Physics

Tutorial Solutions
Advanced Higher

6773

Spring 2000

HIGHER STILL

Physics

Tutorial Solutions
Advanced Higher

Support Materials

CONTENTS

DATA
Common Physical quantities

QUANTITY	SYMBOL	VALUE
Gravitational acceleration	g	9.8 m s^{-2}
Radius of Earth	R_E	$6.4 \times 10^{6} \text{ m}$
Mass of Earth	M_E	$6.0 \times 10^{24} \text{ kg}$
Mass of Moon	M_M	$7.3 \times 10^{22} \text{ kg}$
Mean radius of Moon orbit		$3.84 \times 10^{8} \text{ m}$
Universal constant of gravitation	G	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Speed of light in vacuum	c	$3.0 \times 10^{8} \text{ m s}^{-1}$
Speed of sound in air	v	$3.4 \times 10^{2} \text{ m s}^{-1}$
Mass of electron	m_e	$9.11 \times 10^{-31} \text{ kg}$
Charge on electron	e	$-1.60 \times 10^{-19} \text{ C}$
Mass of neutron	m_n	$1.675 \times 10^{-27} \text{ kg}$
Mass of proton	m_p	$1.673 \times 10^{-27} \text{ kg}$
Planck's constant	h	$6.63 \times 10^{-34} \text{ J s}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12} \text{ F m}^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ H m}^{-1}$

The solutions to the tutorial questions use the data values given above.

TUTORIAL 1

Solutions for Equations of Motion and Relativistic Dynamics

1 (a)

Horizontally

$u_{horiz} = 20 \cos 30^\circ = 17.3 \text{ m s}^{-1}$

Vertically

Let upwards be positive direction

$u_{vert} = 20 \sin 30^\circ = 10 \text{ m s}^{-1}$

$s_{vert} = +30 \text{ m}$

$a = -9.8 \text{ m s}^{-2}$

Consider the vertical motion to find the time

$$s = ut + \frac{1}{2} a t^2 \quad \text{directly.}$$

$$30 = 10\,t - \frac{1}{2}\, 9.8\, t^2$$

$$4.9\, t^2 - 10\, t - 30 = 0$$

$$t = \frac{+10 \pm \sqrt{100 + 4 \times 4.9 \times 30}}{2 \times 4.9} \quad \text{(using the quadratic formula)}$$

$$t = \underline{\underline{3.7}} \text{ s} \qquad \text{(the negative solution is not applicable)}$$

Alternative solution

Consider the vertical motion in two parts. Let upwards be positive direction.
Time from start to highest point using $v = u + at$ gives $t = 1.02$ s
Time from highest point to the ground:
height above 30 m using; $s = \frac{(u + v)}{2} t = 5.1$ m, total height = 35.1 m.

Hence time from highest point using $s = ut + \frac{1}{2} a t^2$ is $t = 2.68$ s

Thus total time to reach the ground $= 1.02 + 2.68 = \underline{\underline{3.7}}$ s (to 2 sig. figs.)

(b)

Horizontally

At impact,

$v_{horiz} = 17.3 \text{ m s}^{-1}$

Vertically

Let upwards be positive direction.

At impact

$v_{vert} = u_{vert.} + at$

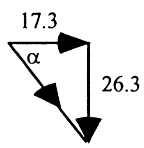
$= 10 - 9.8 \times 3.7$

$= -26.3 \text{ m s}^{-1}$ (velocity is downwards)

magnitude of velocity $= \sqrt{17.3^2 + 26.3^2}$

$= \underline{\underline{31}} \text{ m s}^{-1}$

$\tan \alpha = \frac{26.3}{17.3}$ thus $\underline{\underline{\alpha = 57^\circ}}$



Note: The values given in the question are to two significant figures. Hence the answers are given to two significant figures. However interim calculations must retain three or more significant figures

2 Use $m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \dfrac{9.11 \times 10^{-31}}{\sqrt{1 - \frac{(1 \times 10^8)^2}{(3 \times 10^8)^2}}} = \dfrac{9.11 \times 10^{-31}}{\sqrt{1 - \frac{1}{9}}}$

$= \underline{\underline{9.66 \times 10^{-31}}}$ kg

3 $m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ but $m = 1.5\ m_o$ giving $\dfrac{m_0}{m} = \dfrac{1}{1.5}$

$$\frac{m_0}{m} = \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{1.5}$$ **(use of $\frac{m_0}{m}$ can be simpler)**

$$1 - \frac{v^2}{c^2} = \frac{1}{(1.5)^2}$$

$$1 - \frac{1}{(1.5)^2} = \frac{v^2}{9 \times 10^{16}}$$

$$9 \times 10^{16}\left(1 - \frac{1}{(1.5)^2}\right) = v^2$$

$$v = \underline{\underline{2.24 \times 10^8}}\ \text{m s}^{-1}$$

4 (a) $$m = \frac{8000}{\sqrt{1 - \dfrac{(680)^2}{(3 \times 10^8)^2}}} = \frac{8000}{\sqrt{1 - 5.14 \times 10^{-12}}}$$ ($\frac{v^2}{c^2}$ is negligible)

$$= \underline{\underline{8000}}\ \text{kg}$$

(b) The velocity is too small to have any significant effect on the rest mass. The equation still applies, but 680 m s^{-1} does not cause any significant rise in mass. Most large scale objects on the Earth show no relativistic effects.

5 (a) $$\frac{m_0}{m} = \sqrt{1 - \frac{v^2}{c^2}}$$ and $\dfrac{m_0}{m} = \dfrac{1}{4}$ (since $\dfrac{m}{m_0} = 4$)

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{4}$$ and $1 - \dfrac{v^2}{c^2} = \dfrac{1}{4^2} = \dfrac{1}{16}$

$$\frac{v^2}{c^2} = 1 - \frac{1}{16} = \frac{15}{16}$$

$$v = \sqrt{\frac{15}{16}} \times 3 \times 10^8 = \underline{\underline{2.90 \times 10^8}}\ \text{m s}^{-1}$$ (which is 0.97c)

(b) The velocity cannot possibly double, to 5.80×10^8 m s^{-1}, because speeds greater than c are not possible.
By a similar calculation the new speed is $\underline{\underline{2.98 \times 10^8}}$ m s^{-1} (0.99c)

6 $$p = mv = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} \times v = \frac{9.11 \times 10^{31}}{0.9989} \times 1 \times 10^7 = \underline{\underline{9.1 \times 10^{-24}}}\ \text{kg m s}^{-1}$$ (start)

Similarly momentum at the end = $\underline{\underline{4.1 \times 10^{-22}}}$ kg m s^{-1}

TUTORIAL 2

Solutions for Angular Motion

1 2π radians $= 360°$

$$1 \text{ radian} = \frac{360}{2\pi} = \underline{\underline{57.3°}}$$

2 $\theta = 2\pi$ $\quad\omega = \dfrac{\theta}{t} = \dfrac{2\pi}{60}$

$t = 60$ s $\quad\omega = \underline{\underline{0.10}}$ rad s^{-1}

3 (a) angular displacement θ = "area" under ω,t graph

$$\text{total area} = (5 \times 3) + \tfrac{1}{2}(3 \times 5) = 15 + 7.5$$

$$\theta = \underline{\underline{22.5}} \text{ radians}$$

(b) Similar calculation to (a) $\quad\theta = \underline{\underline{60}}$ radians

(c)
$$\alpha = \frac{\omega - \omega_o}{t} = \frac{15 - 5}{6}$$

$$\alpha = \underline{\underline{1.67}} \text{ rad s}^{-2}$$

4 (a) 100 revs = 100 x 2π radians; 1 min = 60 s

$$100 \text{ r.p.m.} = \frac{100 \times 2\pi}{60} = \underline{\underline{10.5}} \text{ rad s}^{-1}$$

(b)
$$\alpha = \frac{\omega - \omega_o}{t} = \frac{10.5 - 0}{12} = \underline{\underline{0.88}} \text{ rad s}^{-1}$$

5 (a)
$$\alpha = \frac{\omega - \omega_o}{t} = \frac{300 - 100}{10} = \underline{\underline{20}} \text{ rad s}^{-2}$$

(b)
$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$= (100 \times 10) + \frac{1}{2} \times 20 \times (10)^2$$

$$= \underline{\underline{2000}} \text{ radians}$$

(c) distance = circumference x no. of revs

$$= 2\pi r \times \frac{\theta}{2\pi} = r\theta$$

$$\text{distance} = 0.12 \times 2000$$

$$= \underline{\underline{240}} \text{ m}$$

TUTORIAL 3

Solutions for Circular Motion

1 (a) At this distance the central force is provided by gravitational attraction.

$g = 7.0 \text{ m s}^{-2} \qquad \dfrac{m v^2}{r} = m g; \qquad$ m cancels

$$v^2 = g r = 7.0 \times 7.5 \times 10^6 = 7.246$$

Thus $v = \underline{\underline{7.2 \times 10^3}} \text{ m s}^{-1}$

(b) $v = \dfrac{2 \pi r}{T} = \dfrac{2\pi \times 7.5 \times 10^6}{T}$

$$T = \frac{2\pi \times 7.5 \times 10^6}{7.2 \times 10^3} = \underline{\underline{6.5 \times 10^3}} \text{ s} \quad (\text{or } T = 7.25 \times 10^3 \text{ may have been used})$$

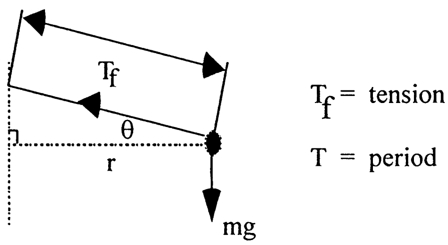
2 This means that the Earth would be rotating so quickly that **all** of the body's weight is required to supply the central force.

$mg = \dfrac{m v^2}{r}$ and $\dfrac{(2 \pi r/T)^2}{r} = \dfrac{4 \pi^2}{T^2}$ m cancels

$$T^2 = \frac{4 \pi^2 r}{g} = \frac{4 \pi^2 \times 6.4 \times 10^6}{9.8}$$

$$T = 5.08 \times 10^3 \text{ s} = \underline{\underline{85}} \text{ minutes}$$

3



(a) Here we are allowed to assume $\theta = 0°$ and $r = L$

T_f supplies central force $= \dfrac{mv^2}{r}$

$$T_f = \frac{m(2\pi r/T)^2}{r} = \frac{m\, 4 \pi^2 r}{T^2} = \frac{0.20 \times 4 \pi^2 \times 0.80}{0.25^2}$$

Thus tension $= \underline{\underline{101}}$ N

(b) In practice $\theta \neq 0$ because the weight will always pull the string down unless the orbital speed is infinite.

(c) Resolving forces: $T_f \cos\theta = \dfrac{m v^2}{r} = \dfrac{m\, 4 \pi^2 r}{T^2}$ and

$T_f \sin\theta = mg$ (and $r = L \cos\theta$)

Dividing: $\dfrac{T_f \sin\theta}{T_f \cos\theta} = \dfrac{mg \times T^2}{m\, 4 \pi^2 L \cos\theta}$

$\sin\theta = \dfrac{g \times T^2}{4 \pi^2 L}$ and $\sin\theta = \dfrac{9.8 \times 0.25^2}{4 \pi^2 \times 0.80}$

thus $\theta = \underline{\underline{1.1}}°$

4 The gravitational attraction supplies the central force.

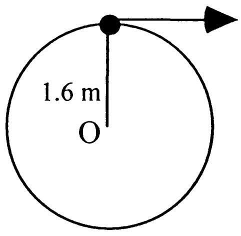
$$m\,g = \frac{m\,v^2}{r} \qquad \text{m cancels}$$

$$g = \frac{(2\,\pi\, r/T)^2}{r} = \frac{4\,\pi^2\, r}{T^2} = \frac{4 \times \pi^2 \times 4.0 \times 10^8}{(2.0 \times 10^6)^2} \qquad \text{(Using T = 2.0 x } 10^6 \text{ s)}$$

Thus the value of "g" at the moon's orbit = $\underline{\underline{3.9 \times 10^{-3}}}$ m s^{-2}

(You may have noticed that 27.3 days is 2.36 x 10^6 s. The 2.0 x 10^6 s is an approximation)

5



(a) $E_k = \frac{1}{2}\, m\, v^2 = \frac{1}{2} \times 2 \times 5^2$

$= \underline{\underline{25}}$ J

(b) $E_p = m\,g\,h = 2 \times 9.8 \times 3.2$

$= \underline{\underline{62.7}}$ J

(c) Change in E_k = change in gravitational E_p

E_k at bottom - E_k at top = mgh

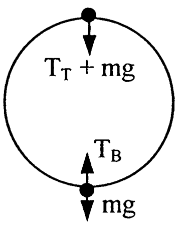
E_k at bottom - 25 = 62.7

E_k at bottom = $\underline{\underline{87.7}}$ J

(d) $87.7\text{ J} = \frac{1}{2}\, m\, v^2$

$$v = \sqrt{\frac{2 \times 87.7}{2}} = \sqrt{87.7} = \underline{\underline{9.36}} \text{ m s}^{-1}$$

(e)



At the top, central force $= T_T + mg = \frac{m\,v^2}{r}$

hence $T_T = \frac{m\,v^2}{r} - mg = \frac{2 \times 5^2}{1.6} - 2 \times 9.8 = \underline{\underline{11.7}}$ N

At the bottom, central force $= T_B - mg = \frac{m\,v^2}{r}$

thus $T_B = \frac{2 \times 9.36^2}{1.6} + 2 \times 9.8 = 109.5 + 19.6 = \underline{\underline{129}}$ N

(f) For this to occur; the force of gravity should **just** equal $\frac{m\,v^2}{r}$

$$mg = \frac{m\,v^2}{r} \quad \text{and} \quad v = \sqrt{g\,r} = \sqrt{9.8 \times 1.6} \quad v = \underline{\underline{3.96}} \text{ m s}^{-1}$$

(In this question all answers are given to three significant figures.)

6 Again $mg = \frac{m\,v^2}{r}$

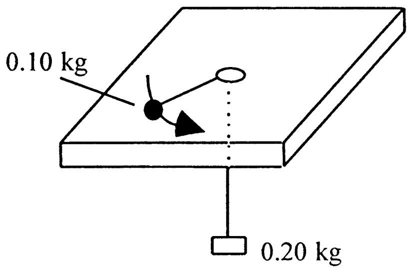
$$v = \sqrt{g\,r} = \sqrt{9.8 \times 20} \qquad v = \underline{\underline{14}} \text{ m s}^{-1}$$

7 (a) Once more, the gravitational force supplies the central force:

$$mg = \frac{m\,v^2}{r} \quad \text{and} \quad v = \sqrt{g\,r} = \sqrt{9.8 \times 1.2} = \underline{\underline{3.4}} \text{ m s}^{-1}$$

(b) $\omega = \frac{v}{r} = \frac{3.4}{1.2} = \underline{\underline{2.8}}$ rad s^{-1}

8



The tension in the string supplies the central force. The hanging mass supplies the tension.

$$\text{tension} = m\,g = 0.20 \times 9.8 = 1.96\text{ N}$$

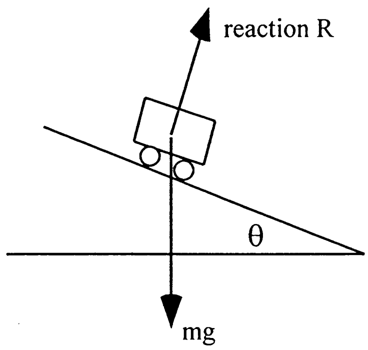
$$1.96 = \frac{m\,v^2}{r} = m\omega^2 r$$

$$\text{thus } \omega^2 = \frac{1.96}{0.10 \times 0.15}$$

$$\omega = 11.43\text{ rad s}^{-1}$$

$$\text{thus r.p.m.} = \frac{11.43}{2\pi} \times 60 = \underline{\underline{109}}\text{ r.p.m.}$$

9 (a)



(b) In vertical direction there is no acceleration $R\cos\theta = mg$

In radial direction there is a central acceleration $R\sin\theta = \dfrac{m\,v^2}{r}$

Dividing the two equations gives $\tan\theta = \dfrac{v^2}{g\,r}$

$$\tan\theta = \frac{20^2}{9.8 \times 60}$$

$$\underline{\underline{\theta = 34^\circ}}$$

10 (a) Horizontally: central force $\dfrac{m\,v^2}{r} = T_f \sin 30^\circ$

Vertically: $mg = T_f \cos 30^\circ$

[*Notice that the upper angle is 30°*]

also $v = \dfrac{2\,\pi\,r}{T}$ where T is period of pendulum

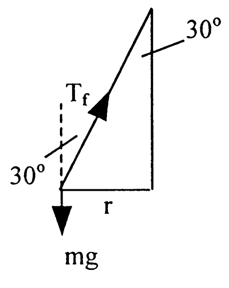
note that $r = 1.2 \times \sin 30^\circ = 0.6$

dividing the above equations

$$\frac{v^2}{g\,r} = \frac{\sin 30}{\cos 30} \quad \text{or} \quad \frac{4\,\pi^2}{gT^2}\,r = \tan 30$$

$$T^2 = \frac{4\pi^2\,r}{g\tan 30} = \frac{4\pi^2 \times 0.6}{9.8 \times \tan 30}$$

giving $T = \underline{\underline{2.05}}\text{ s}$



(b) $v = \dfrac{2\,\pi\,r}{T} = \dfrac{2\pi \times 0.6}{2.05} = \underline{\underline{1.84}}\text{ m s}^{-1}$

TUTORIAL 4

Solutions for Torque, Moments of Inertia and Angular Momentum

1 (a) $T = I\alpha$ $(T = 0.8 \text{ N m and } I = 1.2 \text{ kg m}^2)$

$$\alpha = \frac{T}{I} = \frac{0.8}{1.2} = 0.667$$

$$\alpha = \underline{\underline{0.67}} \text{ rad s}^{-2}$$

(b) $\omega = \omega_o + \alpha t$ $\omega_o = 0 \text{ rad s}^{-1}$ and $t = 5$ s

$$= 0 + 0.667 \times 5$$

$$\omega = \underline{\underline{3.3}} \text{ rad s}^{-1}$$

2 (a) Change in E_p = change in $E_{k(rot)}$ flywheel + change in $E_{k(linear)}$ weight

$$mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} m v^2$$

$$0.1 \times 9.8 \times 2 = \frac{1}{2} I \times \frac{v^2}{r^2} + \frac{1}{2} \times 0.1 \times v^2$$

$$2 \times 0.1 \times 9.8 \times 2 = I \times \frac{v^2}{0.1^2} + 0.1 \times v^2$$

$$3.92 = v^2 \times 100\,I + 0.1 \times v^2 = v^2 (0.1 + 100\,I)$$

$$v^2 = \frac{3.92}{0.1 + 100\,I}$$

thus $$v = \sqrt{\frac{3.92}{0.1 + 100\,I}}$$

(b) Now assume that the 0.10 kg mass falls with a uniform acceleration and that $u = 0 \text{ m s}^{-1}$

In the vertical plane: $s = \dfrac{v+u}{2}\, t$

$$2 = \frac{v+0}{2} \times 8$$

velocity of mass at the bottom $v = 0.5 \text{ m s}^{-1}$

using the expression determined above $v^2 = \dfrac{3.92}{0.1 + 100\,I}$

gives $(0.5)^2 = \dfrac{3.92}{0.1+100\,I}$ and $100\,I + 0.1 = \dfrac{3.92}{0.5^2}$

and $$I = \frac{1}{100}\left(\frac{3.92}{0.5^2} - 0.1\right)$$

$$I = \underline{\underline{0.156}} \text{ kg m}^2$$

Alternatively the terms, mgh, and $\frac{1}{2}$ m v^2 can be calculated separately and the energy relationship used to determine I from the $\frac{1}{2} I\omega^2$ term.

