Physics
Student Material
Advanced Higher

5959

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HIGHER STILL

Physics

Student Material
Advanced Higher

Support Materials

CONTENTS

PART 1

This pack contains the following material:

- Staff Notes
- Checklists (for all three units)
- Uncertainties - Student Material
- Mechanics - Student Material

PART 2

This pack will contain:

- Electrical Phenomena - Student Material
- Wave Phenomena - Student Material
- Summary
- Solutions to Tutorial questions

IMPORTANT NOTICE
Part 2 will be published in due course

ADVANCED HIGHER PHYSICS - STAFF NOTES

Advanced Higher Physics course and units

The Advanced Higher Physics course is divided into the following four units.

- Mechanics (40 hours)
- Electrical Phenomena (40 hours)
- Wave Phenomena (20 hours)
- Physics Investigation (20 hours)

The Support Materials

This student support material covers the first three of the above four units. Separate support material is available for the Physics Investigation unit.

For each of these three units the student material contains the following sections:

- Summary Notes
- Tutorials
- Activities

A checklist for the three units is included at the beginning of this pack.

In addition there is a separate section dealing with uncertainties, which is placed after the checklist. It should be noted that the Content Statements associated with uncertainties are part of *each* of the three units, see the Arrangements for Physics.

Full worked solutions to all the questions in the tutorials will be published.

The student materials are to provide assistance to the teacher or lecturer delivering a unit or the course. They require to be supplemented by learning and teaching strategies. This is to ensure that all of the unit or course content is covered and that the students are given the support they need to acquire the necessary knowledge, understanding and skills demanded by the unit or the course. However these notes are somewhat fuller than those supplied for units of courses at other levels.

Checklists

These are lists of the content statements taken directly from the Arrangements in Physics documentation, published by SQA.

Summary Notes

These notes are a summary of all the essential content and include a few basic worked examples. They are intended to aid students in their revision for unit and course assessment. Further explanation of the concepts and discussion of applications are for the teacher or lecturer to include as appropriate. In some cases details of applications have been included for interest.

Tutorials

A variety of problems have been collated to give the student opportunity for practice and to aid the understanding of the unit or course content.

Activities

The activity pages provide suggestions for experimental work. The instruction sheet can be adapted to suit the equipment available in the centre. Some activities are more suitable for teacher/lecturer demonstration and these can be used where appropriate.

For Outcome 3 of each unit one report of a practical activity is required. Some activities suitable for the achievement of Outcome 3 have been highlighted and should be seen as an opportunity to develop good practice.

Use of the materials

The checklists may be issued at the end or at the beginning of a unit depending on the discretion of the teacher/lecturer. The material for each unit is numbered consecutively through the summary notes, tutorials and activities.

The uncertainties section is numbered separately. Some staff may wish to cover this material at the start of the course, others may prefer to introduce the concepts more gradually during early experimental work. (For unit assessment, uncertainties are covered in Outcome 3. The course assessment will contain questions which will sample uncertainties within the context of any of the units.)

Learning and Teaching

A variety of teaching methods can be used. Direct teaching whether it be to a whole class or small groups is an essential part of the learning process. A good introduction to a topic; for example, a demonstration, activity or video, is always of benefit. This can capture the minds of the students and generate interest in the topic. Applications should be mentioned and included wherever possible.

Further materials

- A staff guide covering the content of the three units is being published by SCCC.
- An Uncertainties Memorandum is to be published by SCCC.
- A Physics Investigation: Staff Guide is to be published. This contains advice on both the unit and course requirements for the investigation.

Outcome 3

The Handbook: Assessing Outcome 3 - Advanced Higher Physics contains specific advice for this outcome together with exemplar instruction sheets and sample student reports.

SQA

A specimen course question paper together with marking scheme is issued by SQA. A NAB pack for each unit is issued by SQA. The NAB for the Physics Investigation unit contains Advice to Candidates which covers both the course and unit requirements.

MECHANICS

The Content Statements for this unit are given below.

Kinematic relationships and relativistic motion

1. Derive from $a = \frac{dv}{dt}$ i.e. $a = \frac{d^2s}{dt^2}$ the kinematic relationships:
   $v = u + at$, $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$ where a is a constant acceleration.
2. Carry out calculations involving constant accelerations.
3. State that the greatest possible speed of any object is that of light in vacuo.
4. State that the relativistic mass m of a moving object is not constant, but increases with its speed.
5. Carry out calculations involving mass and speed, given the formula:

   $$m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$ where m_o is the rest mass.
6. State that the relativistic energy E of an object is mc^2.

Angular motion

1. State that angular velocity ω is the rate of change of angular displacement, $\omega = \frac{d\theta}{dt}$.
2. State that angular acceleration $\alpha = \frac{d\omega}{dt}$ i.e. $\alpha = \frac{d^2\theta}{dt^2}$.
3. State the following relationships:
   • $\omega = \omega_0 + \alpha t$; $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$; $\omega^2 = \omega_0^2 + 2\alpha\theta$ where α is a constant angular acceleration.
4. Carry out calculations involving the relationships in 3 above.
5. State and derive the expression $v = r\omega$ for a particle in circular motion.
6. State that $a = r\alpha$
7. Explain that a central force is required to maintain circular motion.
8. State that the central force required depends on mass, speed and radius of rotation.
9. State and derive the expressions $\frac{v^2}{r}$ or $r\omega^2$ for the radial acceleration.
10. Carry out calculations using $F = \frac{mv^2}{r} = mr\omega^2$.

