius of earth = 6.38×10^{6} m	If a space vehicle is to escape from the surface of venus, what minimum take off **velocity** must it achieve? ● mass of venus = 4.87×10^{24} kg ● radius of venus = 6.05×10^{6} m
An object is positioned 2 200 km from the centre of pluto. Calculate the **escape velocity** for the mass. ● mass of pluto = 1.27×10^{22} kg	A particle is 35 000 km distant from the centre of uranus. Determine the particle's **escape velocity**. ● mass of uranus = 8.69×10^{25} kg
A small mass is positioned 2 500 km ***above*** the surface of neptune. Determine the **escape velocity** for the mass. ● mass of neptune = 1.02×10^{26} kg ● radius of neptune = 2.47×10^{7} m	Calculate the **escape velocity** for a point mass positioned 5 000 km ***above*** the surface of saturn. ● mass of saturn = 5.69×10^{26} kg ● radius of saturn = 6.03×10^{7} m

Satellite Motion

- **Carry out calculations involving orbital speed, period of rotation and radius of orbit of satellites.**

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, mass of earth = 5.97×10^{24} kg, radius of earth = 6.38×10^6 m

Calculate the **tangential velocity** (**speed**) of a satellite which orbits earth at the stated distance above its surface. (Hint - Take into account the radius of the earth).	
2.50×10^7 m	**3.25×10^7 m**

Calculate the **period** (**T**) of a satellite which orbits earth at the stated distance above its surface. (Hint - Take into account the radius of the earth).	
2.00×10^7 m	**3.00×10^7 m**

Calculate the **height** of a satellite above the earth's surface if the satellite has the period stated. (Hint - Take into account the radius of the earth).	
6 500 s	**9 200 s**

Sputnik 1, the first artificial satellite to orbit earth, had a period of 96 minutes.
Calculate the **height** above the earth's surface at which Sputnik 1 orbited and its **tangential velocity** (**speed**).

Calculate the **height** of a geostationary satellite above the earth's surface and its **tangential velocity** (**speed**).

A spy satellite orbits earth at a distance of 2.50×10^5 m above the surface.
Determine the satellite's **tangential velocity** (**speed**) and **period**.

A weather satellite takes 100 s to travel over a 5^{o} sector of the earth's surface.
Calculate the satellite's **period**, **height** above the earth's surface and **tangential velocity** (**speed**).