3 (a) Frictional torque $T = r \times F = -5 \times 0.50 = -2.5$ N m

$$T = I\alpha$$

$$\alpha = \frac{-2.5}{2.0} = -1.25 \text{ rad s}^{-2}$$

using $\omega = \omega_o + \alpha t$ $\quad(\omega_o = 10 \text{ rev s}^{-1} = 20\pi \text{ rad s}^{-1})$

$$0 = 20\pi - (1.25 \times t)$$

$$t = \frac{20\pi}{1.25} = \underline{\underline{50.3}} \text{ s}$$

(b) Use $\omega^2 = \omega_o^2 + 2\alpha\theta$

$$0 = (20\pi)^2 - 2 \times 1.25 \times \theta$$

$$\theta = \frac{(20\pi)^2}{2.5} = \underline{\underline{1.58 \times 10^3}} \text{ rad } (= 251 \text{ rev})$$

(c) Work $= T\theta = 2.5 \times 1580$

$= \underline{\underline{3.95}}$ kJ (= heat generated)

[*Note: calculations involving work done are **not** specified in the syllabus.*]

4 (a) Assume all the mass is at the rim, i.e. the spokes have negligible mass.

Moment of inertia $= MR^2$ all parts of the rim are at the same radius R

$$I = 2.0 \times (0.50)^2$$

$$I = \underline{\underline{0.50}} \text{ kg m}^2$$

(b) (i) Driving torque $T = r \times F$

$= 0.50 \times 20$

$= \underline{\underline{10}}$ N m

(ii) $T = I\alpha$

$$\alpha = \frac{T}{I} = \frac{10}{0.5} = \underline{\underline{20}} \text{ rad s}^{-2}$$

(c) (i) $\theta = \omega_o t + \frac{1}{2}\alpha t^2$

$= 0 + \frac{1}{2} \times 20 \times (5.0)^2$

$= \underline{\underline{250}}$ rad

(ii) Angular momentum, $L = I\omega$

$= 0.50 \times (\alpha t)$

$= 0.50 \times 100$

$L = \underline{\underline{50}} \text{ kg m}^2 \text{ s}^{-1}$

(iii) $E_k = \frac{1}{2} I\omega^2$

$= \frac{1}{2} \times 0.50 \times 100^2$

$= \underline{\underline{2.5}}$ kJ

5 (a)

$$L = m v r = m \omega r^2$$
$$= 0.20 \times 2\pi \times 0.40^2$$

Thus angular momentum of the mass $= \underline{\underline{0.20}}$ kg m^2 s^{-1}

(b)

$$v = \omega r = 2\pi \times 0.40 = 2.51 \text{ m s}^{-1}$$
$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} \times 0.20 \times 2.51^2 \quad \text{(or use } E_k = \frac{1}{2} I \omega^2\text{)}$$
$$= \underline{\underline{0.63}} \text{ J}$$

(c) Total angular momentum before = Total angular momentum after

$$m_1 \omega_1 r_1^2 = m_1 \omega_2 r_2^2 \qquad m_1 \text{ cancels}$$
$$2\pi \times 0.40^2 = \omega_2 \times 0.20^2$$
$$\omega_2 = 2\pi \times \frac{(0.40)^2}{(0.20)^2}$$
$$\omega_2 = \underline{\underline{8\pi}} \text{ rad s}^{-1} \; (= 25 \text{ rad s}^{-1})$$

(d)

$$\text{New } E_k = \frac{1}{2} m v^2 = \frac{1}{2} \times 0.2 \times 0.2^2 \times 25^2 \quad \text{(or use } E_k = \frac{1}{2} I \omega^2\text{)}$$
$$= \underline{\underline{2.5}} \text{ J}$$

The increase in energy was supplied when moving the object inwards.
The 'push' would have to be radial, so that no external torque is given.

6 (a)

$$\text{Total moment of inertia} = \frac{1}{12} M_{rod} L^2 + 2 \times [M_{mass} R^2]$$
$$= \frac{1}{12} \times 1.2 \times 1.0^2 + 2 \times [0.5 \times (0.5)^2]$$
$$= 0.10 + 0.25$$
$$I_{tot} = \underline{\underline{0.35}} \text{ kg m}^2$$

(b) (i)

$$T = r \times F$$
$$= 0.50 \times 10$$
$$= \underline{\underline{5.0}} \text{ N m}$$

(ii)

$$T = I\alpha$$
$$5.0 = 0.35\,\alpha$$
$$\alpha = \frac{5.0}{0.35} = \underline{\underline{14.3}} \text{ rad s}^{-2}$$

(iii)

$$\omega = \omega_o + \alpha t = 0 + 14.3 \times 4.0 = 57.2 \text{ rad s}^{-1}$$
$$E_k = \frac{1}{2} I \omega^2 = \frac{1}{2} \times 0.35 \times 57.2^2$$
$$= \underline{\underline{573}} \text{ J}$$

7

Driving Torque = Frictional Torque (since constant angular velocity)

$$T = I\alpha \quad \text{and} \quad 7.7 = 1.5 \times \alpha$$
$$\alpha = \frac{7.69}{1.5} = 5.13 \text{ rad s}^{-2}$$
$$\omega = \omega_o - \alpha t \qquad \text{(deceleration caused by friction)}$$
$$0 = 52 - (5.13 \times t)$$
$$t = \underline{\underline{10.1}} \text{ s}$$

[**Note**: the corrected version of this question has the driving torque of 7.7 N m given in the question and not the power of the motor.]

8 (a) The solid cylinder will reach the bottom of the slope first.

(b) Change in gravitational E_p = change in rotational E_k + change in linear E_k

$$mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} m v^2 = \frac{1}{2} I\left(\frac{v^2}{r^2}\right) + \frac{1}{2} m v^2$$

$$mgh = \frac{1}{2} v^2 \left(\frac{I}{r^2} + m\right)$$

$$v^2 = \frac{2mgh}{\left(\frac{I}{r^2} + m\right)} = \frac{2mgh}{\left(\frac{I + mr^2}{r^2}\right)} = \frac{2mghr^2}{I + mr^2}$$

The two cylinders have the same values for m, g,h and r. From the equation, if the moment of inertia increases then v^2 decreases and the speed v decreases. The solid cylinder has most of its mass at a radius less than the radius of the cylinder. The hollow cylinder has the *greater* moment of inertia because all the mass is at the radius of the cylinder, hence $v_{hollow} < v_{solid}$. Thus the solid cylinder reaches the bottom first because its linear speed will be greater.

9 (a) (i) solid cylinder: $I = \frac{1}{2} MR^2 = \frac{1}{2} \times 10 \times (0.10)^2$

$= \underline{\underline{0.05}}$ kg m^2

(ii) hollow cylinder: $I = \frac{1}{2} M(R^2 + r^2) = \frac{1}{2} \times 10 \times (0.10^2 + 0.05^2)$

$= \underline{\underline{0.0625}}$ kg m^2

(b) $Mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2$ but $\omega^2 = \frac{v^2}{R^2}$

$$Mgh = \frac{1}{2} I \frac{v^2}{R^2} + \frac{1}{2} Mv^2$$

$$v^2 = \frac{2MghR^2}{(I + MR^2)}$$ divide top and bottom by MR^2

$$v^2 = \frac{2gh}{\left[1 + \frac{I}{MR^2}\right]} \quad \text{and} \quad v = \sqrt{\frac{2gh}{\left[1 + \frac{I}{MR^2}\right]}}$$

(c) solid cylinder: $v = \sqrt{\frac{2 \times 9.8 \times 0.04}{\left[1 + \frac{0.05}{10 \times 0.10^2}\right]}} = \underline{\underline{0.723}}$ m s^{-1}

hollow cylinder: $v = \sqrt{\frac{2 \times 9.8 \times 0.04}{\left[1 + \frac{0.0625}{10 \times 0.10^2}\right]}} = \underline{\underline{0.695}}$ m s^{-1}

Time for a cylinder to roll down slope is given by $s = \frac{v + u}{2} t$ where $u = 0$

Time for solid cylinder to roll down the slope = 5.53 s.
Time for hollow cylinder to roll down the slope = 5.76 s.
Solid cylinder arrives at the bottom $\underline{\underline{0.23}}$ s ahead of the hollow cylinder.

TUTORIAL 5
Solutions for Gravitation

1 Use $F = \dfrac{G m_1 m_2}{r^2} = \dfrac{6.67 \times 10^{-11} \times (50000 \times 10^3)^2}{20^2}$

$$F = \underline{\underline{417}}\ \text{N}$$

2 $F = \dfrac{G m_p m_e}{r^2} = \dfrac{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 9.11 \times 10^{-31}}{(5.3 \times 10^{-11})^2}$

$$= \underline{\underline{3.6 \times 10^{-47}}}\ \text{N}$$

3 (a) The central force is supplied by the force of gravity

$$\frac{m v^2}{r} = \frac{G M m}{r^2}$$

$$v^2 = \frac{G M}{r} \qquad \text{but } v = \frac{2 \pi r}{T}$$

$$T^2 = \frac{4 \pi^2 r^3}{G M} \qquad \text{(Notice } r = R_E + 160 \text{ km)}$$

$$T = \sqrt{\frac{4 \pi^2 r^3}{G M}} = \sqrt{\frac{4\pi^2 \times (6.4 \times 10^6 + 160 \times 10^3)^3}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}}$$

$$= 5277 \text{ seconds}$$

$$= \underline{\underline{88}} \text{ minutes}$$

(b) $E_{tot} = E_p + E_k = -\dfrac{G M m}{2 r} = -\dfrac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 1500}{2 \times (6.4 \times 10^6 + 160 \times 10^3)}$

$= \underline{\underline{-4.58 \times 10^{10}}}$ J **[Note:** the corrected version gives the mass]

(c) E_{tot} at 36000 km $= -\dfrac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 1500}{2 \times (6.4 \times 10^6 + 36000 \times 10^3)}$

$$= -0.71 \times 10^{10} \text{ J}$$

Minimum energy required is the difference between these two.

Energy required $= -0.71 \times 10^{10} - (-4.58 \times 10^{10})$

$$= \underline{\underline{3.87 \times 10^{10}}} \text{ J}$$

4 (a) Density, $\rho = \dfrac{M}{V}$; volume of sphere $= \dfrac{4}{3} \pi r^3$

$$\rho_{mars} = \frac{M_m}{V_m} = \frac{0.11 M_e}{\frac{4}{3} \pi r_e^3} \qquad \rho_{earth} = \frac{M_e}{V_e} = \frac{M_e}{\frac{4}{3} \pi r_e^3}$$

$$\frac{\rho_{mars}}{\rho_{earth}} = \frac{M_m \times r_e^3}{r_m^3 \times M_e} = \frac{0.11 \times M_e \times r_e^3}{r_m^3 \times M_e} = 0.11 \times \left[\frac{r_e}{r_m}\right]^3$$

$$= 0.11 \times (6.4 \times 10^6 / 3.4 \times 10^6)^3$$

$$\rho_{mars} = \underline{\underline{0.73}}\ \rho_{earth}$$

(b) At the surface of any planet: $mg = \frac{G M m}{r^2}$ m cancels

$$g_{mars} = \frac{G M_{mars}}{r^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 0.11}{(3.4 \times 10^6)^2}$$

$$= \underline{\underline{3.8}} \text{ m s}^{-2}$$

(c) $$v_{esc} = \sqrt{\frac{2 G M}{r}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 0.11}{3.4 \times 10^6}}$$

$= 5.1 \times 10^3 \text{ m s}^{-1}$

escape velocity on Mars is $\underline{\underline{5.1}}$ km s^{-1}

5 $$E_p = -\frac{G M m}{r}$$ M = mass of Saturn; m = mass of rings

$$= -\frac{6.67 \times 10^{-11} \times 5.72 \times 10^{26} \times 3.5 \times 10^{18}}{1.1 \times 10^8}$$

$$= -\underline{\underline{1.21 \times 10^{27}}} \text{ J}$$

6 $$E_p \text{ at highest point} = -\frac{G M m}{r} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times m}{(6.4 \times 10^6 + 125 \times 10^6)}$$

$$= -(3.05 \times 10^6 \times m) \text{ J}$$

$$E_p \text{ at height of atmosphere} = -\frac{G M m}{r} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times m}{(6.4 \times 10^6 + 130 \times 10^3)}$$

$$= -(61.29 \times 10^6 \times m) \text{ J}$$

Energy difference -> E_k thus $\Delta E_p = \frac{1}{2} m v^2$

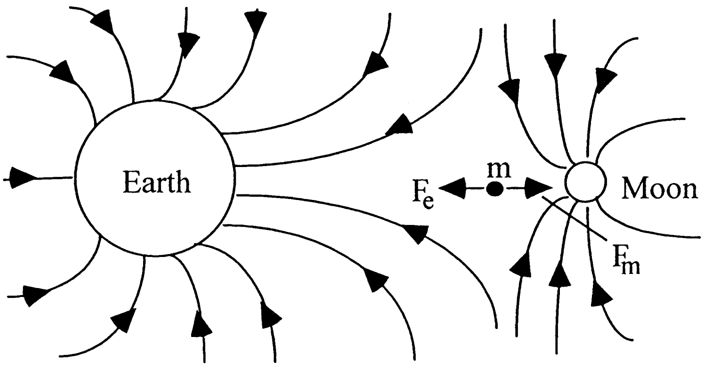
$$-(3.05 \times 10^6 \times m) - (-61.29 \times 10^6 \times m) = \frac{1}{2} m v^2$$

$58.24 \times 10^6 \times m = \frac{1}{2} m v^2$ m cancels

$v^2 = 2 \times 58.24 \times 10^6$

$v = \underline{\underline{10.8}}$ km s^{-1}

7 (a)



(b) Let r_1 = distance of m from centre of the Earth and
let r_2 = distance of m from centre of the Moon.

Thus $r_1 + r_2$ = separation of the Earth and Moon = 3.84×10^8 m

If m is at the 'null' point: $F_e = F_m$

$$\frac{G M_e m}{r_1^2} = \frac{G M_m m}{r_2^2} \qquad \text{(G and m cancel)}$$

$$\frac{r_1^2}{r_2^2} = \frac{M_e}{M_m} \quad \text{(take square roots of both sides)}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{M_e}{M_m}} = \sqrt{\frac{6 \times 10^{24}}{7.3 \times 10^{22}}} = 9.07$$

$$r_1 = 9.07 \times r_2 \qquad \text{and} \quad r_2 = 3.84 \times 10^8 - r_1$$

$$r_1 = 9.07 \times (3.84 \times 10^8 - r_1) = 9.07 \times 3.84 \times 10^8 - 9.07\, r_1$$

$$r_1 = \frac{9.07 \times 3.84 \times 10^8}{(9.07 + 1)}$$

$$= \underline{\underline{3.5 \times 10^8}} \text{ m}$$

The null point in this field, (ignoring the effect of the Sun), is approximately $\frac{9}{10}$ of the distance from the centre of the Earth to the moon.