Rotational dynamics

1. State what is meant by the moment of a force.
2. State that the torque $T = Fr$.
3. State that an unbalanced torque produces an angular acceleration.
4. State that the angular acceleration produced by an unbalanced torque depends on the moment of inertia of the body.
5. Explain that the moment of inertia of a body depends on the mass of the body and the distribution of the mass about a fixed axis.
6. State that the moment of inertia of a mass m at a distance r from a fixed axis is mr^2.
7. State the relationship $T = I\alpha$.
8. Carry out calculations using $T = I\alpha$, given I where required.
9. State that the angular momentum of a rigid body is $I\omega$.
10. State that in the absence of external torques, the angular momentum of a rotating rigid body is conserved.
11. State the expression $E_{rot} = \frac{1}{2}I\omega^2$ for the rotational kinetic energy of a rigid body.
12. Carry out calculations using the above relationship.

Gravitation

1. State the inverse square law of gravitation $F = \frac{Gm_1m_2}{r^2}$

where G is the gravitational constant.

2. Carry out calculations using the relationship in 1.
3. Define gravitational field strength.
4. Sketch gravitational field lines for an isolated point mass and for two point masses.
5. State that the gravitational potential at a point in a gravitational field is the work done by external forces in bringing unit mass from infinity to that point.
6. State that the zero of gravitational potential is taken to be at infinity.
7. State the expression for the gravitational potential $-\frac{Gm}{r}$ at a distance r from a mass m.
8. Carry out calculations involving the gravitational potential energy of a mass in a gravitational field.
9. Explain what is meant by a conservative field.
10. State that a gravitational field is a conservative field.
11. Explain the term 'escape velocity'.
12. Derive the expression $v = \sqrt{\frac{2GM}{r}}$ for the escape velocity.
13. State that the motion of photons is affected by gravitational fields.
14. State that, within a certain distance from a sufficiently dense body, the escape velocity is greater than c, hence nothing can escape from such a body – a black hole.
15. Carry out calculations involving orbital speed, period of rotation and radius of orbit of satellites.

Simple harmonic motion

1. Describe examples of simple harmonic motion (SHM).
2. State that in SHM the unbalanced force is proportional to the displacement of the body and acts in the opposite direction.
3. State and explain the equation $\frac{d^2y}{dt^2} = -\omega^2 y$ for SHM.
4. Show that $y = a\cos\omega t$ and $y = a\sin\omega t$ are solutions of the equation for SHM.
5. Show that $v = \pm\,\omega\sqrt{(a^2 - y^2)}$ for the relationships in 4
6. Derive the expressions $\frac{1}{2}m\omega^2(a^2 - y^2)$ and $\frac{1}{2}m\omega^2y^2$ for the kinetic and potential energies of a particle executing SHM.
7. State that damping on an oscillatory system causes the amplitude of oscillation to decay.

Wave–particle duality

1. State that electrons can behave like waves.
2. Describe evidence which shows that electrons and electromagnetic radiation exhibit wave–particle duality.
3. State that the wave and particle models are related by the expression:
   $\lambda = h/p$ where p is the associated momentum.
4. State that the wavelength found for a particle using $\lambda = h/p$ is small compared with the dimensions of any physical system (except on the atomic or sub–atomic scale).
5. Carry out calculations using the relationship in 3 above.
6. State that the angular momentum of an electron about the nucleus is quantised.
7. Describe qualitatively the Bohr model of the atom.
8. State that the quantisation of angular momentum is given by $mvr = \frac{nh}{2\pi}$.
9. Carry out calculations using the relationship in 8 above.
10. State that a more far-reaching model of atomic and nuclear structure interprets waves in terms of probabilities.
11. State that quantum mechanics provides methods to determine probabilities.

ELECTRICAL PHENOMENA

The Content Statements for this unit are given below.

Electric fields

1. State Coulomb's inverse square law for the force between two point charges.

i.e. $F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$ where ε_0 is the permittivity of free space.

2. Carry out calculations involving the electrostatic force between point charges.
3. Describe how the concept of an electric field is used to explain the forces that charged particles at rest exert on each other.
4. State that the electric field strength E at any point is the force per unit positive charge placed at that point.
5. State that the expression for the electric field strength E at a distance r from a point charge Q is $E = \frac{Q}{4\pi\varepsilon_0 r^2}$.
6. State that the units of electric field strength are newton per coulomb.
7. Carry out calculations involving the electric fields due to point charges.
8. State and derive the expression $V = Ed$ for a uniform electric field.
9. Carry out calculations using the above relationship.
10. Describe what happens during the process of charging by induction.
11. Describe the effect of placing a conducting shape in an electric field: the induced charge resides on the surface of the conductor, inside the shape E is zero, and outside the shape E is perpendicular to the surface of the conductor.
12. State that the electrostatic potential at a point is the work done by external forces in bringing unit positive charge from infinity to that point.
13. State that the expression for the electrostatic potential V at a distance r from a point charge Q is $V = \frac{Q}{4\pi\varepsilon_0 r}$.
14. Carry out calculations involving potentials due to point charges.
15. Describe the energy transformation associated with the movement of a charge in an electric field.
16. Describe the motion of charged particles in uniform electric fields.
17. Carry out calculations concerning the motion of charged particles in uniform electric fields.
18. State that relativistic effects must be considered when the velocity of a charged particle is more than 10% of the velocity of light.
19. Carry out calculations involving the head-on collision of a charged particle with a fixed nucleus.
20. Explain how the results of Millikan's experiment lead to the idea of quantisation of charge.