8 Use Kepler's Third Law: $\frac{r^3}{T^2}$ = constant for a gravitational system

Thus, for Mars: $\frac{r_P{}^3}{T_P{}^2} = \frac{r_D{}^3}{T_D{}^2}$ P = Phobos, D = Deimos

$$T_D = \sqrt{\frac{(2.4 \times 10^7)^3 \times (2.8 \times 10^4)^2}{(9.4 \times 10^6)^3}}$$

$$= \underline{\underline{1.14 \times 10^5}} \text{ s} \quad (= 31.7 \text{ hours})$$

9 Change in $E_k = \Delta E_k = \frac{1}{2}\ m \times (5374)^2 - \frac{1}{2}\ m \times (3560)^2$

$$\Delta E_k \text{ per unit mass, } \frac{\Delta E_k}{m} = (14.44 - 6.34) \times 10^6 \text{ J kg}^{-1}$$

$$= 8.10 \times 10^6 \text{ J kg}^{-1}$$

This must be equivalent to the change in gravitational potential.

$$\Delta V = \underline{\underline{8.10 \times 10^6}} \text{ J kg}^{-1}$$

10 For escape velocity: $\frac{1}{2}\ m\, v^2 = \frac{G\,M\,m}{R}$

also $mg = \frac{G\,M\,m}{R^2}$ m cancels in both equations

$$\frac{1}{2}\ v^2 = \frac{G\,M}{r} \text{ ---------- } 1 \qquad \& \qquad g\,R = \frac{G\,M}{r} \text{ ------}2$$

combine 1 and 2: $\frac{1}{2}\ v^2 = g\,R$ giving $v^2 = 2\,g\,R$

$$v = \sqrt{2\,g\,R} \quad \text{as required.}$$

11 $$v_{esc} = \sqrt{\frac{2\,G\,M_E}{r_E}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6}}$$

$$v_{esc} = \underline{\underline{1.1 \times 10^4}} \text{ m s}^{-1}$$

or $v_{esc} = \sqrt{2\,g\,r_E} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = 11 \text{ km s}^{-1}$

12 $$T = 2\,\pi\sqrt{\frac{r^3}{G\,M}}$$ (see question 3 (a) for obtaining equation)

$$= 2\,\pi\sqrt{\frac{6.4 \times 10^6 + 400 \times 10^3}{6.67 \times 10^{11} \times 6 \times 10^{24}}}$$

$$= = 5569 \text{ s} = \underline{\underline{93}} \text{ minutes}$$

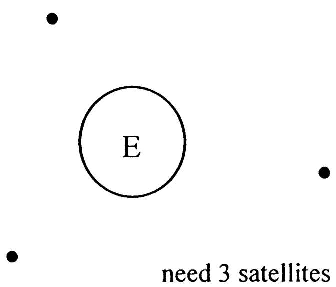
13 $$r^3 = \frac{T^2}{4\,\pi^2}\,.\,G\,M$$ (see question 3 (a) for obtaining equation)

$$= \frac{(24 \times 60 \times 60)^2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{4\,\pi^2}$$

$$r = \underline{\underline{42 \times 10^6}} \text{ m}$$

Thus height $= 42.3 \times 10^6 - 6.4 \times 10^6 = 35.9 \times 10^6$ height $= \underline{\underline{36 \times 10^6}}$ m

(c)



need 3 satellites at least

TUTORIAL 6

Solutions for Simple Harmonic Motion

1 (a) $y = 4\cos 4\pi t$ compare with $y = a\cos\omega t$

thus $a = y_{max} = \underline{\underline{4}}$ cm

(b) $\omega = 4\pi$ and $\omega = 2\pi f$

$4\pi = 2\pi f$ giving $f = \underline{\underline{2}}$ Hz

Also $f = \frac{1}{T}$ thus $T = \underline{\underline{0.5}}$ s

(c) (i) when $t = 0$ s $y = 4\cos 0 = \underline{\underline{4}}$ cm

(ii) when $t = 1.5$ s $y = 4\cos(4\pi \times 1.5)$ (Remember angle in radians)

$= \underline{\underline{4}}$ cm

2 (a) $f = 40$ Hz thus $T = \frac{1}{f} = \frac{1}{40} = \underline{\underline{0.025}}$ s

(b) $y = a\cos\omega t = a\cos 2\pi f t = \underline{\underline{0.05\cos 80\pi t}}$

(c) (i) acceleration $= -\omega^2 y$

at mid-point $y = 0$ thus acceleration $= \underline{\underline{0}}$ m s^{-2}

at max amplitude $y = 0.05$ m

acceleration $= -(2\pi \times 40)^2 \times 0.05$

$= -\underline{\underline{3.2 \times 10^3}}$ m s^{-2} directed towards the midpoint

(ii) $v_{max} = \pm\omega a = \pm 2\pi \times 40 \times 0.5$

$= \underline{\underline{\pm 12.6}}$ m s^{-1} this occurs at the midpoint when $y = 0$.

3 (a) $y = a\cos\omega t$ (at $t = 0$, $y = a$)

$= 0.12\cos\frac{2\pi}{1.5} \times 0.4$ (remember ωt in radians)

$= \underline{\underline{-0.0125}}$ m (or $y = -0.013$ m to 2 sig figs)

The position of object is 0.0125 m on the **opposite** side of the equilibrium position to $y = a$ at $t = 0$.

(b) Use $F = m \times$ acceleration and acceleration $= -\omega^2 y$

$= m \times -\omega^2 y$

$= -0.5 \times (\frac{2\pi}{1.5})^2 \times 0.0125$

$= \underline{\underline{0.11}}$ N

The force is acting in the positive direction, towards the equilibrium position.

(c) $y = -0.06$ m and $-0.06 = 0.12 \times \cos(\frac{2\pi}{1.5} \times t)$

$\cos(\frac{2\pi}{1.5} \times t) = -\frac{0.06}{0.12}$ and $\frac{2\pi}{1.5} \times t = \cos^{-1}(-\frac{0.06}{0.12})$

$t = \frac{1.5}{2\pi}\cos^{-1}(-\frac{0.06}{0.12})$

$t = \underline{\underline{0.5}}$ s

4 (a) $y = a \sin \omega t$ and $y_{max} = a$

$y = 2.0 \sin(3.22 \times 10^3 t)$ thus $y_{max} = \underline{\underline{2.0}}$ mm

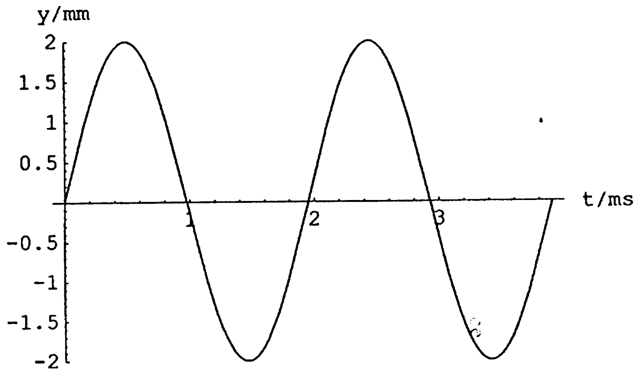
$\omega = 2\pi f$ and $\omega = 3.22 \times 10^3$

$$f = \frac{3.22 \times 10^3}{2\pi} = \underline{\underline{512}} \text{ Hz}$$

(b) $accn_{max} = -\omega^2 y_{max} = -(2\pi \times 512)^2 \times 2.0 \times 10^{-3}$

$= \underline{\underline{2.07 \times 10^4}}$ m s^{-2}

(c) $T = \frac{1}{512} = 1.95 \times 10^{-3}$ s and $y = 0$ when $t = 0$



(d) The period of any SHM is constant even although the amplitude is decreasing.

5 Acceleration will have to be greater than 10 m s^{-2} for this condition to occur.

Use $accn_{max} = -\omega^2 y_{max}$ and $\omega = 2\pi f$

$= (2 \times \pi \times 40)^2 \; y_{max}$

$$y_{max} = \frac{10}{(2 \times \pi \times 40)^2} = \underline{\underline{1.58 \times 10^{-4}}} \text{ m}$$

6 (a)

$$k = \frac{\text{force}}{\text{extension}} = \frac{1.2 \times 9.8}{0.10} = \underline{\underline{118}} \text{ N m}^{-1}$$

(b) (i) amplitude $= \underline{\underline{0.08}}$ m

(ii) $\omega^2 = \frac{k}{m} = \frac{118}{1.2}$ and $T = \frac{2\pi}{\omega}$

$T = 2\pi \sqrt{\frac{1.2}{118}} = \underline{\underline{0.63}}$ s and $f = \frac{1}{T} = \underline{\underline{1.6}}$ Hz

(iii) $v_{max} = \pm \omega a = \pm \sqrt{\frac{118}{1.2}} \times 0.08 = \underline{\underline{\pm 0.79}}$ m s^{-1}

Total energy $= \frac{1}{2} m \omega^2 a^2 = \frac{1}{2} \times 1.2 \times \frac{118}{1.2} \times 0.08^2$

$= \underline{\underline{0.38}}$ J

7 First use the conservation of linear momentum to find the velocity just after the dart embeds.

In the absence of external forces;

$$\text{total momentum before} = \text{total momentum after}$$

$$mv_1 = (m + M) \times v_2$$

$$0.060 \times 120 = (5.0 + 0.06) \times v_2$$

$$v_2 = \frac{0.060 \times 120}{5.06} = 1.42 \text{ m s}^{-1}$$

This is the maximum velocity, v_{max}

(a) $v_{max} = \omega a$ and $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{450}{5.06}}$

$$a = \frac{v_{max}}{\omega} = 1.42\sqrt{\frac{5.06}{450}} = \underline{\underline{0.15}} \text{ m}$$

An alternative solution can be found from an analysis of the energy.

$$E_{tot} \text{ (system)} = \frac{1}{2} m v_{max}^2 = \frac{1}{2} \times 5.06 \times (1.42)^2$$

$$= 5.10 \text{ J}$$

but $E_p = \frac{1}{2} k a^2$ when $v = 0$ i.e. $E_k \to E_p$

thus $a^2 = \dfrac{2 \times E_{tot}}{k}$ and $a = \sqrt{\dfrac{2 \times 5.10}{450}} = \underline{\underline{0.15}}$ m

(b) E_k (total of system) = 5.1 J (see above $E_{tot} = \frac{1}{2} m v_{max}^2$)

$$E_k \text{ (of dart)} = \frac{1}{2} m v^2 = \frac{1}{2} \times 0.06 \times (120)^2$$

$$= 432 \text{ J}$$

percentage of energy in oscillating system $= \dfrac{5.1}{432} \times \dfrac{100}{1} = \underline{\underline{1.2}}$ %

8 An oscillating system will eventually come to rest if there is no driving force. The amplitude of the oscillations decrease due to the presence of friction. This decrease in the amplitude is called damping.

Critical damping occurs when the frictional resistance is just sufficient to prevent any oscillations past the rest position. Critical damping occurs when an oscillating system comes to rest in the shortest possible time.

It is worth noting that in the process of damping the energy of the system ends up as heat energy which is transferred to the surroundings.

TUTORIAL 7

Solutions for Wave Particle Duality

1 (a) electron $\lambda = \frac{h}{p}$ where p = momentum

$$= \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{6}}$$

thus de Broglie wavelength = $\underline{\underline{2.43 \times 10^{-10}}}$ m

(b) proton $\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.673 \times 10^{-27} \times 3 \times 10^{6}}$

$$= \underline{\underline{1.32 \times 10^{-13}}} \text{ m}$$

2 (a) $QV = \frac{1}{2} m v^2$

thus $v = \sqrt{\frac{2QV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 200}{9.11 \times 10^{-31}}}$

$v = \underline{\underline{8.38 \times 10^{6}}}$ m s^{-1}

(b) $\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 8.38 \times 10^{6}}$

$$= \underline{\underline{8.68 \times 10^{-11}}} \text{ m}$$

(c) This gap of 1 mm is much bigger than the wavelength. No diffraction will be observed. The electrons in this case will behave like particles.

3 Consider both the equations, (also see question 8 below):

$\frac{1}{2} m v^2 = QV$ and $\lambda = \frac{h}{mv}$

$m^2 v^2 = 2mQV$ giving $\lambda = \frac{h}{\sqrt{2meV}}$ for an electron

Thus $\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 20 \times 10^{3}}}$

$\lambda = \underline{\underline{8.68 \times 10^{-12}}}$ m

Alternatively v can be found first, then the wavelength as shown for question 2.

4 (a) λ is the de Broglie wavelength
h is the Planck constant
p is the momentum of the particle/wave

(b) (i) Photons behave like waves if diffraction of light takes place, for example when a laser beam strikes a grating.
(ii) Photons behave like particles in the photoelectric effect where electromagnetic radiation above a certain frequency can eject electrons from some metals.
(iii) Electrons can behave like waves if they have a very high speed and their de Broglie wavelength is similar in size to any gap or obstacle they meet.
(iv) Electrons behave like particles in a cathode ray tube.

5 (a) Using a non-relativistic calculation. **[Note:** this is stated on the revised page]

$$\lambda = \frac{h}{\sqrt{2\,m\,e\,V}} \qquad \text{for an electron}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 100 \times 10^{3}}} = \underline{\underline{3.88 \times 10^{-12}}}\ \text{m}$$

For interest it may be noted that the non relativistic value of v is 1.87×10^{8} m s^{-1}, which is **more** than 10% of the speed of light. In the Electrical Phenomena unit, Content Statement 1.18 indicates that relativistic effects should be considered for such speeds.
A relativistic calculation gives v = 1.64×10^{8} m s^{-1} and a wavelength of 3.71×10^{-12} m. Page 5 of the Student Material indicates the method: $E_k = m c^2 - m_0 c^2$ where $E_k = q V$

giving $m_0 c^2 + q V = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} c^2$ from which v is then calculated.

Then $\lambda = \dfrac{h}{m\,v}$ is used, but remembering that m is the relativistic mass!
Such calculations are **not** required for examination purposes.)

(b) Dimensions on the atomic scale are around 1×10^{-10} m. Electrons will therefore diffract on collision with atoms of this spacing. In practice the de Broglie wavelength will not be as small as the value calculated.

6 The wave nature of matter is not very evident in everyday life because typical speeds of ordinary objects are too low for any effects to be observable. For example, the wavelength of a moving car is simply to small to show any diffraction effects.

7 $$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{70 \times 10} = \underline{\underline{9.5 \times 10^{-37}}}\ \text{m}$$

8 (a)
$$\tfrac{1}{2}\, m\, v^2 = Q\,V$$
$$m^2 v^2 = 2mQV \qquad \text{both sides multiplied by m}$$
$$mv = \sqrt{2\,m\,e\,V} \qquad \text{for an electron } Q = e$$
$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2\,m\,e\,V}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19}}} \times \frac{1}{\sqrt{V}}$$
$$\text{thus} \quad \lambda = \frac{1.23 \times 10^{-9}}{\sqrt{V}}$$

(b) $$\lambda = = \frac{1.23 \times 10^{-9}}{\sqrt{1000}} = \underline{\underline{3.89 \times 10^{-11}}}\ \text{m}$$

(c) For good diffraction, gaps can be of the order of the wavelength or less. Thus the approximate spacing is around 3.9×10^{-11} m.

9 The angular momentum of the electron round the nucleus can only take certain discrete values. The angular momentum is quantised in units of $\dfrac{h}{2\pi}$. The values of angular momentum are not continuously variable.

TUTORIAL 1

Coulomb's Inverse Square Law

Note: the following examples use $\frac{1}{4\pi\varepsilon_o} = 9.0 \times 10^9$ N m^2 C^{-2}

1 (a) Use Coulomb's Law

$$F = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{r^2}$$

$$= 9.0 \times 10^9 \times \frac{2 \times 10^{-8} \times 4 \times 10^{-8}}{(2 \times 10^{-2})^2}$$

$$= \underline{\underline{0.018}}\ \text{N}$$

This is a force of attraction since the charges have opposite sign.

(b) $F = 1.0 \times 10^{-4}$ N

$$r^2 = \frac{1}{4\pi\varepsilon_o} \times \frac{Q_1Q_2}{F}$$

$$= 9.0 \times 10^9 \times \frac{(2 \times 10^{-8} \times 4 \times 10^{-8})}{1.0 \times 10^{-4}}$$

thus $r = \underline{\underline{0.27}}$ m

2 $F_g = \frac{GM_1M_2}{r^2}$ $F_e = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{r^2}$

We could work out the forces separately. However it is easier to simply take the ratio $\frac{F_e}{F_g}$. Then the r^2 will cancel.

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_o} \times Q_1\,Q_2}{G\,M_1\,M_2}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{6.67 \times 10^{-11} \times 1.673 \times 10^{-27} \times 9.11 \times 10^{-31}}$$

$$= \underline{\underline{2.3 \times 10^{39}}}$$

Thus the electrostatic force is almost 10^{40} times bigger than the gravitational force between sub-atomic particles. We can therefore safely neglect gravitational effects for such particles.

3 Here $F_e = F_g$

thus $\frac{1}{4\pi\varepsilon_o}\frac{Q\,Q}{r^2} = \frac{GMm}{r^2}$ r^2 cancels

$$9 \times 10^9 \times Q^2 = 6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.3 \times 10^{22}$$

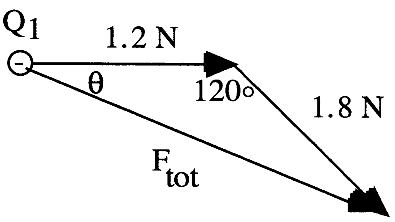
$$Q^2 = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.3 \times 10^{22}}{9 \times 10^9}$$

and $Q = \underline{\underline{5.7 \times 10^{13}}}$ C

This is a huge charge and as you will see later it is not possible to create a positive charge in isolation. Such a possibility for the Earth could not arise.

4
$$F_2 = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{r^2}$$
$$= 9 \times 10^9 \times \frac{1 \times 10^{-6} \times 3 \times 10^{-6}}{(0.15)^2}$$
$= 1.2$ N in a direction from Q_1 towards Q_2
$$F_3 = 9 \times 10^9 \times \frac{1 \times 10^{-6} \times 2 \times 10^{-6}}{(0.10)^2}$$
$= 1.8$ N in a direction away from Q_1, along the line Q_3Q_1

The resultant force is the vector sum of these two forces.