Electromagnetism

1. State that a magnetic field exists around a moving charge in addition to its electric field.
2. State that a charged particle moving across a magnetic field experiences a force.
3. Describe how the concept of a magnetic field is used to explain the magnetic force exerted by current-carrying conductors on each other.
4. State that one tesla is the magnetic induction of a magnetic field in which a conductor of length one metre, carrying a current of one ampere perpendicular to the field is acted on by a force of one newton.
5. State the relationship $F = IlB\sin\theta$.
6. Carry out calculations involving the relationship in 5.
7. State the relative directions of current, magnetic field and force for a current-carrying conductor in a magnetic field.
8. State that the magnetic induction at a perpendicular distance r from an 'infinite' straight conductor carrying a current I is $\frac{\mu_0 I}{2\pi r}$ where μ_0 is the permeability of free space.
9. Derive the expression $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$ for the force per unit length between two parallel current carrying wires a distance r apart, using the above relationships

Motion in a magnetic field

1. State and derive the relationship $F = qvB$ for the magnitude of the force acting on a charge q moving with speed v perpendicular to a magnetic field B, using the relationship $F = IlB\sin\theta$.
2. State that if the charge particle's velocity vector is not perpendicular to the field, then the component of v perpendicular to the field $v_{\perp}$ must be used in the above equation.
3. State the relative directions of magnetic field, velocity and force for positive and negative charges.
4. Explain how the helical movement of a charged particle in a magnetic field arises.
5. Carry out calculations on the motion of charged particles moving with non-relativistic velocities in uniform magnetic fields.
6. Describe the principles of J. J. Thomson's method for measuring the charge to mass ratio (specific charge) of the electron

Self-inductance

1. Sketch qualitative graphs of the growth and decay of current in a d.c. circuit containing an inductor.
2. Describe the principles of a method to illustrate the growth of current in a d.c. circuit.
3. State that an e.m.f. is induced across a coil when the current through the coil is varying.
4. Explain the production of the induced e.m.f. across a coil.
5. Explain the direction of the induced e.m.f. in terms of energy.
6. State that the self-induced e.m.f. in a coil is $e = -L\frac{dI}{dt}$, where L is the self-inductance of the coil.
7. State that the inductance of an inductor is one henry if an e.m.f. of one volt is induced when the current changes at a rate of one ampere per second.
8. Explain that the work done in building up the current in an inductor is stored in the magnetic field of the inductor.
9. Explain that the energy stored in the magnetic field of an inductor may be a source of e.m.f.
10. State that the energy stored in an inductor is $\frac{1}{2}LI^2$.
11. Carry out calculations involving the above relationship.
12. Describe the principles of a method to show how the current varies with frequency in an inductive circuit.
13. Describe and explain the possible functions of an inductor-sources of high e.m.f., blocking a.c. signals while transmitting d.c. signals.
14. Compare the dependence on frequency of the capacitive and inductive reactances. (No numerical calculations required.)

Forces of nature

1. State that nuclear particles attract each other with a force called the strong force.
2. State that the strong force has a short range < 10^{-14} m.
3. State that the weak force is associated with beta decay.
4. State that there are a number of 'elementary' particles.
5. State that neutrons and protons can be considered to be composed of quarks.

WAVE PHENOMENA

The Content Statements for this unit are given below.

Waves

1. State that in wave motion energy is transferred with no net mass transport.
2. State that the intensity of a wave is directly proportional to $(\text{amplitude})^2$.
3. State that the sine or cosine variation is the simplest mathematical form of a wave.
4. State that all waveforms can be described by the superposition of sine or cosine waves.
5. Explain that the relationship $y = a \sin 2\pi(ft - \frac{x}{\lambda})$, represents a travelling wave.
6. Carry out calculations on travelling waves using the above relationship.
7. Explain the meaning of phase difference.
8. Explain what is meant by a stationary wave.
9. Define the terms 'node' and 'antinode'.
10. State that the Doppler effect is the change in frequency observed when a source of sound waves is moving relative to an observer.
11. Derive the expression for the apparent frequency detected when a source of sound waves moves relative to a stationary observer.
12. Derive the expression for the apparent frequency detected when an observer moves relative to a stationary source of sound waves.
13. Carry out calculations using the above relationships.

Interference – division of amplitude

1. State in simple terms the condition for two light beams to be coherent.
2. State the reasons why the conditions for coherence are usually more difficult to satisfy for light than for sound and microwaves.
3. Define the term 'optical path difference' and relate it to phase difference.
4. State what is meant by the principle of interference by division of amplitude.
5. Describe how the division of amplitude enables an extended source to be used.
6. State that there is a phase change of π on reflection at an interface where there is an increase in optical density and that there is no change in phase on reflection at an interface where there is a decrease in optical density.
7. Derive the expressions for maxima and minima in the fringes, formed by reflection and transmission of monochromatic light or microwaves in a 'thin film'.
8. Carry out calculations using the above expressions.
9. Derive the expression for the distance between the fringes which are formed by reflection of light at normal incidence from a 'thin wedge'.
10. Carry out calculations using the above expression.
11. Explain how lenses are made non-reflecting for a specific wavelength of light.
12. Derive the expression $d = \lambda/4n$ for the thickness of a non-reflecting coating.
13. Carry out calculations using the above expression.
14. Explain why coated (bloomed) lenses have a coloured hue when viewed in reflected light.
15. Explain the formation of coloured fringes in a thin film illuminated by white light

Interference – division of wavefront

1. State what is meant by the principle of interference by division of a wavefront.
2. Explain why the principle of division of a wavefront requires the use of a 'point' or 'line' source.
3. Derive the expression $\Delta x = \lambda D/d$ for the fringe spacing in the Young's slit experiment for $\Delta x << D$.
4. Carry out calculations using the above expression.

Polarisation

1. Explain the difference between polarised and unpolarised waves.
2. State that only transverse waves can be polarised.
3. State that light can be linearly polarised using a polaroid filter.
4. Explain how a combination of a ‘polariser’ and ‘analyser’ can prevent the transmission of light.
5. State that light reflected from any electrical insulator may be polarised.
6. Explain what is meant by the polarising angle i_p (Brewster’s angle).
7. Derive the expression $n = \tan i_p$.
8. Carry out calculations using the above expression.
9. Explain how polaroid sunglasses can remove glare.

MECHANICS, ELECTRICAL PHENOMENA AND WAVE PHENOMENA

The Content Statements given below are required for each of the above units.

Units, prefixes and scientific notation

1. Use SI units of all physical quantities appearing in the ‘Content Statements’.
2. Give answers to calculations to an appropriate number of significant figures.
3. Check answers to calculations.
4. Use prefixes (p, n, μ, m, k, M, G).
5. Use scientific notation.

Uncertainties

1. State that all instruments are subject to calibration uncertainty.
2. Express the numerical result of an experiment and its uncertainty, making appropriate use of significant figures.
3. Combine the calibration uncertainty, reading uncertainty, and random uncertainty to obtain the total uncertainty.
4. Calculate the uncertainty in a quantity raised to a power.
5. Calculate the uncertainty in a product or quotient of quantities.
6. Calculate the uncertainty in a sum or difference of quantities.
7. Estimate the uncertainty in the gradient and intercept of a straight-line graph.
8. Represent, in graphical analysis, the uncertainties in readings as error bars for the points on the graph representing the readings.
9. Compare critically the numerical result of one experiment with that of another experiment.

UNCERTAINTIES

Summary of the Basic Theory associated with Uncertainty

It is important to realise that whenever a physical quantity is being measured there will always be a degree of inaccuracy associated with the measurement. Thus, whenever experimental measurements are made these inaccuracies or **uncertainties** should be estimated.

Calibration Uncertainty

All measuring instruments have an associated inaccuracy known as the calibration uncertainty. For instance when a wooden metre stick is used to measure a length in the laboratory it is a fair estimate that the metre length of wood itself will be accurate to within 0.5 mm. The table below gives some typical examples of calibration uncertainties:

Instrument	**Calibration Uncertainty**
Metre Stick (wood)	0.5 mm
Ruler made of Steel	0.1 mm
Digital Meter	0.5% of reading + 1 in last digit

Thus for an ammeter reading (from a digital meter) of 3.54 A the uncertainty will be:

$$(0.5\% \text{ of } 3.54 \text{ A}) + 0.01 = 0.018 + 0.01 = 0.02 + 0.01 = 0.03 \text{ A}$$

Thus final value of current should be quoted as: current = 3.54 ± 0.03 A.

Systematic Effects

As the name suggests, uncertainties can arise because of the system used to gather the information. The measurement of time is a good example of this. If you were using a stopwatch which after much use now runs slow, the uncertainty in its use may in fact be worse than its calibration uncertainty. This effect would only be detected by using an independent instrument to check the stop watch. Similarly if a student consistently measured the oscillation of a pendulum wrongly e.g. started the stopwatch at the wrong point in the first swing, then the period of the pendulum would have a systematic uncertainty. This uncertainty can be detected if several different numbers of swings are timed and T is plotted against $\sqrt{l}$. The graph will not pass through the origin as it should, if the experiment had been carried out properly.

Scale Reading Uncertainty

This value indicates how well an instrument scale can be read.
An estimate of the reading uncertainty for an analogue scale is taken as ± half the smallest scale division. For very widely spaced scales a reasonable estimate should be made. For a digital scale, the reading uncertainty is taken as ± 1 in the least significant digit. This has been mentioned above under calibration uncertainty.

Random Uncertainties

It is always advisable to repeat measurements if it is possible. This allows us to check that nothing has gone wrong in taking the first measurement. We usually find that there is a spread of values for the quantity being measured and the random uncertainty in the measurements can be determined from this spread.

Mean and random uncertainty in the mean

The **mean** of a number, n, of measurements of quantity P is found in the usual way:

$$P_{mean} = \frac{\text{sum of all the measurements}}{\text{number of measurements}} \qquad (P_{mean} = \frac{\sum P_i}{n}).$$

The **approximate random uncertainty** in the mean is found from:

$$\text{uncertainty in P} = \frac{P_{maximum} - P_{minimum}}{n}$$

This method is suitable if we have more than about five readings.

Combining Uncertainties

Addition and Subtraction

When two quantities, A and B, are added (or subtracted) the uncertainty (ΔS) in the sum (or difference) is given by:

$$\Delta S = \sqrt{(\Delta A)^2 + (\Delta B)^2}$$

where ΔA is the uncertainty in A and ΔB is the uncertainty in B.