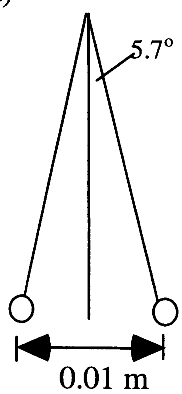
Use the cosine rule:
$$F^2_{tot} = 1.2^2 + 1.8^2 - 2 \times 1.2 \times 1.8 \cos 120°$$
$$= 6.84$$
thus $F_{tot} = \sqrt{6.84} = \underline{\underline{2.6}}$ N

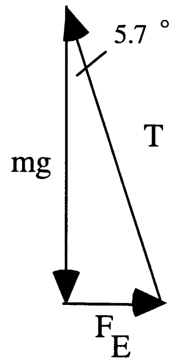
To find the direction, θ, use the sine rule:
$$\frac{\sin\theta}{1.8} = \frac{\sin 120°}{2.6}$$
$$\theta = \underline{\underline{37}}°$$

The resultant force is 2.6 N at 37° as shown in the sketch opposite.

This problem could have been done by scale drawing.

5 (a)





$$F_E = mg \tan 5.7°$$
$$= 0.10 \times 10^{-3} \times 9.8 \times \tan 5.7$$
$$= 9.78 \times 10^{-5}$$
$= \underline{\underline{9.8 \times 10^{-5}}}$ N (2 sig. figs.)

Alternatively use the components of the tension T.

(b) $F_E = \frac{1}{4\pi\varepsilon_o}\frac{QQ}{r^2}$ thus $Q^2 = \frac{9.8 \times 10^{-5} \times (0.01)^2}{9 \times 10^9}$

$Q = \underline{\underline{1.04 \times 10^{-9}}}$ C

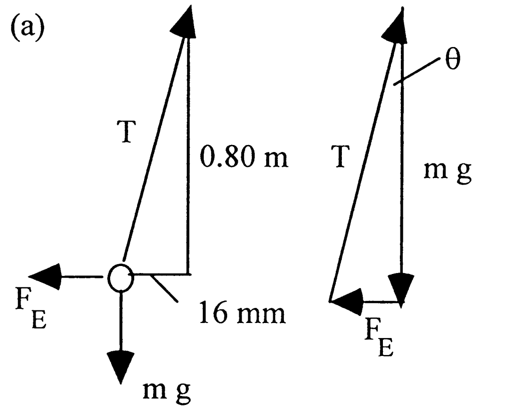
(c) $Q = It$ thus $t = \frac{Q}{I}$

$$t = \frac{1.04 \times 10^{-9}}{1 \times 10^{-11}} = \underline{\underline{104}} \text{ s}$$

(d) Charge one of the spheres by touching it to the dome of a Van de Graaff generator. Now touch the uncharged sphere to the charged sphere. If both spheres are the same size, the charge will be shared equally.

6

(a)

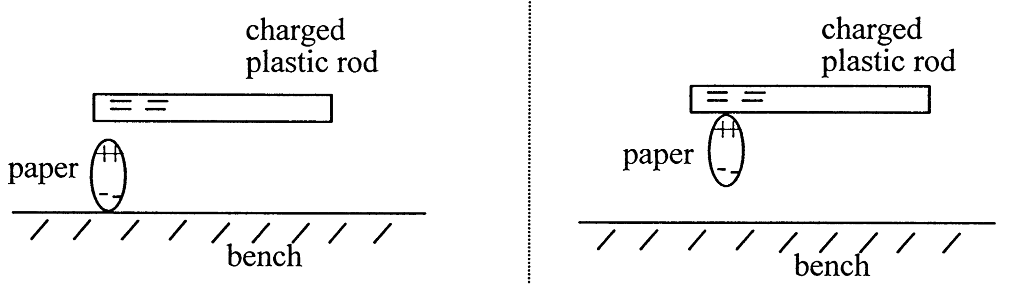


(b)

$$\tan\theta = \frac{0.016}{0.80} = 0.02$$

$$F_E = mg\tan\theta$$
$$= 0.50 \times 10^{-3} \times 9.8 \times 0.02$$
$$= \underline{\underline{9.8 \times 10^{-5}\ \text{N}}}$$

7



The charged plastic rod causes the polar molecules in the paper to line up in the way shown. Note that the paper is overall neutral. The paper is attracted because the positive charge is closer to the negative rod.

TUTORIAL 2
Electric Field Strength

1 $$E = \frac{F}{Q} = \frac{0.02}{4.0 \times 10^{-6}}$$
$$= \underline{\underline{5000}}\ \text{N C}^{-1}$$

2 (a) $$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$
$$1.0 = 9 \times 10^9 \times \frac{Q}{(1.0)^2}$$
thus $$Q = \frac{1}{9 \times 10^9} = \underline{\underline{1.1 \times 10^{-10}}}\ \text{C}$$

(b) $E \ \alpha \ \frac{1}{d^2}$ the distance has been doubled thus field strength quarters.

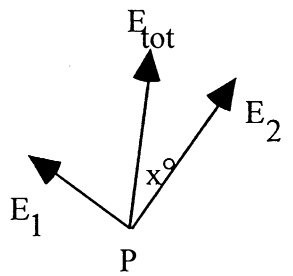
E at 2.0 m will be $\underline{\underline{0.25}}$ N C^{-1}

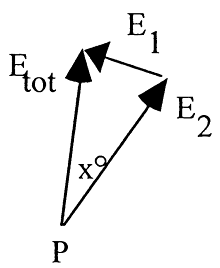
3 (a) $$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$
α-particle has 2 protons $Q = 2 \times 1.6 \times 10^{-19}$ C

thus $$E = 9 \times 10^9 \times \frac{3.2 \times 10^{-19}}{(5 \times 10^{-3})^2}$$
$$= \underline{\underline{1.2 \times 10^{-4}}}\ \text{N C}^{-1}$$

(b) for one proton the charge is halved compared to part (a) and $E \ \alpha \ Q$
thus E will also be halved: $E = \underline{\underline{6.0 \times 10^{-5}}}$ N C^{-1}

4 (a)





The angle at P is a right angle.

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q_1}{r^2} = 9 \times 10^9 \times \frac{18.8 \times 10^{-9}}{(0.12)^2}$$
$= 1.2 \times 10^4$ N C^{-1} (2 sig figs)
in direction shown, from 18.8 nC charge

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{Q_2}{r^2} = 9 \times 10^9 \times \frac{10 \times 10^{-9}}{(0.05)^2}$$
$= 3.6 \times 10^4$ N C^{-1}
in direction shown, from 10 nC charge

$$E_{tot} = \sqrt{E_1^2 + E_2^2} \quad \text{(magnitude)}$$
$$= \underline{\underline{3.8 \times 10^4}}\ \text{N C}^{-1}$$

(b) From the sketch above:
$$\tan x° = \frac{E_1}{E_2} = \frac{1.2 \times 10^4}{3.6 \times 10^4}$$
$$x = \underline{\underline{18}}°$$

5 (a) For a stationary charged sphere:
the downward force of gravity = upward electric force

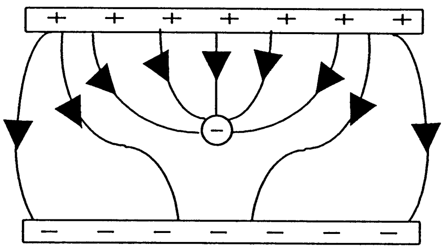
thus $m\,g = E\,Q$

$$E = \frac{m\,g}{Q}$$

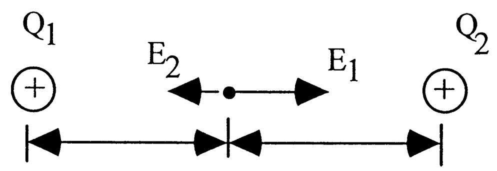
$$E = \frac{2.0 \times 10^{-5} \times 9.8}{5.0 \times 10^{-9}}$$

$$= \underline{\underline{3.9 \times 10^{4}}}\ \text{N C}^{-1}$$

(b)



6 (a)



E_1 will be double the size of E_2 because Q_1 is double the size of Q_2.

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q_1}{r_1^2} = 9 \times 10^9 \times \frac{8.0 \times 10^{-9}}{(0.10)^2} = 7.2 \times 10^3\ \text{N C}^{-1}$$

thus $E_2 = 3.6 \times 10^3$ N C^{-1} Directions of E_1 and E_2 are shown in sketch

$E_{tot} = E_1 - E_2 = \underline{\underline{3.6 \times 10^3}}$ N C^{-1} in the direction from Q_1 to Q_2.

(b) When $E_{tot} = 0$ N C^{-1} $E_1 = -E_2$ and in magnitude $E_1 = E_2$

$$\frac{1}{4\pi\varepsilon_o}\frac{Q_1}{r_1^2} = \frac{1}{4\pi\varepsilon_o}\frac{Q_2}{r_2^2} \quad \text{and } r_2 = (0.20 - r_1)$$

$$\frac{Q_1}{r_1^2} = \frac{Q_2}{(0.20 - r_1)^2}$$

thus $\dfrac{Q_1}{Q_2} = \dfrac{r_1^2}{(0.20 - r_1)^2}$ but $Q_1 = 2\,Q_2$

thus $2 = \dfrac{r_1^2}{(0.20 - r_1)^2}$ take square root of each side

$$\sqrt{2} = \frac{r_1}{(0.20 - r_1)}$$

thus $r_1 = \sqrt{2} \times (0.20 - r_1)$ and $0.20 \times \sqrt{2} = (1 + \sqrt{2})r_1$

$$r_1 = \frac{0.20 \times \sqrt{2}}{(1 + \sqrt{2})} = \underline{\underline{0.12}}\ \text{m}$$

(c) (i) Use Coulomb's Law: $F = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{r^2}$

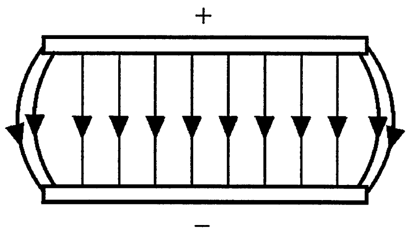
$$= 9 \times 10^9 \times \frac{8.0 \times 10^{-9} \times 4.0 \times 10^{-9}}{(0.2)^2}$$

$$= \underline{\underline{7.2 \times 10^{-6}}}\ \text{N}$$

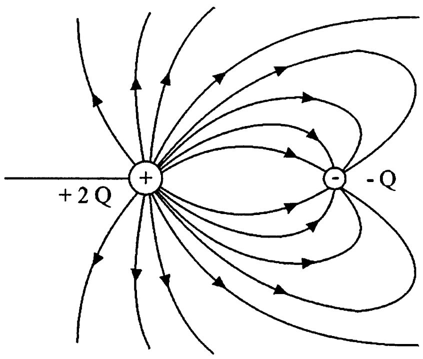
(ii) Using $F = mg$ $\quad F = 5\ 0 \times 10^{-4} \times 9.8$

$$F = \underline{\underline{4.9 \times 10^{-3}}}\ \text{N}$$

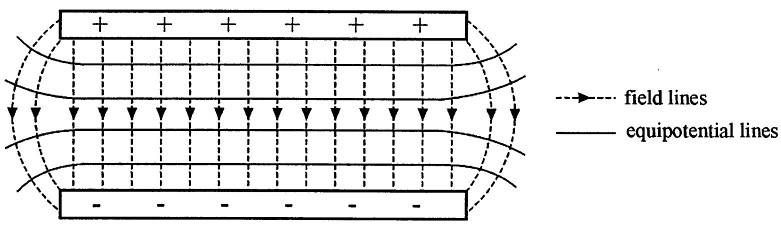
7 (a)



(b)



8



field lines and equipotential lines are always at right angles

TUTORIAL 3

Electrostatic Potential

1 $$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} = 9 \times 10^9 \times \frac{3 \times 10^{-9}}{005}$$
$$= \underline{\underline{540}}\ V$$

2 (a) Potential is a scalar - therefore there is no need to consider any directions.
Note that all the charges are equidistant from C at 0.10 m.
Potential due to a +1.0 nC charge at C:
$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} = 9 \times 10^9 \times \frac{1.0 \times 10^{-9}}{0.10} = 90\ V$$
Potential due to a negative charge is negative.
$$V_C = (2 \times 90) + (-2 \times 90) + (6 \times 90) + (3 \times 90)$$
$$= 180 - 180 + 540 + 270$$
$$= \underline{\underline{810}}\ V$$

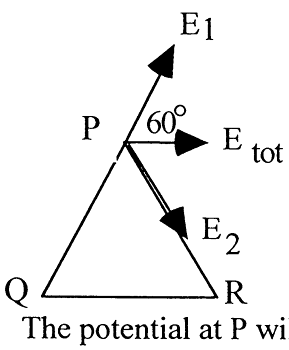
(b) Distance from +2.0 nC and +6.0 nC to D = 0.158 m (by Pythagoras)
Distance from -2.0 nC and +3.0 nC to D = 0.0707 m (by Pythagoras).

thus $$V \text{ due to } +6.0 \text{ nC at D} = 9 \times 10^9 \times \frac{6.0 \times 10^{-9}}{0.158} = 341.8\ V$$
$$V \text{ due to } +2.0 \text{ nC at D} = 9 \times 10^9 \times \frac{2.0 \times 10^{-9}}{0.158} = 113.9\ V$$
$$V \text{ due to } -2.0 \text{ nC at D} = 9 \times 10^9 \times \frac{2.0 \times 10^{-9}}{0.0707} = -254.6\ V$$
$$V \text{ due to } +3.0 \text{ nC at D} = 9 \times 10^9 \times \frac{2.0 \times 10^{-9}}{0.0707} = -381.9\ V$$
$$V_{tot} = 583\ V \text{ at D}$$

thus potential difference between C and D = 810 - 583 = $\underline{\underline{+227}}$ V

3 $$\text{Energy} = QV = Q \times \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}$$
$$= 1.6 \times 10^{-19} \times 9 \times 10^9 \times \frac{1.6 \times 10^{-19}}{5 \times 10^{-11}}$$
$$= \underline{\underline{4.6 \times 10^{-18}}}\ J$$

4 (a)

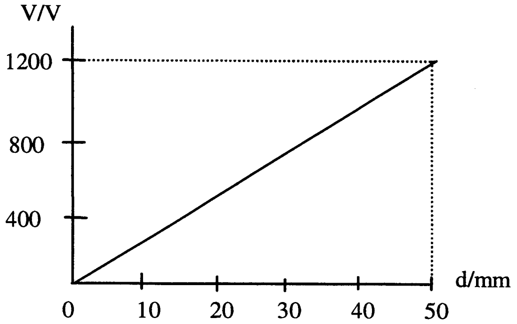


$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} = 9 \times 10^9 \times \frac{1 \times 10^{-8}}{(2.0 \times 10^{-2})^2}$$
$= 2.25 \times 10^5\ N\ C^{-1}$ in direction away from Q
$E_2 = 2.25 \times 10^5\ N\ C^{-1}$ from P towards R

$$E_{tot} = E_1 \cos 60° + E_2 \cos 60°$$
$= \underline{\underline{2.25 \times 10^5}}\ N\ C^{-1}$ in the direction shown

(b) The potential at P will be zero because charges at Q and R are equal in size and opposite in sign and both points are equidistant from P.

5 (a)



(b) $E = \frac{V}{d} = \frac{1200}{5 \times 10^{-2}}$

$= \underline{\underline{2.4 \times 10^{4}}}\ \text{V m}^{-1}$

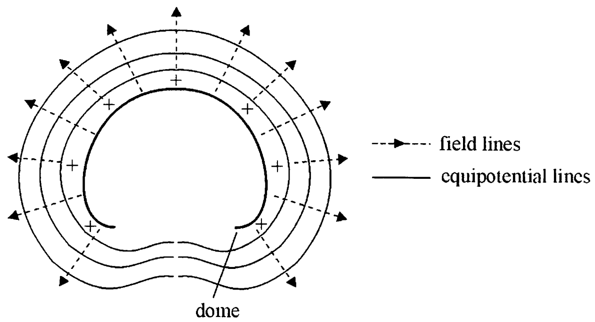
Direction of field is toward the lower plate

(c) Take the gradient of the graph

$E = -\frac{dV}{dx}$

6 (a) Equipotential surfaces have the same potential at all points. Note that moving a charge between two points on an equipotential surface needs no work.

(b) In the sketch below the solid lines show the electric field and the dotted lines show the equipotential surface lines.



7 $E = \frac{V}{d} = \frac{1500}{0.02}$

$= \underline{\underline{7.5 \times 10^{4}}}\ \text{V m}^{-1}$

8 (a) $E = \frac{V}{d} = \frac{2000}{0.15}$

$= \underline{\underline{1.33 \times 10^{4}}}\ \text{V m}^{-1}$

(b) (i) Energy change: electrical potential energy into kinetic energy

(ii) work done $= Q V$

$= 1.6 \times 10^{-19} \times 2000$

$= \underline{\underline{3.2 \times 10^{-16}}}\ \text{J}$

(iii) $Q V = \frac{1}{2} m v^{2}$

$v = \sqrt{\frac{2QV}{m}}$

$= \underline{\underline{2.7 \times 10^{7}}}\ \text{m s}^{-1}$

9 (a) The proton will have a uniform acceleration of 1.27×10^{-12} m s^{-2} towards the negative plate, assuming air resistance is negligible. Any downward force due to gravity has been ignored.

(b) (i) The work done will be the same as in the previous question because the charge on the proton is the same as the charge on the electron.