Thus subtracting two quantities which are nearly the same can result in very high percentage uncertainty.

Multiplication and Division

When two quantities, A and B, are multiplied or divided the **fractional uncertainties** are important. Thus if P = A x B or if $P = \frac{A}{B}$

$$\frac{\Delta P}{P} = \sqrt{[\frac{\Delta A}{A}]^2 + [\frac{\Delta B}{B}]^2}$$

This must also apply to **percentage** uncertainties.

$$\text{\% uncertainty in P} = \sqrt{(\text{\% uncertainty in A})^2 + (\text{\% uncertainty in B})^2}$$

Powers

If $P = A^n$ then: % uncertainty in P = n x % uncertainty in A.

For example, if the % uncertainty in a distance s is 1.5% and our formula involved s^2 then the % uncertainty in s^2 would be 3%.

Graphs

Individual points should include 'error bars' where appropriate. These are used to enable the best straight line or curve to be drawn.

When plotting a straight line graph it is possible to get the uncertainty in the gradient by employing the "centroid" method. This involves finding the maximum and minimum gradients from the scatter of points which make up the graph.

First the centroid is found. This is the mean of all the x co-ordinates and the mean of all the y co-ordinates. The best straight line is drawn through this centroid. A top line is then drawn, parallel to this best line, so that it passes through the point (not its error bar) that lies furthest above the best line. A similar line is drawn below, to give a parallelogram. The gradients of the two diagonals of this parallelogram, the 'worst lines', are then calculated. Let these be m_1 and m_2.

The uncertainty in the gradient is given by: $\Delta m = \dfrac{m_1 - m_2}{2\sqrt{(n-2)}}$

where n is the number of points on the graph (excluding the centroid).

The uncertainty in the intercept is found by noting where the two 'worst lines' cut the y axis. Let these be c_1 and c_2.

The uncertainty in the intercept is given by: $\Delta c = \dfrac{c_1 - c_2}{2\sqrt{(n-2)}}$

Dominant Uncertainty in an Expression

Consider three quantities multiplied together in an expression. If one quantity has a much larger percentage uncertainty than the other two, then this largest uncertainty can be applied to the quantity in question.

For example, if in equation $E_p = m g h$, the % uncertainty in m is 1%; the % uncertainty in g is 1% and the % uncertainty in h is 5% then we can safely say that the % uncertainty in E_p is 5%.

Check: % uncertainty in $E_p = \sqrt{1^2 + 1^2 + 5^2} = \sqrt{27} = 5.2\%$.

Thus, taking 5% as the overall estimate of uncertainty is a statistically acceptable approximation as long as the dominant uncertainty is considerably more than the other uncertainties. As a general rule a dominant uncertainty should be three times any other uncertainty. Thus if an uncertainty is less than a third of another uncertainty it can be neglected.

Comparing Results of Experiments

If we arrive at a numerical result in the form $x \pm \Delta x$, this allows us to compare the results for other experiments measuring the same quantity. Doing this may allow us to evaluate how successful or otherwise the method has been.

A good example of this is an analysis of two different methods of measuring g, the acceleration due to gravity.

Method A - pendulum	result: $g = 9.5 \pm 0.4$ m s^{-2}
Method B - oscillating spring	result: $g = 9.82 \pm 0.09$ m s^{-2}

Both of these values lie within the accepted value for g in Scotland which is between 9.815 m s^{-2} and 9.819 m s^{-2}. However we can say that the pendulum method is obviously more inaccurate but nevertheless still a valid measure. If the value had been 9.5 ± 0.2 m s^{-2} then this would have indicated that the method used could have been improved since it lies outside the accepted value for g. A repeat of the measurements should be carried out.

ACTIVITY 1

Title **Density of Glass from a Microscope Slide**

Apparatus Microscope slides, micrometer, vernier callipers, ruler
top-pan balance (accurate to at least 0.1 g).

Instructions

- Using the measuring instruments available, measure the dimensions of a glass microscope slide.
- Record the results and estimate the uncertainty attached to each of these measurements.
- Calculate the volume of the microscope slide. Give your value for the volume in the form: value ± uncertainty.
- Find the mass of the microscope slide using the top-pan balance. Again estimate the uncertainty associated with this measurement of mass.
- Calculate a value for the density of glass.
- Give your final answer in the form: density value ± uncertainty.
- In your report show clearly how you arrived at your estimate of uncertainty in volume, mass and density.
- Look up a data book and find a value for the density of glass. Compare your answer to those listed in the data book.

Theory Density $(\rho) = \dfrac{\text{mass}}{\text{volume}}$

ACTIVITY 2

Title **Density of Steel from Ball Bearings**

Apparatus Ball bearings, micrometer, vernier callipers
top-pan balance (accurate to at least 0.1 g).

Instructions

- Using the measuring instruments available, measure the diameter of one of the ball bearings
- Record the results and estimate the uncertainty attached to each of these measurements.
- Calculate the volume of the ballbearing.
Give your value for the volume in the form: value ± uncertainty.
- Find the mass of the ballbearing using the top-pan balance. Again estimate the uncertainty associated with this measurement of mass.
- Calculate a value for the density of the ballbearings.
- Give your final answer in the form: density value ± uncertainty.
- In your report show clearly how you arrived at your estimate of uncertainty in volume, mass and density.
- Look up a data book and find a value for the density of steel. Compare your answer to those listed in the data book.