(ii) Using $QV = \frac{1}{2} m v^2$

$$v = \sqrt{\frac{2QV}{m}}$$

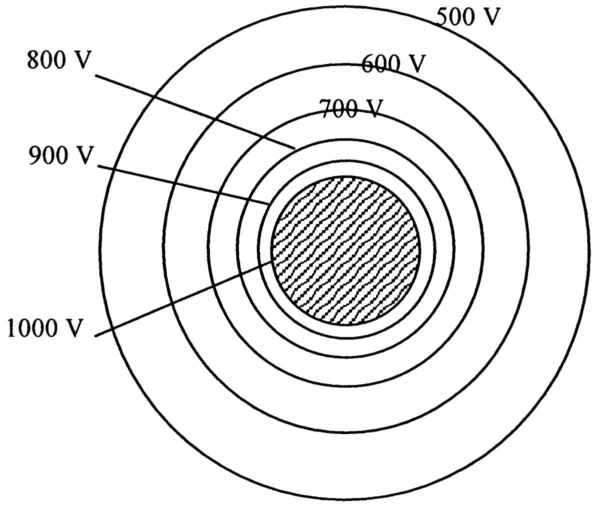
$$= \underline{\underline{6.2 \times 10^5}} \text{ m s}^{-1}$$

10 (a) $V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}$ thus $V \alpha \frac{1}{r}$ or $V_1 r_1 = V_2 r_2$

thus $r_2 = \frac{V_1 r_1}{V_2} = \frac{0.05 \times 1000}{900} = 0.056$ m

$r_3 = \frac{0.05 \times 1000}{800} = 0.063$ m

similarly, $r_4 = 0.071$ m; $r_5 = 0.083$ m; $r_6 = 0.10$ m



Notice that equipotential lines which are separated by the same amount of p.d., in this case 100 V, become further apart as the radius increases.

(b) $V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}$

thus $1000 = 9 \times 10^9 \times \frac{Q}{0.05}$

$Q = \frac{1000 \times 0.05}{9 \times 10^9} = \underline{\underline{5.6 \times 10^{-9}}}$ C

11 (a) $v = \sqrt{\frac{2eV}{m}} = \underline{\underline{5.9 \times 10^8}}$ m s^{-1}

(b) This calculated velocity is greater than the speed of light which is not physically possible. A relativistic calculation is needed, an example is shown for interest at the end of the Mechanics unit solutions.

TUTORIAL 4

Charges in Motion

1 (a)

$$F = EQ$$
$$= 1.2 \times 10^6 \times 1.6 \times 10^{-19}$$
$$a = \frac{F}{m} = \frac{1.2 \times 10^6 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} = 2.11 \times 10^{-17}$$
$$= \underline{\underline{2.1 \times 10^{17}}} \text{ m s}^{-2}$$

(b) (i) Using $v = u + at \quad u = 0 \quad$ gives $t = \frac{v}{a}$

$$t = \frac{3.0 \times 10^7}{2.11 \times 10^{17}} = \underline{\underline{1.42 \times 10^{-10}}} \text{ s}$$

(ii) Using $s = ut + \frac{1}{2} a t^2$

$$= 0 + \frac{1}{2} \times 2.11 \times 10^{17} \times (1.42 \times 10^{-10})^2$$
$$= 2.1 \times 10^{-3} \text{ m} = \underline{\underline{2.1}} \text{ mm}$$

2 (a) The oil drop must be **negatively** charged.

(b) Calculate the size of the electric field: $E = \frac{V}{d} = \frac{2000}{0.02} = 1.0 \times 10^5 \text{ N C}^{-1}$

At balance: $F_{elect.} = F_{grav.}$

$$EQ = mg$$
$$Q = \frac{mg}{E} = \frac{4.9 \times 10^{-15} \times 9.8}{1.0 \times 10^5}$$
$$= \underline{\underline{4.8 \times 10^{-19}}} \text{ C}$$

This is equivalent to an **excess** of three electrons on the oil drop.

3 (a) Use the principle of conservation of energy:

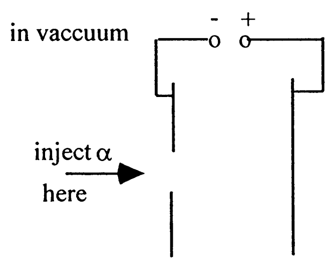
thus work done on a charge by the electric field = E_k 'lost'

$F \times d = \frac{1}{2} m v^2 \quad$ also: $F = EQ$: $\quad EQd = \frac{1}{2} m v^2$

$$\text{thus} \quad E = \frac{\frac{1}{2} m v^2}{Qd} = \frac{\frac{1}{2} \times 6.7 \times 10^{-27} \times (5 \times 10^6)^2}{3.2 \times 10^{-19} \times 6.0 \times 10^{-2}}$$
$$E = \underline{\underline{4.4 \times 10^6}} \text{ N C}^{-1}$$

(Alternatively use: $v^2 = u^2 + 2as$, $F_{un} = ma$ and $E = \frac{F_{un}}{m}$)

(b)

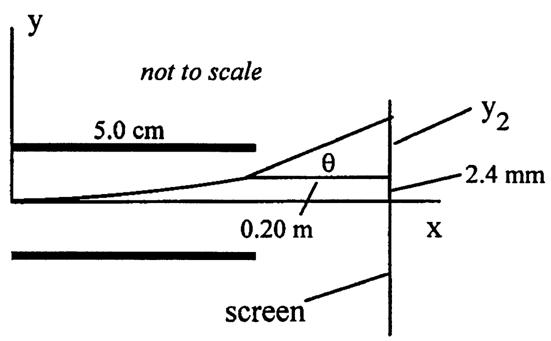


(c) Gamma rays cannot be stopped by an electric field because gamma rays do not have any electric charge. Gamma rays are electromagnetic radiation of very high frequency.

4 (a)
$$t = \frac{d}{v} = \frac{5.0 \times 10^{-2}}{6.0 \times 10^{6}}$$
$$= \underline{\underline{8.33 \times 10^{-9}}}\ \text{s} \quad \text{(keeping 3 sig figs for parts (b) to (d))}$$

(b) Let the displacement in the vertical plane be y.
$$y = u_y t + \frac{1}{2} a t^2 \quad \text{where} \quad a = \frac{F}{m} = \frac{EQ}{m}$$
$$u_y = 0 \text{ thus } y = \frac{1}{2} \times \frac{4 \times 10^2 \times 1.6 \times 10^{-19} \times (8.33 \times 10^{-9})^2}{9.11 \times 10^{-31}} = \underline{\underline{2.4 \times 10^{-3}}}\ \text{m}$$
Thus the vertical displacement experienced by an electron is 2.4 mm.

(c)



θ can be worked out from the combination of vertical and horizontal velocities at the plate edge.

(From above $a = \frac{EQ}{m} = 7.025 \times 10^{13}\ \text{m s}^{-2}$)

$v_{vert} = a\,t = 7.025 \times 10^{13} \times 8.33 \times 10^{-9}$

$= 5.85 \times 10^5\ \text{m s}^{-1}$

$v_{hor} = 6.0 \times 10^6\ \text{m s}^{-1}$

$$\tan\theta = \frac{v_{vert}}{v_{hor}} = \frac{5.85 \times 10^5}{6.0 \times 10^6}$$
$$\theta = \underline{\underline{5.6^\circ}}$$

(d) Also $\quad \tan\theta = \frac{y_2}{0.20}$ thus $y_2 = \tan\theta \times 0.20$

$y_2 = 1.96 \times 10^{-2}\ \text{m} = 19.6\ \text{mm}$

thus total vertical displacement at screen = 2.4 mm + 19.6 mm = 22.0 mm

thus $\quad y_{tot} = \underline{\underline{22\ \text{mm}}}$ $\quad$ (2 sig figs)

5
$$QV = \frac{1}{2} m v^2$$
giving
$$v = \sqrt{\frac{2QV}{m}}$$
$$= \underline{\underline{5.13 \times 10^8\ \text{m s}^{-1}}}$$

This speed is greater than the speed of light. Hence this speed is not possible. A relativistic calculation is needed which gives $v = 2.7 \times 10^8\ \text{m s}^{-1}$. An example is shown, for interest, at the end of the solutions to the Mechanics unit. Such calculations are **not** required for examination purposes.

6 (a) Head on collisions of α-particles with gold nuclei happen very rarely because the nucleus of the gold atom is so very small compared to the overall size of the atom. Most of the atom is empty space.

(b) At closest approach $E_p = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{r}$ q = electronic charge and

Q = charge of gold nucleus

E_k of α–particle $= \frac{1}{2}\, m\, v^2$

Change of E_k = change of E_p where r is the closest distance of approach.

$\frac{1}{2}\, m\, v^2 - 0 = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{r} - 0$ (initial $E_p = 0$ and final $E_k = 0$)

thus $r = \frac{1}{4\pi\varepsilon_o}\frac{2qQ}{mv^2}$

$$= 9 \times 10^9 \times \frac{2 \times 3.2 \times 10^{-19} \times 79 \times 1.6 \times 10^{-19}}{6.7 \times 10^{-27} \times (2.0 \times 10^7)^2}$$

$$= \underline{\underline{2.7 \times 10^{-14}}}\text{ m}$$

7 (a) $m = \rho V = 870 \times \frac{4}{3}\pi \times (1.62 \times 10^{-6})^3$

$= \underline{\underline{1.55 \times 10^{-14}}}$ kg

(b) At balance $F_{elect} = F_{grav}$

$E\,Q = m\,g$

thus $Q = \frac{m\,g}{E} = \frac{1.55 \times 10^{-14} \times 9.8}{1.9 \times 10^5}$

$= \underline{\underline{8.0 \times 10^{-19}}}$ C

(c) e = one electronic charge $= 1.6 \times 10^{-19}$ C

thus no. of charges $= \frac{Q}{e} = 5$ a whole number.

8 $\frac{1}{2}\, m\, v^2 = Q\,V$

$$v^2 = \frac{2QV}{m}$$

$$v = \sqrt{2\frac{Q}{m}V}$$

$$= \sqrt{2 \times 1.8 \times 10^{11} \times 250}$$

$$= \underline{\underline{9.5 \times 10^6}}\text{ m s}^{-1}$$

Notice that this speed is much less than the speed of light.

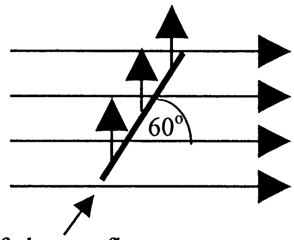
TUTORIAL 5

Force on a Conductor

1 (a)

$$F = Il\ B \sin\theta$$
$$= 7.5 \times 0.05 \times 0.04 \times \sin 60°$$
$$= \underline{\underline{0.013}}\ \text{N}$$

(b)



(c) For maximum force, $\theta = 90°$ and all the conductor, 50 mm, is in the field.

2

$$F = Il\ B \sin\theta$$
$$4.5 \times 10^{-3} = 1.4 \times 50 \times 10^{-3} \times 0.09 \times \sin\theta$$
$$\theta = \underline{\underline{46°}}$$

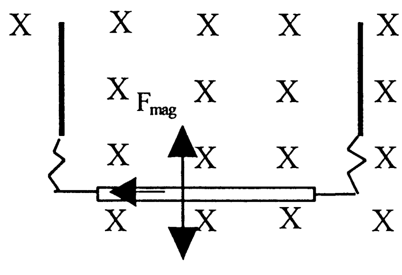
3 (a) To remove tension in the supporting leads the magnetic force has to be equal and opposite to the weight of the wire.

$$W = mg = 0.025 \times 9.8$$
$$= 0.245\ \text{N}$$

using $F = I\,l\ B \sin\theta$ and $F = 0.245$ N and $\theta = 90°$

$$I = \frac{F}{l\,B \sin\theta} = \frac{0.245}{0.75 \times 0.50 \times 1}$$
$$I = \underline{\underline{0.65}}\ \text{A}$$

(b) Apply Right Hand Rule



4 (a) $$F = IlB\sin\theta$$

For $\theta = 90°$ the 0.25 m sides are perpendicular to the field

$$F = 0.25 \times 0.25 \times 0.40 \times 1$$

$$F = \underline{\underline{0.025}}\ \text{N}$$

(b) $T = Fr$ for **each** force $T = 0.025 \times 0.075$

Total torque $= 2 \times 0.025 \times 0.075 = 3.75 \times 10^{-3}$ N m (for each turn of wire)

Thus for the whole coil: Torque $= 120 \times 3.75 \times 10^{-3}$

$= \underline{\underline{0.45}}$ N m

(c) As the coil rotates the 0.25 m sides of the coil make angles less than 90° with the field. The force on the wire decreases so the torque decreases. When the coil is perpendicular to the field these sides are momentarily parallel to the field and the torque will be zero.

5 (a) Since the balance reading is less, this suggests that there is an **upward** force on the magnet assembly exerted by the current in the wire.

difference in readings = 95.6 g - 93.2 g = 2.4 g

$$F = 2.4 \times 10^{-3} \times 9.8 = 0.02352$$

$$= \underline{\underline{0.024}}\ \text{N}$$

(b) $$F = IlB\sin\theta \qquad (\theta = 90°)$$

$$0.024 = 4.0 \times 0.06 \times B \times 1$$

$$B = \underline{\underline{0.1}}\ \text{T}$$

(c) Reversing the direction of the current in the wire reverses the direction of the force. This direction is now downward and will **increase** the reading by 2.4 g.

new reading on balance = 95.6 + 2.4 = $\underline{\underline{98.0}}$ g

(d) When north faces north, the field is zero between the magnets (in centre). There will be no magnetic force on the wire. The balance will read 95.6 g.

6 (a) $$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

thus $$F = \frac{4\pi \times 10^{-7} \times 16 \times (2500)^2}{2\pi \times 0.20}$$

$$= \underline{\underline{100}}\ \text{N}$$

(b) If the wires were suspended freely they would attract each other. If they touched there would be a short circuit which could start a fire.

TUTORIAL 6

Charged Particles in Magnetic Fields

1 $$F = q\,v\,B\sin\theta$$

thus $$B = \frac{F}{q\,v\sin\theta} = \frac{3.0 \times 10^{-17}}{1.6 \times 10^{-19} \times 1.0 \times 10^{7} \times \sin 45^\circ}$$

$$= \underline{\underline{2.7 \times 10^{-5}}}\ \text{T}$$

2 $$F_m = q\,v\,B\sin\theta \qquad \text{and } \theta = 90^\circ$$

$$= 1.6 \times 10^{-19} \times 2.8 \times 10^{8} \times 3.3 \times 10^{-5} \times 1.0$$

$$= 1.478 \times 10^{-15}\ \text{N}$$

$F_g = m\,g$ notice the speed of proton is > 10% speed of light

Use $m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ to find the relativistic mass m

$$m = 4.660 \times 10^{-27}$$

$$F_g = 4.660 \times 10^{-27} \times 9.8 = 4.567 \times 10^{-26}\ \text{N}$$

thus $$\frac{F_m}{F_g} = \frac{1.478 \times 10^{-15}}{4.567 \times 10^{-26}} = \underline{\underline{3.2 \times 10^{10}}}$$

If the relativistic mass is not used an answer of 9.0×10^{-10} is obtained!

3 $$F = q\,v\,B = \frac{mv^2}{r} \qquad \text{giving} \quad r = \frac{m\,v}{q\,B}$$

$\frac{1}{2}\,m\,v^2 = 4.2 \times 10^{-12}$ to find v from value given for E_k.

$$v = \sqrt{\frac{2 \times 4.2 \times 10^{-12}}{1.673 \times 10^{-27}}} = 7.086 \times 10^{7}\ \text{m s}^{-1}$$

from above $$r = \frac{m\,v}{q\,B} = \frac{1.673 \times 10^{-27} \times 7.086 \times 10^{7}}{1.6 \times 10^{-19} \times 0.28}$$

$$= \underline{\underline{2.6}}\ \text{m}$$

4 from $qvB = \dfrac{mv^2}{r}$ $\quad r = \dfrac{m\,v}{q\,B}$ and $v = r\omega = r \times 2\pi f$

giving $f = \dfrac{qB}{2\pi m}$ use this to compare frequency for α and electron

$$\frac{f_\alpha}{f_e} = \frac{q_\alpha B}{2\pi m_\alpha} \times \frac{2\pi m_e}{q_e B} = \frac{q_\alpha m_e}{q_e m_\alpha} = \frac{2m_e}{m_\alpha} \qquad \text{since. } q_\alpha = 2q_e$$

$$= \frac{2 \times 9.11 \times 10^{-31}}{6.68 \times 10^{-27}} = \underline{\underline{2.73 \times 10^{-4}}}$$

Alternatively: $f_e = 3.67 \times 10^{3}\, f_\alpha$

5 (a) The magnetic force supplies the central acceleration.

$qvB = \dfrac{mv^2}{r}$ giving $r = \dfrac{m\,v}{q\,B}$ also $v = r\omega$ and $\omega = 2\pi f$

hence $\dfrac{v}{r} = 2\pi f$ and $\dfrac{v}{r} = \dfrac{qB}{m}$ giving $f = \dfrac{qB}{2\pi m}$

(b) From the equation in (a) above: $B = \frac{2\pi mf}{q}$

$$B = \frac{2\pi \times 3.34 \times 10^{-27} \times 1.2 \times 10^{7}}{1.6 \times 10^{-19}} = 1.574$$

$= \underline{\underline{1.6}}$ T

(c) At maximum radius R: $v = \frac{qBR}{m}$

E_k of deuterons emerging: $E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \times \left[\frac{qBR}{m}\right]^2$

$$E_k = \frac{q^2B^2R^2}{2m}$$

$$= \frac{(1.6 \times 10^{-19})^2 \times (1.574)^2 \times (0.50)^2}{2 \times 3.34 \times 10^{-27}}$$

$= \underline{\underline{2.4 \times 10^{-12}}}$ J

6 (a) If undeflected:

$$F_{mag} = F_{elect}$$
$$q\,v\,B = q\,E$$
$$v = \frac{E}{B}$$

(b)

$$v = \frac{1.4 \times 10^5}{0.70}$$

$= \underline{\underline{2.0 \times 10^5}}$ m s^{-1}

(c) (i) $r = \frac{m\,v}{q\,B}$ thus $m = \frac{q\,r\,B}{v}$

$$m = \frac{1.6 \times 10^{-19} \times 0.07 \times 0.7}{2.0 \times 10^5}$$

$= \underline{\underline{3.9 \times 10^{-26}}}$ kg

(ii) The ions of the different isotopes will have different radii. They will therefore show up at different points on the photographic record. The less massive particles will have a larger radius.