Theory Density (ρ) = $\frac{\text{mass}}{\text{volume}}$ Volume of sphere = $\frac{4}{3}\pi r^3$

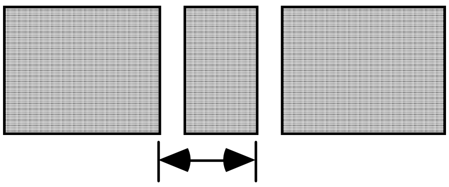
ACTIVITY 3A

Title **The Travelling Microscope**

Apparatus Travelling microscope (reading to 0.01 mm),
double slit slide (from Young's Interference Experiment),
low voltage bulb and battery.

Instructions

- Place the double slit slide on the stage of the microscope. Focus the cross hairs in the objective of the microscope on the edge of one of the slits ruled on the slide (see below).
- If illumination is a problem try to put the low voltage bulb beneath the microscope stage below the position of the slide. The edge of the slit rulings may now be visible.
- Read this position on the vernier scale.
- Now rack the microscope along the frame until the crosshairs are at the point which gives the slit separation (see below).
- Read the new position on the vernier scale. To avoid backlash in the mechanism, do not rack the microscope back and forth; i.e. only move it in one direction when taking both readings.



slit separation

- Find the separation.
- Estimate the uncertainty in the slit separation.
- Give your final answer in the form: separation ± uncertainty.
- In your report show clearly how you arrived at your estimate of uncertainty in the slit separation
- Check your value against the known value for slit separation.

ACTIVITY 3B

Title **Measurement of a Small Object**

Apparatus A small objects, if you do not have access to a double slit.

Instructions

- Place your small object (e.g. a coin or strand of wool) on the stage.
- Focus the crosshairs on one edge and note the scale reading.
- Rack along until the crosshairs are at the other edge of the object. Note the new reading on the scale.
- Calculate the size of the object.
- Estimate the uncertainty in your measurement.
- Give your final answer in the form: separation ± uncertainty.
- In your report show clearly how you arrived at your estimate of uncertainty in the size of the object.

ACTIVITY 4

Title **The Simple Pendulum**

This activity illustrates a graphical method to arrive at a measurement of a physical constant - the acceleration due to gravity.

Apparatus Pendulum bob, string, stopwatch, clamp stand, metre stick, two slotted masses (to grip the string).

Instructions

- Set up a length of string around 2 metres long and clamp it so that a pendulum bob tied to the end can swing freely.
- Time 20 complete to and fro small amplitude swings. This is best done by placing a vertical mark behind the rest position of the pendulum bob and starting the stopwatch as the bob obscures the mark: this is time zero and swing number zero. Stop the timer after the 20th swing as the bob passes the mark in the same direction.
- Repeat this timing and take an average of the two times.
- Measure the length, L, of the pendulum, taking care to measure the length from suspension point to the centre of the bob.
- Decrease the length of the pendulum in 0.10 m steps, repeating the timing of different oscillating lengths. Take 5 or 6 sets of times and lengths.
- Do not allow the swings to become elliptical - if this happens stop the measurements and repeat with the pendulum swinging in one plane.
- When the graph has been plotted use the "centroid" method to estimate the uncertainty in the gradient. From this, the uncertainty in "g" can be found.
- Give your final answer in the form: value ± uncertainty.
- In your report show clearly how you arrived at your estimate of uncertainty in the value of "g".

Theory From simple harmonic motion, the period of oscillation of a pendulum of length L is given by: $T = 2\pi\sqrt{\frac{L}{g}}$

Thus $T^2 = 4\pi^2\frac{L}{g} = \frac{4\pi^2}{g}\ L$

If a graph of T^2 against L is plotted the gradient will be numerically equal to $\frac{4\pi^2}{g}$, from which a value for "g" can be calculated.

Note that T^2 should be on the ordinate (i.e. y - axis) and that L should be the abscissa (i.e. x - axis).

TUTORIAL

Uncertainties

1 Three packages have to be added to the payload of the Space Shuttle. Their masses have been measured as follows:
$m_1 = (112 \pm 1)$ kg $m_2 = (252 \pm 2)$ kg and $m_3 = (151 \pm 1)$ kg.
Calculate the total mass to be added and the uncertainty in the total.

2 When using a travelling microscope the following measurements were made.
Reading 1 = (112.1 ± 0.2) mm Reading 2 = (114.5 ± 0.2) mm.
Calculate:
(a) the percentage uncertainty in the sum of these readings
(b) the percentage uncertainty in the difference of these readings
(c) Which of these, sum or difference, is usually needed for the travelling microscope?

3 A block of building material has been carefully machined to undergo tests. Its dimensions and mass are as follows:

length = 0.050 ± 0.001 m
breadth = 0.100 ± 0.001 m
height = 0.040 ± 0.001 m
mass = 0.560 ± 0.002 kg

(a) From this data, calculate the density of this material.
(b) Find the uncertainty in this value of density and express it as a percentage.

4 The radius of a sphere is measured to be $(1.2 \pm 0.1) \times 10^{-2}$ m.
If the volume of a sphere is given as $\frac{4}{3} \pi r^3$, where r is the radius of the sphere, calculate the volume of the sphere, quoting the uncertainty in your answer.

5 A uniform disc is to be used as a flywheel in a new design of small engine. Its moment of inertia has to be known. The following method is used:

The diameter of the disc is measured with a metre stick at 8 different positions round the rim and its mass is measured on a balance which was accurate to 10 g.