7 (a) $v = \frac{q\,r\,B}{m}$ from $qvB = \frac{mv^2}{r}$

$$= \frac{3.2 \times 10^{-19} \times 0.45 \times 1.2}{6.68 \times 10^{-27}} = 2.59 \times 10^7$$

$= \underline{\underline{2.6 \times 10^7}}$ m s^{-1}

(b) $v = \frac{2\pi r}{T}$

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 0.45}{2.59 \times 10^7}$$

$= \underline{\underline{1.1 \times 10^{-7}}}$ s

(c) $E_k = \frac{1}{2} m v^2 = \frac{1}{2} \times 6.68 \times 10^{-27} \times (2.6 \times 10^7)^2$

$= \underline{\underline{2.3 \times 10^{-12}}}$ J

8 (a) Particle X is moving in a direction parallel to the magnetic field. This means that it will not experience a magnetic force. Particle X will therefore carry on in a straight line with no change of speed.

Particle Y follows a circular path because it enters the magnetic field at right angles to the field direction.

Particle Z follows a helical (spiral) path because it enters the magnetic field at an angle.

(b) $$r = \frac{m\,v}{q\,B} \qquad \text{from } qvB = \frac{mv^2}{r}$$

$$r = \frac{2 \times 10^{6} \times 1.673 \times 10^{-27}}{1.6 \times 10^{-19} \times 1.3 \times 10^{-5}}$$

$$= 1.61 \times 10^{3}\ \text{m}$$

$$= \underline{\underline{1.6}}\ \text{km}$$

9 (a) (i)

$$F_{mag} = F_{elect}$$

$$q\,v\,B = q\,E$$

thus $$v = \frac{E}{B} \quad \text{and} \quad E = \frac{V}{d}$$

$$v = \frac{V}{Bd}$$

(ii) $$B = \frac{9 \times 10^{-7}\ NI}{a}$$

$$= \frac{9 \times 10^{-7} \times 320 \times 0.31}{0.073} = 1.22 \times 10^{-3}$$

$$= \underline{\underline{1.2 \times 10}}^{-3}\ \text{T}$$

(iii) thus $$v = \frac{V}{Bd} = \frac{1200}{1.22 \times 10^{-3} \times 0.045} = 2.19 \times 10^{7}$$

$$= \underline{\underline{2.2 \times 10}}^{7}\ \text{m s}^{-1}$$

(b) (i) $$r = \frac{m\,v}{q\,B} \qquad \text{from } qvB = \frac{mv^2}{r}$$

and $$\frac{e}{m} = \frac{v}{rB} \qquad \text{since here } q = e$$

(ii) $$r = \frac{L^2 + y^2}{2y}$$

$$r = \frac{0.055^2 + 0.015^2}{2 \times 0.015}$$

$$= 0.108\ \text{m}$$

$$\frac{e}{m} = \frac{v}{rB} = \frac{2.19 \times 10^{7}}{0.108 \times 1.2 \times 10^{-3}}$$

$$= \underline{\underline{1.7 \times 10}}^{11}\ \text{C kg}^{-1}$$

The accepted value for $\frac{e}{m}$ is 1.76×10^{11} C kg^{-1}

10 (a) $E = \frac{V}{d} = \frac{1200}{0.05} = \underline{\underline{2.40 \times 10^4 \text{ V m}^{-1}}}$

(b) (i) $B = \frac{8\mu_0 NI}{\sqrt{125}\,r}$

$= \frac{8 \times 4\pi \times 10^{-7} \times 320 \times 0.25}{11.2 \times 0.068} = 1.058 \times 10^{-3}$

$= \underline{\underline{1.06 \times 10^{-3}}} \text{ T}$

(ii) $v = \frac{E}{B} = \frac{2.40 \times 10^4}{1.058 \times 10^{-3}}$

$v = \underline{\underline{2.27 \times 10^7}} \text{ m s}^{-1}$ (keeping 3 sig figs)

(c) (i) time taken to cross between plates $t = \frac{L}{v}$

deflection, $y = \frac{1}{2} a t^2 = \frac{1}{2} a \frac{L^2}{v^2}$

(ii) thus $y = \frac{1}{2} \times \frac{eE}{m} \times \frac{L^2}{v^2}$ since $a = \frac{F}{m} = \frac{eE}{m}$

$\frac{e}{m} = \frac{2yv^2}{EL^2} = \frac{2 \times 0.01 \times (2.27 \times 10^7)^2}{2.40 \times 10^4 \times 0.05^2}$

$= \underline{\underline{1.72 \times 10^{11}}} \text{ C kg}^{-1}$

11 (a) Electric potential energy = kinetic energy gained

$eV = \frac{1}{2} m v^2$ and $V = 1000$ V

thus $\frac{e}{m} = \frac{v^2}{2 \times 1000}$

(b) (i) $B = \frac{9 \times 10^{-7} NI}{r}$

$= \frac{9 \times 10^{-7} \times 320 \times 0.26}{0.068}$

$= \underline{\underline{1.10 \times 10^{-3}}} \text{ T}$

(ii) $v = \frac{E}{B} = \frac{2.0 \times 10^4}{1.10 \times 10^{-3}}$ since $E = \frac{1000}{0.05} = 2 \times 10^4$

$= \underline{\underline{1.82 \times 10^7}} \text{ m s}^{-1}$

(c) $\frac{e}{m} = \frac{v^2}{2 \times 1000} = \frac{(1.82 \times 10^7)^2}{2000}$

$= \underline{\underline{1.66 \times 10^{11}}} \text{ C kg}^{-1}$

(d) Accepted value for $\frac{e}{m} = 1.76 \times 10^{11} \text{ C kg}^{-1}$

(i) An uncertainty of 5% in the value in (c) above:

gives $\frac{e}{m} = (1.66 \pm 0.08) \times 10^{11} \text{ C kg}^{-1}$

(ii) The calculated uncertainty does not bring the measured value within range of the accepted value. The measured value is about 6% too low.

TUTORIAL 7

Self Inductance

1 (a) V across R = I R = 8.0 x 1.0 = 8.0 V

thus e.m.f. across inductor = 12 - 8.0 = $\underline{\underline{4.0}}$ V

(b) $$\varepsilon = -L\frac{dI}{dt}$$

$$-4.0 = -L \times 400 \quad \text{(4.0 V is a back e.m.f)}$$

$$L = \frac{4.0}{400} = \underline{\underline{0.01}}\ \text{H}$$

(c) When the switch is closed and current is zero, all of the e.m.f. will be across the inductor: ε = 12 V.

$$\frac{dI}{dt} = \frac{\varepsilon}{L} = \frac{12}{0.01} = \underline{\underline{1200}}\ \text{A s}^{-1}$$

(d) $$I_{final} = \frac{\varepsilon}{R} = \frac{12}{1.0} = \underline{\underline{12}}\ \text{A}$$

(e) $$E = \frac{1}{2} L I^2$$

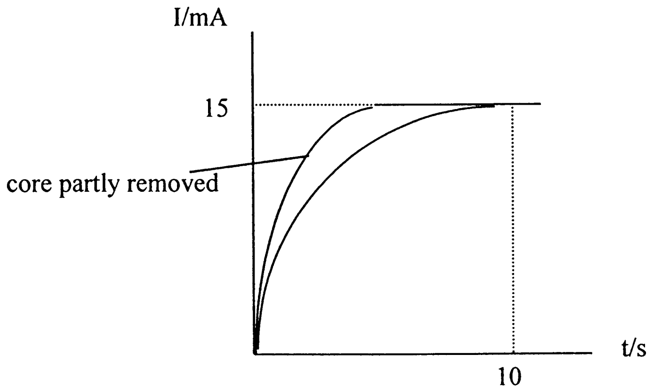
$$= \frac{1}{2} \times 0.01 \times 12^2$$

$$= \underline{\underline{0.72}}\ \text{J}$$

2 (a) (i) When the switch is closed the current starts to increase in the circuit. This *changing* current produces a *changing* magnetic field which in turn induces an e.m.f. across the coil. This e.m.f. opposes the build up of the current (Lenz's law). This is observed as a delay in the current reaching a final steady value.

(ii) When the current reaches its final steady value there is no induced e.m.f. across the inductor and therefore no back e.m.f. generated. The resistance in the circuit and the e.m.f. of the supply determine this steady current.

(b) The removal of the soft iron core reduces the inductance of the coil. This will result in a faster build up of current because the back e.m.f. will be less.



(c) $$R_{coil} = \frac{\text{e.m.f.}}{I_{final}} = \frac{3.0}{0.015} = \underline{\underline{200}}\ \Omega$$

3 (a) (i) p.d. across R at start is $\underline{\underline{0}}$ V

(ii) initial current is also $\underline{\underline{\text{zero}}}$.

(iii) initial induced e.m.f across L is the e.m.f. of the supply, $\underline{\underline{10}}$ V

(iv) $E = \frac{1}{2} L I^2$ but $I = 0$ A thus $E = \underline{\underline{0}}$ J

(b) (i) $V = IR = 0.04 \times 40 = \underline{\underline{1.6}}$ V

(ii) thus $\varepsilon = 10 - 1.6 = 8.4$ V

(iii)

$$\varepsilon = -L\frac{dI}{dt}$$

$$-8.4 = -2.0 \times \frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{8.4}{2.0} = \underline{\underline{4.2}}\ \text{A s}^{-1}$$

(iv)

$$E = \frac{1}{2} L I^2$$

$$= \frac{1}{2} \times 2.0 \times 0.04^2$$

$$= \underline{\underline{1.6 \times 10^{-3}}}\ \text{J}$$

4 (a) The self-inductance of a coil is given by $\varepsilon = -L\frac{dI}{dt}$. The inductance is one henry if an e.m.f. of one volt is induced when the current changes at a rate of one ampere per second.

(b) (i) Lamp X lights more slowly due to the self-inductance of the inductor L. When the circuit is switched on the current grows and produces a changing magnetic field in the inductor. This in turn generates an e.m.f. which by Lenz's law opposes the original current. There is no such effect with a resistor, hence lamp Y lights immediately.

(ii) When the switch is closed $\varepsilon = 10$ V

$$\varepsilon = -L\frac{dI}{dt} \quad \text{and } L = \frac{10}{0.50} = \underline{\underline{20}}\ \text{H}$$

(iii) Current in inductor branch $= \frac{P}{V} = \frac{3}{6} = 0.50$ A

p.d. across L $= 10 - 6.0 = 4.0$ V

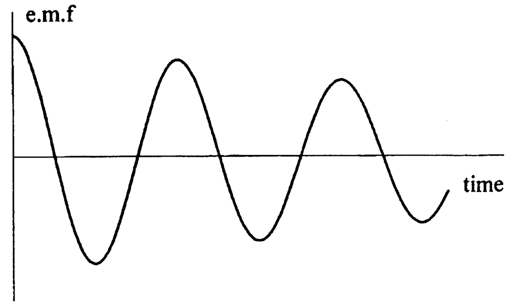
thus $R_L = \frac{V}{I} = \frac{4.0}{0.50} = \underline{\underline{8.0}}\ \Omega$ using V = IR

5 (a) When S is opened the current in the primary collapses. This produces a large change in magnetic field in the primary. This in turn produces a change in magnetic field in the linked secondary coil, which gives an large induced e.m.f. across the spark plug.

(b) If there are more turns in the secondary (step-up), a larger e.m.f. will be produced across the spark plug.

(c) The energy for the spark comes from the battery via the electromagnetic field set up in the coils and core.

6 (a)



The oscillations will be damped due to Lenz's law. The magnetic field in the coil will oppose the movement of the magnet.

(b) When the magnet momentarily stops the induced e.m.f. is zero. Relative movement is needed to induce an e.m.f.

(c) When magnet movement is reversed the induced e.m.f. will also be reversed.

(d) The fastest movement results in the maximum induced e.m.f.

7 Lamp Z: no change in brightness when frequency is altered. The resistance of a resistor does not change with frequency.

Lamp Y: as frequency increases the lamp dims because there is a greater back e.m.f. generated at a higher frequency. The inductive reactance has increased, i.e. the opposition to a.c. increases.

Lamp X: as frequency increases the lamp becomes brighter because the capacitor allows a greater current. to pass. The capacitive reactance has decreased, i.e. the opposition to a.c. decreases.

8 (a) Loudspeaker A will reproduce the high frequency signals while loudspeaker B will reproduce the low frequency signals.

(b) Both high and low frequency signals have a choice of path at the top of the circuit, where the inductor L and capacitor C_2 join. Higher frequency signals will pass through the capacitor, C_2, because there is less opposition (capacitive reactance) for that route. After passing loudspeaker A the high frequency signals take the low opposition route through capacitor C_1.

The low frequency signals will pass through the inductor because this route has a lower opposition (inductive reactance) for low frequencies. The low frequencies signals, after passing through inductor L, will pass through loudspeaker B rather than pass through C_1. A capacitor has a larger opposition to low frequency signals.

TUTORIAL 8

Forces of Nature

1 (a) (i)

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{Q_1 Q_2}{r^2} \quad \text{and } Q_1 = Q_2$$

$$= 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{(4.0 \times 10^{-15})^2}$$

$$= \underline{\underline{14}}\ \text{N} \quad \text{a huge force between nucleons}$$

(ii) This force between two like charges is repulsive.

(b) (i)

$$F = \frac{GM_1M_2}{r^2} = \frac{6.67 \times 10^{-11} \times (1.673 \times 10^{-27})^2}{(4.0 \times 10^{-15})^2}$$

$$= \underline{\underline{1.2 \times 10^{-35}}}\ \text{N}$$

(ii) The gravitational force is attractive.

(iii) This force is very very small compared to the electrostatic force, it is around 1×10^{36} times smaller!

(iv) If the above two forces were the only forces acting, the nucleus would fly apart. There must be another force acting between nucleons which is (a) attractive and (b) stronger than the electrostatic force. We call this the Strong Force.

2 (a) The strong nuclear force acts between nucleons in the nucleus. It is a much greater force than the weak nuclear force. Particles called leptons, for example the electron and muon, do not experience the strong nuclear force but do experience the weak force. Particle called hadrons, for example the proton and neutron, experience both the weak and strong forces.

(b) The weak nuclear force is associated with beta decay in which a neutron decays to a proton, an electron (and an associated anti-neutrino $\overline{\nu_e}$):

$$^{1}_{0}n \dashrightarrow {}^{1}_{1}p + {}^{0}_{-1}e + \overline{\nu_e}$$

The weak force can change the 'flavour' of a quark, i.e. a down quark is changed to an up quark in the above decay.

(c) The range for the strong nuclear force is less than 10^{-14} m.
The range of the weak force is smaller, less than 10^{-17} m.
The diameter of a nucleus is of the order of 10^{-14} m, hence the effects of both the strong and weak force are mainly restricted to subnuclear distances.

3 (a) Each nucleon is made up of 3 quarks.

(b) Protons are composed of two **up** quarks and one **down** quark. The up quark has a charge $+\frac{2}{3}$ e and the down quark $-\frac{1}{3}$ e, giving a total change of +1 e.
Neutrons are composed of one **up** quark and two **down** quarks, giving a total charge of zero.

TUTORIAL 1

Waves

1 (a) $y = 3 \sin 2\pi (10t - 0.2x)$ is compared with $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$a = \underline{\underline{3.0}}$ cm

(b) $f = \underline{\underline{10}}$ Hz

(c) $\frac{1}{\lambda} = 0.2$ thus $\lambda = \frac{1}{0.2} = \underline{\underline{5.0}}$ cm

(d) $v = f\lambda = 10 \times 5.0 = \underline{\underline{50}}$ cm s^{-1}

2 $y = a \sin 2\pi (ft - \frac{x}{\lambda})$ thus $y = 0.30 \sin 2\pi (20t - \frac{x}{0.5})$

3 $y = 0.20 \sin (220\pi t - 30\pi x)$ rewrite in the form $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$y = 0.20 \sin 2\pi (110t - 15x)$

assuming v remains the same, doubling f halves λ

thus $y = 0.40 \sin 2\pi (220t + 30x)$

4 $y = 0.04 \sin [2\pi(\frac{t}{0.04} - \frac{x}{2.0})]$

(a) $y_{max} = \underline{\underline{0.04}}$ m

(b) compare with $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$\lambda = \underline{\underline{2.0}}$ m

(c) $f = \frac{1}{0.04} = \underline{\underline{25}}$ Hz

(d) The movement of a particle will be Simple Harmonic with a maximum amplitude of 0.04 m. The particle will move in a direction perpendicular to the wave direction along the string.