Diameters	0.245 m	0.249 m	0.255 m	0.248 m
	0.243 m	0.247 m	0.251 m	0.246 m

Mass 4.04 kg

Use the formula for the moment of inertia = $\frac{1}{2} M R^2$, where R is the radius of the disc. Find the moment of inertia, quoting a value for the uncertainty associated with your answer.

6 Calculate the refractive index of a glass block from the following information:
Angle of incidence = $(46 \pm 1)°$ Angle of refraction = $(28 \pm 1)°$.
Make sure you quote an uncertainty in your answer.

SOLUTIONS

Uncertainties

1 Total mass = 112 + 252 + 151 = 515 kg

Thus the uncertainty in the sum is given by: $\sqrt{1^2 + 2^2 + 1^2} = 2.4$ kg

Thus total mass = (515 ± 2) kg

Notice that simply adding the uncertainties in the masses is over pessimistic, giving 4 kg. The square root of the sum of the squares is better statistically in that uncertainties of this nature will sometimes cancel each other).

Note: the uncertainty is given as ± 2, not ± 2.4. Giving an excessive number of figures must be avoided. In general the uncertainty is given to one figure unless the leading digit is one, see question 4 below.
Remember to give the value to the same number of decimal places as the uncertainty, see question 3 below where the 2.75 becomes 2.8.

2 (a) & (b) uncertainty in the sum and difference = $\sqrt{(0.2)^2 + (0.2)^2}$
= 0.3 mm (one sig. fig.)

$$\text{Thus \% uncertainty in sum} = \frac{0.3}{226.6} \times 100 = 0.1\ \%$$

$$\text{Thus \% uncertainty in difference} = \frac{0.3}{2.4} \times 100 = 13\ \%$$

(c) Usually the difference in two readings is needed when using the travelling microscope. Great care has to be taken when measuring very small distances, even with an “accurate” instrument large uncertainties can be incurred.

3 (a) Volume = L x B x H = 0.050 x 0.100 x 0.040 = 2.00 x 10^{-4} m^3

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{0.560}{2.0 \times 10^{-4}} = 2.8 \times 10^3\ \text{kg m}^{-3}$$

(b) $$\text{\% uncertainty in mass} = \frac{0.002}{0.560} \times 100 = 0.4\ \%$$

% uncertainty in length = 2 %, % uncertainty in breadth = 1 %,
% uncertainty in height = 3 %

Thus % uncertainty in volume = $\sqrt{2^2 + 1^2 + 3^2}$ = 3.7 % or 4%

The dominant uncertainty is in the volume. Thus the % uncertainty in density will be 4 %.

density = 2.8 x 10^3 ± 4 % kg m^{-3}
= (2.8 ± 0.1) x 10^3 kg m^{-3}

4 $$\text{\% uncertainty in radius, } r = \frac{0.1}{1.2} \times 100 = 8\ \%$$

The volume depends on the cube of r:

the % uncertainty in the volume = 3 x 8 % = 24 %

$$\text{volume} = \frac{4}{3}\pi (1.2 \times 10^{-2})^3\ \text{m}^3 = 7.24 \times 10^{-6}\ \text{m}^3$$

volume = (7.2 ± 1.7) x 10^{-6} m^3

5 % uncertainty in mass $= \frac{0.01}{4.04} \times 100 = 0.25\ \%$

mean diameter = 0.248 m

random uncertainty in mean $= \frac{0.255 - 0.243}{8} = 0.0015$

$= 0.002$ m

% uncertainty in mean diameter $= \frac{0.002}{0.248} \times 100 = 0.8\ \%$

The % uncertainty will be the same for the radius.

Thus moment of inertia of the disc $= \frac{1}{2} M R^2 = 0.5 \times 4.04 \times (0.124)^2$

$= 0.0311$ kg m^2

The dominant uncertainty here is in the radius, which is squared:

total % uncertainty $= 2 \times 0.8\ \% = 1.6\ \%$

Thus moment of inertia $= 0.0311 \pm 1.6\ \%$

$= (0.0311 \pm 0.0005)$ kg m^2

6 refractive index, $n = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin 46}{\sin 28} = \frac{0.7193}{0.4695} = 1.53$

The easiest way to work out the uncertainty in a sine value is to work out the maximum and minimum values. Find the difference between these values and halve it.

Thus for θ_1 : $\sin 47° = 0.7314$ $\sin 45° = 0.7071$

range = 0.0243 uncertainty $= \frac{0.0243}{2} = 0.0122$

Thus % uncertainty in $\sin \theta_1 = \frac{0.0122}{0.7193} \times 100 = 1.7\ \%$.

For θ_2 : $\sin 29° = 0.4848$ $\sin 27° = 0.4540$

range = 0.0308 uncertainty $= \frac{0.0308}{2} = 0.0154$

Thus % uncertainty in $\sin \theta_2 = \frac{0.0154}{0.4695} \times 100 = 3.3\ \%$.

To find the overall uncertainty in refractive index these two uncertainties have to be combined.

% uncertainty in refractive index, $n = \sqrt{1.7^2 + 3.3^2} = \sqrt{13.8}$

$= 3.7\ \%$ or $4\ \%$

Final value : $n = 1.53 \pm 4\ \%$

$= 1.53 \pm 0.06$

KINEMATIC RELATIONSHIPS AND RELATIVISTIC MOTION

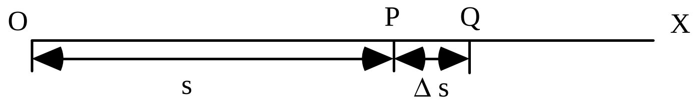
Throughout this course calculus techniques will be used. These techniques are very powerful and a knowledge of integration and differentiation will allow a deeper understanding of the nature of physical phenomena.