5 (a) $y = 0.01 \sin \pi (2.0t - 0.01x)$

$= 0.01 \sin 2\pi (t - \frac{0.01}{2} x)$

thus $f = 1.0$ Hz and $\lambda = \frac{2}{0.01} = 200$ m

$v = f\lambda = 1.0 \times 200 = \underline{\underline{200}}$ m s^{-1}

(b) at $x = 0$ $y = 0.01 \sin \pi (2.0t)$

$= 0.01 \sin 2\pi t$

$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.01 \sin 2\pi t) = [0.01 \cos 2\pi t]\, 2\pi$

thus $v_{ymax} = 0.01 \times 1.0 \times 2\pi = \underline{\underline{0.063}}$ m s^{-1}

6 (a) $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

substitute $f = \frac{1}{T}$ giving $y = a \sin 2\pi (\frac{t}{T} - \frac{x}{\lambda})$

(b) substitute $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$

$y = a \sin 2\pi (ft - \frac{x}{\lambda}) = y = a \sin (2\pi ft - \frac{2\pi x}{\lambda})$

$y = a \sin (\omega t - kx)$

(c) substitute $\lambda = \frac{v}{f}$ into $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$y = a \sin 2\pi (ft - \frac{x}{\lambda}) = y = a \sin 2\pi (ft - \frac{xf}{v})$

$y = a \sin 2\pi f(t - \frac{x}{v})$

(d) substitute $f = \frac{v}{\lambda}$ into $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$y = a \sin 2\pi (\frac{vt}{\lambda} - \frac{x}{\lambda})$

$y = a \sin \frac{2\pi}{\lambda} (vt - x)$

7 (a) A phase difference of 2π occurs in one wavelength $\lambda = \frac{v}{f} = \frac{350}{500} = 0.70$ m

thus a phase difference of $\frac{\pi}{3}$ occurs in $\frac{1}{6}\lambda$

the two points are separated by $\frac{0.70}{6} = \underline{\underline{0.12}}$ m

(Alternatively use $\phi = \frac{2\pi x}{\lambda}$)

(b) 2 π phase difference occurs in one period (T)

$T = \frac{1}{f} = \frac{1}{500} = 0.002$ s

0.002 s is equivalent to a phase difference of 2π

0.001 s is equivalent to a phase difference of π

8 (a) $\lambda = \frac{v}{f} = \frac{30}{250} = 0.12$ m

phase difference $= \frac{2\pi x}{\lambda} = 2\pi \times \frac{10}{12} = \underline{\underline{1.67\,\pi}}$ radians $= \underline{\underline{5.24}}$ radians

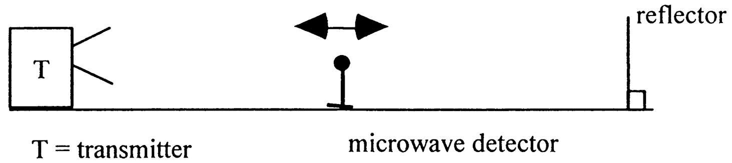
(b) $y = a \sin 2\pi (ft - \frac{x}{\lambda}) = 0.03 \sin 2\pi (250t - \frac{x}{0.12})$

(c) distance between nodes $= \frac{\lambda}{2} = \frac{0.12}{2} = \underline{\underline{0.06}}$ m

9 (a) A travelling wave is a wave which moves through a material transferring energy in the direction of travel. All particles of the material which transmits the energy perform Simple Harmonic Motion.

A stationary wave has parts of the material at rest, the nodes, and energy does not travel along the material. Energy is effectively trapped between the nodes. The particles between the nodes vibrate in phase with SHM but have different amplitudes.

(b) Wavelength of microwaves



Set up the apparatus as above. Move the microwave detector between transmitter and reflector and note that nodes are detected.
Measure the distance, d, between the first and eleventh node for example.

The distance between adjacent nodes $= \frac{\lambda}{2}$

Thus distance between first and eleventh nodes is 5λ giving $\lambda = \frac{d}{5}$.

10 (a) (i) The microphone has been moved a distance of $\frac{\lambda}{2}$.

thus $\lambda = 2 \times 0.24 = \underline{0.48}$ m

(ii) $v = f\lambda = 700 \times 0.48 = 336 \text{ m s}^{-1}$

velocity of sound $= \underline{\underline{340}} \text{ m s}^{-1}$ (2 sig figs)

(b) As the listener walks across the room he will hear alternately quiet and loud sounds of the same frequency. This is because there are two sources of sound producing coherent waves and as they overlap, constructive and destructive interference takes place. Constructive interference will be positions of loud sound and destructive interference positions of quiet.

At constructive interference the path difference is $n\lambda$, the waves are in phase, the amplitude is bigger so there is more energy. At destructive interference the path difference is $(n + ½)\lambda$, the waves are completely out of phase, the amplitude is zero so there is no energy. The quiet patches are not completely silent because there is a degree of reflection of the sound from the walls in the room.

TUTORIAL 2

The Doppler Effect

1 (a) In this case the source of the sound is moving at speed v_S and the observer is stationary. v is the speed of sound.

Thus $f_{obs} = f_S \frac{v}{(v - v_S)}$ for a source moving **towards** a stationary observer

$$f_{obs} = 1000 \times \frac{340}{340 - 20} = \underline{\underline{1060}}\ \text{Hz}$$

(b) $f_{obs} = f_S \frac{v}{(v + v_S)}$ for a source moving **away from** a stationary observer

$$= 1000 \times \frac{340}{340 + 20} = \underline{\underline{944}}\ \text{Hz}$$

2 (a) Thus $f_{obs} = f_S \frac{v + v_O}{v}$ for an observer moving **towards** a stationary source

$$f_{obs} = 1500 \times \frac{340 + 30}{340} = \underline{\underline{1630}}\ \text{Hz}$$

(b) $f_{obs} = f_S \frac{v - v_O}{v} = 1500 \times \frac{340 - 30}{340} = \underline{\underline{1370}}\ \text{Hz}$

3 (a) As the siren is moving towards the observer the frequency heard increases. The speed of the siren relative to the observer changes continually, therefore the observed frequency will change continuously: higher as it approaches and lower as it recedes.

(b) velocity of siren $v_s = r\omega = 0.8 \times 2\pi \times 3 = 15\ \text{m s}^{-1}$

$f_{obs} = f_S \frac{v}{(v - v_S)}$ for a source moving **towards** a stationary observer

$$f_{obs} = 1200 \times \frac{340}{(340-15)} = 1255\ \text{Hz}$$

$$f_{obs} = f_S \frac{v}{(v + v_S)} = 1200 \times \frac{340}{340+15} = 1149\ \text{Hz}$$

Thus range of frequencies heard is $\underline{\underline{1150}}$ Hz to $\underline{\underline{1260}}$ Hz.

(c) The girl will hear a constant frequency of 1200 Hz. The source of sound is not moving towards or away from her but remains at the same distance from her.

[*In questions 1-3 the speed of movement is given to two figures and the frequency to four figures. The answers have been rounded to three significant figures for clarity.*]

4 (a) Let v be the speed of sound. The source produces waves of wavelength, $\lambda = \frac{v}{f_s}$

For a source moving away from an observer, the observer will receive waves of a **longer** wavelength, λ_{obs}. $\lambda_{obs} = \frac{v}{f_s} + \frac{v_s}{f_s} = \frac{1}{f_s}(v + v_s)$

but the observed frequency, $f_{obs} = \frac{v}{\lambda_{obs}} = \frac{v}{\frac{1}{f_s}(v + v_s)} = f_s \frac{v}{(v + v_s)}$

$\boxed{f_{obs} = f_s \frac{v}{(v + v_s)}}$ for a source moving **away from** a stationary observer

(b) Source at rest relative to a moving observer
Consider in this case the number of waves received in a given time, t.

The number of waves produced by the source in time t $= f_s \times t = \frac{v}{\lambda} \times t$

If the observer is moving **towards** the source at speed v_o, an **extra** $\frac{v_o}{\lambda}$ x t waves will be heard in that time.

Thus total number of waves received in time $t = \frac{v}{\lambda}t + \frac{v_o}{\lambda}t$

Number of waves per second, $f_{obs} = \frac{\frac{v}{\lambda}t + \frac{v_o}{\lambda}t}{t} = \frac{v + v_s}{\lambda} = f_s \frac{v + v_o}{v}$.

Thus $\boxed{f_{obs} = f_s \frac{v + v_o}{v}}$ for an observer moving **towards** a stationary source.

5 $f_{obs} = f_s \frac{v}{(v - v_s)}$ for a source moving **towards** a stationary observer

$$1600 = 1100 \times \frac{v}{(v - 0.26)}$$

thus $1600v - 1600 \times 0.26 = 1100v$

$(1600 - 1100)\, v = 1600 \times 0.26$

$v = \frac{1600 \times 0.26}{500} = 0.83 \text{ m s}^{-1}$ Speed of sound on Hsdu is $\underline{\underline{0.83}}$ m s^{-1}

6 Source moving towards the observer $f_{obs} = f_s \frac{v}{(v - v_s)}$

$540 = 500 \frac{340}{340 - v_s}$ and $540\,(340 - v_s) = 500 \times 340$ giving $v_s = 25.2 \text{ m s}^{-1}$

Source moves away from the observer $f_{obs} = f_s \frac{v}{(v + v_s)}$

$f_{obs} = 500 \frac{340}{340 + 25.2} = \underline{\underline{465}}$ Hz

TUTORIAL 3

Interference - division of amplitude

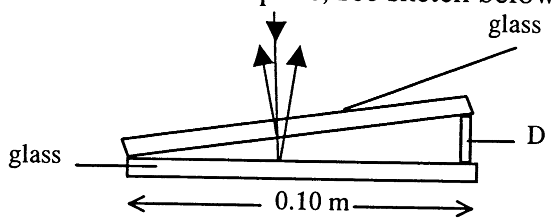
1 (a) Coherent sources must have a constant phase relationship. The two or more sources will come from the same original source.

(b) When we try to produce an interference pattern from two separate light sources it does not work because light is produced in small wave packets and not as a continuous wave. This is not the case for sound waves. We can have two separate loudspeakers, connected to the same signal generator, and produce an interference pattern.

2 (a) Division of amplitude involves splitting a single beam into two beams by producing a reflected beam and a transmitted beam at a surface between two materials of different refractive index. They may be multiple reflections and transmissions.

(b) An extended beam of light can be used because the beam is sub-divided by reflection and transmission at a surface. Hence there will always be a fixed phase relation between the sub-divided parts.

3 (a) Fringes are formed when reflections from the bottom surface of the top glass plate interfere with the beam transmitted through the top plate and reflected from the top surface of the bottom plate, see sketch below.



(b) $\tan\theta = \dfrac{\lambda}{2\Delta x}$ where λ = wavelength and Δx = fringe spacing

$= \dfrac{6.9 \times 10^{-7}}{2 \times 1.2 \times 10^{-3}}$

from sketch $D = \tan\theta \times 0.10 = \dfrac{6.9 \times 10^{-7}}{2 \times 1.2 \times 10^{-3}} \times 0.10$ D = thickness of foil

$= \underline{\underline{2.9 \times 10^{-5}}}$ m

(c) new value of $D = 2.9 \times 10^{-5} \times 1.1 = 3.2 \times 10^{-5}$ m i.e. 10% bigger

new $\tan\theta = \dfrac{3.2 \times 10^{-5}}{0.10}$

new $\Delta x = \dfrac{\lambda}{2\tan\theta} = \dfrac{6.9 \times 10^{-7} \times 0.10}{2 \times 3.2 \times 10^{-5}}$

$= \underline{\underline{1.1 \times 10^{-3}}}$ m

4 (a) For cancellation of reflected light: optical path in fluoride = $\frac{\lambda}{2}$

thus $2nd = \frac{\lambda}{2}$ where n = refractive index and d is the thickness of the film

$$d = \frac{\lambda}{4n}$$

(b) $d = \frac{\lambda}{4n} \quad \frac{4.8 \times 10^{-7}}{4 \times 1.25} = \underline{\underline{9.6 \times 10^{-8}}}\ \text{m}$

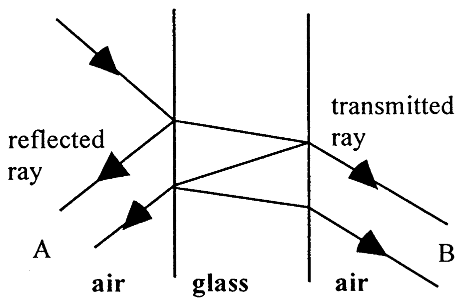
5 $d = \frac{\lambda}{4n} = \frac{6.7 \times 10^{-7}}{4 \times 1.3} = \underline{\underline{1.3 \times 10^{-7}}}\ \text{m}$

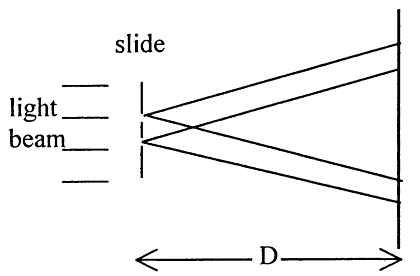
6 $d = \frac{\lambda}{4n} \quad \frac{6.2 \times 10^{-7}}{4 \times 1.3} = \underline{\underline{1.2 \times 10^{-7}}}\ \text{m}$

TUTORIAL 4
Interference - division of wavefront

1 An example of **division of amplitude** involves splitting a single beam into two beams by producing a reflected beam and a transmitted beam at thin parallel sided film. The reflected rays at A will give interference. Similarly the transmitted rays at B will give interference. In both cases the rays can be brought to a focus with the eye.

An example of **division of wavefront** involves a single source incident on a double slit producing two secondary sources. The two secondary beams will give interference fringes on a screen, see sketch below which is not to scale. The distance between the slits must be very small and the distance D is a few metres.





An extended source of light can be used. Any transparent boundary between two media of different refractive index can produce division of amplitude.

The source of light must be a point or line source. The slits size must be the order of the wavelength to act as secondary sources.

2 (a) The two narrow slits act as coherent sources by division of wavefront. When light from each of the sources meet in phase areas of constructive interference are produced, giving a bright fringe. When the light from the two sources are completely out of phase there is destructive interference and almost darkness.

(b) White fringes have coloured edges because white light is composed of the colours of the spectrum. The position of the nth interference fringe is given by: $x_n = \dfrac{n\lambda D}{d}$

Notice that the position of a fringe is dependent on the wavelength, λ; as λ increases x_n increases. Thus red will be deviated most and violet least. These two colours will appear at the edges of the white fringes.

3 (a) $\Delta x = \dfrac{\lambda D}{d}$ $\qquad \Delta x = \dfrac{695 \times 10^{-9} \times 0.92}{2.0 \times 10^{-4}}$

$\Delta x = 3.2 \times 10^{-3}$ m $= \underline{\underline{3.2}}$ mm

(b) The new double slit is half the size. From the relationship in (a) above the fringe separation and the slit separation are inversely proportional. This means that decreasing d will increase x. The pattern will spread out and the fringes will be further apart. In this case the spacing of the fringes will double to 6.4 mm.

4 $\Delta x = \frac{\lambda D}{d}$ thus $\lambda = \frac{\Delta x d}{D}$

$$\lambda = \frac{8 \times 10^{-3} \times 5.0 \times 10^{-4}}{7.2}$$

$$= 5.56 \times 10^{-7} = \underline{\underline{556}} \text{ nm}$$

5 (a) Two coherent sources are produced by the double slit. Interference takes place between these two sources and red and black lines called fringes will be seen on the screen. With a red filter only red fringes are seen because all of the other wavelengths, except red, are absorbed by the filter.
A bright red line is formed by constructive interference. The path difference is $n\lambda$, the waves arrive in phase, there is a larger amplitude and more energy.
A dark line is formed by destructive interference. The path difference is $(n + \frac{1}{2})\lambda$, the waves are completely out of phase, the amplitude is zero and there is no energy.

(b) The blue fringes will be closer together because the fringe separation Δx is proportional to the wavelength λ and $\lambda_{blue} < \lambda_{red}$.

(c) White fringes have coloured edges because white light is composed of the colours of the spectrum. The position of the nth interference fringe is given by: $x_n = \frac{n\lambda D}{d}$
Notice that this is dependent on λ the wavelength. Thus red will be deviated most and violet least. These two colours will appear at the edges of the white fringes.

(d) $\lambda = \frac{\Delta x d}{D}$

$$= \frac{5 \times 10^{-3} \times 0.25 \times 10^{-3}}{2.0}$$

$$= 6.25 \times 10^{-7} = \underline{\underline{625}} \text{ nm}$$

6 (a) (i) $\Delta x = \frac{\lambda D}{d}$

Δx is proportional to D. Thus making D smaller will also reduce Δx.

(ii) Δx is inversely proportional to d. Thus decreasing d will increase Δx, making the fringes further apart.

(b) (i) Covering one of the slits will cause the interference patterns to disappear. Two sources are needed to produce interference in division of wavefront.

(ii) $\Delta x \propto \lambda$
Thus a longer wavelength will produce a larger Δx and the fringes will be further apart.

(iii) White light fringes will be seen. Depending on the values of λ, D and d the fringes may have red and violet coloured edges, see answer to 5 (c).

(c) $$\lambda = \frac{\Delta x d}{D}$$
$$= \frac{10 \times 10^{-3} \times 0.5 \times 10^{-3}}{8.0}$$
$$= 6.25 \times 10^{-7} = \underline{\underline{625}} \text{ nm}$$

(d) The fringe separation would have the greatest percentage uncertainty because this is likely to be measured using a metre stick giving (10 ± 1) mm which is 10%. The slit separation can be measured to a higher accuracy with a travelling microscope. The distance to the screen is much larger than the fringe separation so the percentage uncertainty for this will be much less, namely 8 m ± 1 mm which is 0.01%

7 (a) (i) $$\lambda = \frac{\Delta x d}{D} = \frac{7 \times 10^{-3} \times 0.2 \times 10^{-3}}{2.4}$$
$$= 5.8 \times 10^{-7} = \underline{\underline{580}} \text{ nm}$$

(ii) uncertainty in $\Delta x = \frac{1}{7} \times 100 = 14\ \%$

uncertainty in d $= \frac{0.01}{0.20} \times 100 = 5\ \%$

uncertainty in D $= \frac{0.10}{2.4} \times 100 = 4\ \%$

The uncertainty in D can be neglected, since it is less than $\frac{1}{3}$ of 14 %.

Total % uncertainty $= \sqrt{14^2 + 5^2} = 14.9\ \%$

Thus uncertainty in the wavelength is $\underline{\underline{86}}$ nm and $\lambda = (\underline{5.8} \pm \underline{0.9}) \times 10^{-7}$ m

(b) (i) Place the double slit slide on the stage of a travelling microscope. Focus the cross hairs in the objective of the microscope on the edge of one of the slits ruled on the slide. Sometimes illumination can be a problem, so try putting a low voltage bulb beneath the microscope stage below the position of the slide. The edge of the slit rulings may now be visible.
Read this position on the vernier scale. Now rack the microscope along the frame until the crosshairs are at the point which gives the slit separation. Read the new position on the vernier scale. To avoid backlash in the mechanism, do not rack the microscope back and forth; i.e. only move it in one direction when taking both readings.

(ii) This could be improved by counting a number of fringes and dividing by the number rather than simply measuring the separation of adjacent fringes.

(c) (i) Reducing d increases the fringe separation Δx because $\Delta x \propto \frac{1}{d}$.

(ii) Blue light has a shorter wavelength than yellow light. The blue fringes will therefore be closer together than the yellow fringes because $\Delta x \propto \lambda$.

(iii) Covering one of the slits will cause the interference patterns to disappear. Two coherent sources are needed to produce interference.

TUTORIAL 5

Polarisation

1 (a)

$$n = \tan i_p$$

thus

$$\tan i_p = 1.52$$

$$i_p = \underline{\underline{57^\circ}}$$

$$r = 90 - i_p$$

$$= \underline{\underline{33^\circ}}$$

(b) Hold a polaroid filter, an analyser, at the appropriate angle of 57° and rotate it. At some point in the rotation no light will be transmitted through the analyser.

2 (a)

$$r = 90 - i_p$$

$$= \underline{\underline{38^\circ}}$$

(b)

$$n = \frac{\sin i}{\sin r} = \frac{\sin 52^\circ}{\sin 38^\circ}$$

$$= \underline{\underline{1.28}}$$

3 (a)

$$n = \tan i_p$$

red - 650 nm

$$i_p = \tan^{-1} 1.52$$

$$= 56.7^\circ$$

green - 510 nm

$$i_p = \tan^{-1} 1.53$$

$$= 56.8^\circ$$

violet - 400 nm

$$i_p = \tan^{-1} 1.54$$

$$= 57^\circ$$

(b)

$$r = 90^\circ - i_p$$

maximum r:

$$= 90^\circ - 56.7^\circ$$

$$= 33.3^\circ$$

4 (a)

$$f = \frac{v}{\lambda} = \frac{3 \times 10^8}{0.028} = \underline{\underline{1.1 \times 10^{10}}}\text{ Hz}$$

(b) The microwave emitter gives out plane polarised waves. The grid in this arrangement acts as an analyser. The metal grid only transmits when the oscillations of the electric field strength vector are perpendicular to the metal rods. Since the initial reading is low, this suggests that the emitted microwaves are plane polarised with their electric vector in the vertical plane.

As the grid is rotated to the horizontal plane, the reading on the receiver will increase because the microwaves will be transmitted. When the grid is rotated further so that it is upside down, to its initial direction, the reading will be a minimum again. There will be a further maximum when the grid is again horizontal.

(c) The venetian blind is acting like the grid in (a) above. Some of the TV waves are being absorbed by the metal slats of the blind.

5 $$n = \frac{1}{\sin c} = \frac{1}{\sin 38^\circ} = 1.62$$

$$i_p = \tan^{-1} 1.62 = \underline{\underline{58^\circ}}$$

6. (a) When light is incident at a boundary between air and an electrical insulator, the polarising angle i_p is the angle of incidence in air which causes the reflected light to be linearly polarised.

(b) Brewster's angle

(c) Assumption: at the polarising angle the refracted and reflected rays are separated by 90°

$$n = \frac{\sin i_p}{\sin r} \qquad \text{and } r = 90 - i_p$$

$$n = \frac{\sin i_p}{\cos i_p} \qquad \text{since } \sin(90 - i_p) = \cos i_p$$

thus $n = \tan i_p$

(d) (i) $r = 90 - i_p = \underline{\underline{34^\circ}}$

(ii) $n = \dfrac{\sin i}{\sin r} = \underline{\underline{1.48}}$

7 (a) (i)

$$n = \tan i_p$$

$$i_p = \tan^{-1} 1.33$$

$$= \underline{\underline{53^\circ}}$$

(ii) angle of refraction $= 90^\circ - 53^\circ = \underline{\underline{37^\circ}}$

(b) (i) The path of the light is reversed for the same polarising condition:

$$i_p = \underline{\underline{37^\circ}}$$

Alternatively $${}_{water}n_{air} = \frac{1}{{}_{air}n_{water}} = \frac{1}{1.33}$$

$$i_p = \tan^{-1} \frac{1}{1.33} = 37^\circ$$

(ii) The refracted ray in air will be $\underline{\underline{53^\circ}}$

MECHANICS UNIT

KINEMATIC RELATIONSHIPS AND RELATIVISTIC MOTION

Calculus notation	$v = \frac{ds}{dt}$; $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
	derive $v = u + at$; $v^2 = u^2 + 2as$; $s = ut + \frac{1}{2}at^2$
Rest Mass (m_o)	The mass of an object which is at rest relative to an observer. (The mass of an object increases with its velocity).
Relativistic Mass (m)	The mass of an object which is travelling at a velocity comparable to the velocity of light.
	$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$ [*equation will be given*]
Relativistic Energy	$E = mc^2$

ANGULAR MOTION

Angular Displacement (θ)	measured in radians. (2π radians = 360°)
Angular Velocity (ω)	$\omega = \frac{d\theta}{dt}$ (rad s^{-1})
Angular Acceleration (α)	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ (rad s^{-2})

Equations of Motion

CIRCULAR MOTION [**no** derivations required]	***LINEAR MOTION*** [derivations required]
$\omega = \omega_o + \alpha t$	$v = u + at$
$\theta = \omega_o t + \frac{1}{2}\alpha t^2$	$s = ut + \frac{1}{2}at^2$
$\omega^2 = \omega_o^2 + 2\alpha\theta$	$v^2 = u^2 + 2as$

$v = r\omega$ [*derivation required*]

$a = r\alpha$ [***no** derivation required*]

Central Force	The force required to maintain a particle in circular motion.
Central acceleration	$a = \frac{v^2}{r}$ and $a = r\omega^2$
	[*derivation required*]
Central Force equations	$F = \frac{mv^2}{r}$ and $F = m\omega^2 r$

ROTATIONAL DYNAMICS

Moment of a Force	The magnitude of the moment of a force (or the turning effect) is force x perpendicular distance
Torque (T)	$T = F \times r$ where r is the perpendicular distance from the force to the axis of rotation
Moment of Inertia (I)	The moment of inertia depends on the mass and the distribution of the mass about a fixed axis. $I = m r^2$ mass m at distance r from axis of rotation [$I = \Sigma m r^2$ (Σ is the 'sum of') *equation not required*]
Torque (T)	$T = I \alpha$
Angular Momentum(L)	$L = I \omega$ (for a rigid body) $L = mr^2\omega = mrv$ (for a particle)
Rotational Kinetic Energy	$E_{rot} = \frac{1}{2} I \omega^2$ (for a rigid body)

GRAVITATION

Law of Gravitation	$F = \dfrac{G m_1 m_2}{r^2}$
Gravitational Potential	$V = -\dfrac{Gm}{r}$ (zero of V is at infinity)
Conservative Field	The gravitational field is an example of a conservative field where the total work done moving a mass around any closed path is zero.
Equipotentials	Lines joining points of equal gravitational potential.
Escape Velocity	The velocity a projectile must have in order to escape from a planet's gravitational field. $v_{esc} = \sqrt{\dfrac{2G\,M}{r}}$ [*derivation required*]
Black Hole	A body with a sufficiently high density to make the escape velocity greater than c, the speed of light.

SIMPLE HARMONIC MOTION

SHM	The unbalanced force, or acceleration, is proportional to the displacement of the object and acts in the opposite direction.
SHM Equation	$\frac{d^2y}{dt^2} = -\omega^2 y$ and $\omega = \frac{2\pi}{T}$
SHM Solutions	$y = a \sin \omega t$ if $y = 0$ at $t = 0$ $y = a \cos \omega t$ if $y = a$ at $t = 0$
Velocity $(\frac{dy}{dt})$	$v = \pm \omega\sqrt{a^2 - y^2}$ a = amplitude of motion. $v_{max} = \pm \omega a$ and occurs at the centre of the motion, $v_{min} = 0$ at extremes.
Acceleration $(\frac{d^2y}{dt^2})$	$acc = -\omega^2 y$ $acc_{max} = -\omega^2 a$ and occurs at y = a. $acc_{min} = 0$ at centre.
Energy	$E_k = \frac{1}{2} m \omega^2 (a^2 - y^2)$ [*derivation required*] $E_p = \frac{1}{2} m \omega^2 y^2$ [*derivation required*] $E_{tot} = E_k + E_p = \frac{1}{2} m \omega^2 a^2$
Damping	Damping causes the amplitude of the oscillation to decay.

WAVE PARTICLE DUALITY

Particles as Waves	Particles such as electrons can exhibit wave properties, such as diffraction.
de Broglie Wavelength	$\lambda = \frac{h}{p}$ (h is the Planck constant and p is momentum)
The Bohr Model of the Atom	The electrons occupy only certain allowed orbits. Angular momentum is quantised. Radiation is emitted when electrons move from higher energy levels to lower energy levels.
Quantisation of Angular Momentum	$mvr = \frac{nh}{2\pi}$
Quantum Mechanics and Probability	Quantum mechanics provides methods to determine probabilities.

ELECTRICAL PHENOMENA UNIT

ELECTRIC FIELDS

Coulomb's Inverse Square Law

$F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$ or $\left[\dfrac{1}{4\pi\varepsilon_o}\right] \cdot \dfrac{Q_1 Q_2}{r^2}$

(ε_o is the permittivity of free space)

Electric Field Strength (E)

Force on one coulomb of positive charge at that point.

$E = \dfrac{F}{Q}$

Electric Field Strength for a uniform electric field

$E = \dfrac{V}{d}$ [*derivation required*]

Electric Field Strength for a point charge

$E = \dfrac{Q}{4\pi\varepsilon_o r^2}$ or $\left[\dfrac{1}{4\pi\varepsilon_o}\right] \cdot \dfrac{Q}{r^2}$

[*no derivation required*]

Charging by Induction

Conducting objects can be charged by separating the positive and negative charges on the objects and then removing one set of charges by earthing.

Conducting Shapes

When a conducting shape is in an electric field the induced charge stays on its surface and the electric field inside the conducting shape is zero.

Electrostatic Potential

Work done by an external force to bring one coulomb of positive charge from infinity to that point.

$V = \dfrac{Q}{4\pi\varepsilon_o r}$ or $\left[\dfrac{1}{4\pi\varepsilon_o}\right] \cdot \dfrac{Q}{r}$

[*no derivation required*]

Charged Particles in uniform electric fields
- non relativistic

$\dfrac{1}{2} mv^2 = QV$ (kinetic energy to electrical energy)

Charged Particles in uniform electric fields
- relativistic case

Relativistic effects must be considered when the velocity of the charged particle is more than 10% of the velocity of light.

[*no relativistic calculations required*]

Particle head-on collisions

Change in E_k = change in E_p

$\dfrac{1}{2} mv^2 = \dfrac{qQ}{4\pi\varepsilon_o} \cdot \dfrac{1}{r}$

where r is closest distance of approach

Millikan's Experiment

Quantisation of charge.

$E q = mg$ (neglecting upthrust)

ELECTROMAGNETISM

Tesla	The tesla is the magnetic induction of a magnetic field in which a conductor of length one metre, carrying a current of one ampere perpendicular to the field is acted on by a force of one newton
Magnetic Induction (B)	$F = IlB \sin \theta$ (θ is the angle between B and l) The direction of F is perpendicular to the plane containing B and I.
The Magnetic Induction around an 'infinite', straight conductor	$B = \frac{\mu_o I}{2\pi r}$ (μ_o is the permeability of free space) (r is the perpendicular distance from conductor)
Force between parallel conductors	$\frac{F}{l} = \mu_o \frac{I_1 I_2}{2 \pi r}$ [*derivation required*]

MOTION IN A MAGNETIC FIELD

Force on charge q, speed v, in field B:	$F = q v B \sin\theta$ (θ is the angle between v and B) **The direction of F is perpendicular to the plane containing v and B.**
Helical path	This is the spiral path followed by a charge when its velocity makes an angle θ with the direction of B. $v \sin \theta$ is the component perpendicular to the direction B, while $v \cos \theta$ is the component parallel to the direction of B.
J.J. Thomson	Measured the charge to mass ratio of the electron by using electric and magnetic deflection of an electron beam.
'Crossed' fields	Electric and magnetic fields are applied at right angles to each other. Charged particles of certain speeds will pass through undeviated - velocity selector: $v = \frac{E}{B}$

SELF-INDUCTANCE

Growth and Decay of current	The current takes time to grow and decay in a d.c. circuit containing an inductor
Self-Induction	An e.m.f. is induced across a coil when the current in the coil changes.
Self Inductance (L)	$e = - L \frac{dI}{dt}$ (L is the self inductance of the coil)
Henry	The inductance of an inductor is one henry if an e.m.f. of one volt is induced when the current changes at a rate of one ampere per second.
Direction of induced e.m.f.	The direction of the induced e.m.f. is such that it opposes the change of current. This is known as Lenz's Law. The negative sign in the above equation indicates this opposing direction.
Energy stored	The work done in building up the current in an inductor is stored in the magnetic field of the inductor. The magnetic field can be a source of energy when the magnetic field is allowed to collapse.
Energy equation	$E = \frac{1}{2} L I^2$ (energy E stored in inductor L)
Current and frequency in an inductive circuit	Current is inversely proportional to the frequency in an inductive circuit.
Reactance	The opposition to flow of an alternating current is called reactance.
C and L in a.c. circuits	For an inductor the reactance increases as the frequency of the a.c. increases. Conversely the reactance of a capacitor decreases as the frequency of the a.c. increases.
Uses	Inductors can be used to block a.c. signals while allowing d.c signals to pass. Capacitors can block d.c signals, but allow high frequency a.c. signals to pass. Inductors can be used to generate a high voltage when the magnetic field is allowed to collapse suddenly.

FORCES OF NATURE

Strong Force	The force of attraction between nucleons in a nucleus, with a very short range < 1 x 10^{-14} m.
Weak Force	This is the force associated with β-decay.
Quarks	Neutrons and protons are made up of quarks.

WAVE PHENOMENA SUMMARY

WAVES

Wave motion	Energy is transferred with no net mass transport.
Travelling Wave	The displacement, y, of any point on a travelling wave in the positive x direction is given by: $y = a \sin 2\pi (ft - \frac{x}{\lambda})$ [*explain not derive*]
Intensity of a wave	Intensity is directly proportional to $(\text{amplitude})^2$.
Superposition	The displacement at a point, due to two or more waves, is the algebraic sum of the individual displacements.
Phase Difference	For two points separated by distance x, the phase difference is $\phi = 2\pi \frac{x}{\lambda}$ (ϕ is the phase angle)
Stationary Wave	This wave is produced by the interference of two identical waves travelling in opposite directions
Nodes	These are points of zero displacement on a stationary wave separated by a distance of $\frac{\lambda}{2}$.
Antinodes	These are points of maximum displacement on a stationary wave, also separated by $\frac{\lambda}{2}$.
Doppler Effect	This is the change in frequency which is observed when a source of sound waves moves relative to a stationary observer.

Apparent frequency when source of sound moves	$\boxed{f_{obs} = f_s \frac{v}{(v - v_s)}}$	source moving **towards** stationary observer
	$\boxed{f_{obs} = f_s \frac{v}{(v + v_s)}}$	source moving **away from** stationary observer
Apparent frequency when observer moves	$\boxed{f_{obs} = f_s \frac{v + v_o}{v}}$	observer moving **towards** stationary source
	$\boxed{f_{obs} = f_s \frac{v - v_o}{v}}$	observer moving **away from** stationary source

[*derivation of the above expressions for f_{obs} required*]

INTERFERENCE – DIVISION OF AMPLITUDE

Coherent Sources of light	Coherent sources must have a **constant phase difference**.
Optical path length	Optical path length = n x geometrical path length
Optical path difference	For *optical* path lengths S_1P and S_2P: $(S_2P - S_1P) = m\lambda$ for constructive interference $(S_2P - S_1P) = (m + \frac{1}{2})\lambda$ for destructive interference
Phase difference and optical path length	phase difference = $\frac{2\pi}{\lambda}$ x optical path length
Phase change on reflection	When light reflects off an optically more dense medium a phase change of π occurs.
Thin Film	**Destructive** interference: $2nt \cos r = m\lambda$ For viewing at near normal incidence $2nt = m\lambda$ [*derivation required*]
Wedge Fringes	At normal incidence, fringe separation Δx is $\Delta x = \frac{\lambda}{2 \tan\theta} = \frac{\lambda L}{2D}$ [*derivation required*] (D is the wedge separation, and L is the wedge length)
Non-Reflective Coatings	Thickness of coating, $d = \frac{\lambda}{4n}$ [*derivation required*]

INTERFERENCE – DIVISION OF WAVEFRONT

Point or line source	Explain why division of wavefront requires a point or line source. Describe why division of amplitude can use an extended source.
Young's Slits	Fringe separation $\Delta x = \frac{\lambda D}{d}$ [*derivation required*]

POLARISATION

Plane Polarised Light	Linearly polarised light waves consist of vibrations of the electric field strength vector in one plane only.
Polarisers and Analysers	A polariser and analyser held so that their planes of polarisation are at right angles can prevent the transmission of light.
Brewster's angle	At the polarising angle i_p, known as Brewster's angle, the refracted and reflected rays are separated by 90°.
Brewster's law	$n = \tan i_p$ [*derivation required*]