Kinematics is the study of the motion of points, making no reference to what causes the motion.

The displacement s of a particle is the length **and** direction from the origin to the particle.

The displacement of the particle is a function of time: $s = f(t)$

Consider a particle moving along OX.



Δ s is covered in time Δt

At time $t + \Delta t$ particle passes Q.

Velocity

average velocity $v_{av} = \frac{\Delta s}{\Delta t}$

However the **instantaneous** velocity is different, this is defined as :

$$v = \lim \frac{\Delta s}{\Delta t} \quad (\text{as } \Delta t \to 0) \qquad v = \frac{ds}{dt}$$

Acceleration

velocity changes by Δv in time Δt

$$a_{av} = \frac{\Delta v}{\Delta t}$$

Instantaneous acceleration : $a = \lim \frac{\Delta v}{\Delta t}$ (as $\Delta t \to 0$) $\qquad a = \frac{dv}{dt}$

$$\text{if} \quad a = \frac{dv}{dt} \quad \text{then} \quad \frac{dv}{dt} = \frac{d}{dt} \cdot \frac{ds}{dt} = \frac{d^2s}{dt^2}$$

$$\boxed{a = \frac{dv}{dt} = \frac{d^2s}{dt^2}}$$

Note: a change in velocity may result from a change in direction (e.g. uniform motion in a circle - see later).

Mathematical Derivation of Equations of Motion for Uniform Acceleration

$$a = \frac{d^2s}{dt^2}$$

Integrate with respect to time:

$$\int \frac{d^2s}{dt^2}\,dt = \int a\,dt$$

$$\frac{ds}{dt} = at + k$$

when $t = 0$ $\frac{ds}{dt} = u$ hence $k = u$

$t = t$ $\frac{ds}{dt} = v$

$$\mathbf{v = u + at} \quad \ldots\ldots \quad \mathbf{1}$$

integrate again : remember that $v = \frac{ds}{dt} = u + at$

$$\int ds = \int u\,dt + \int a\,t\,dt$$

$$s = ut + \frac{1}{2}at^2 + k$$

apply initial conditions: when $t = 0$, $s = 0$ hence $k = 0$

$$\mathbf{s = ut + \frac{1}{2}at^2} \quad \ldots\ldots \quad \mathbf{2}$$

Equations 1 and 2 can now be combined as follows: square both sides of equation 1

$$v^2 = u^2 + 2uat + a^2t^2$$

$$v^2 = u^2 + 2a\left[ut + \frac{1}{2}at^2\right]$$

(using equation 2) $$\mathbf{v^2 = u^2 + 2as} \quad \ldots\ldots \quad \mathbf{3}$$

A useful fourth equation is

$$\mathbf{s = \frac{(u + v)}{2}\,t} \quad \ldots\ldots \quad \mathbf{4}$$

Variable Acceleration

If acceleration depends on time in a simple way, calculus can be used to solve the motion.

Graphs of Motion

The slope or gradient of these graphs provides useful information. Also the area under the graph can have a physical significance.

Displacement - time graphs

$$v = \frac{ds}{dt}$$ slope = instantaneous velocity.

Area under graph - no meaning.

Velocity - time graphs

$$a = \frac{dv}{dt}$$ slope = instantaneous acceleration.

Also $s = \int v\,dt$ Area under v-t graph gives the displacement.

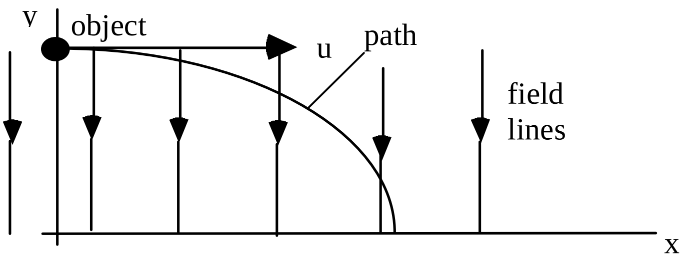
Calculations Involving Uniform Accelerations

Examples of **uniform** acceleration are:

- vertical motion of a projectile near the Earth's surface, where the acceleration is $g = 9.8 \text{ m s}^{-2}$ vertically downwards
- rectilinear (i.e. straight line) motion e.g. vehicle accelerating along a road.

These have been covered previously; however a fuller mathematical treatment for projectiles is appropriate at this level.

Consider the simple case of an object projected with an initial velocity u at right angles to the Earth's gravitational field - (locally the field lines may be considered parallel).



$a = g$, time to travel distance x across field = t

$$t = \frac{x}{u}$$

apply $y = u_y t + \frac{1}{2} a t^2$, $u_y t = 0$ and $a = g$

$$y = \frac{1}{2} \cdot g \cdot \frac{x^2}{u^2}$$

$$y = \left[\frac{1}{2} \cdot \frac{g}{u^2} \right] . x^2$$

Now g and u are constants, $y \propto x^2$ and we have the equation of a **parabola**.

The above proof and equations are **not** required for examination purposes.

Relativistic Dynamics

The greatest possible speed of any object is the speed of light in a vacuum.
The accepted value for the speed of light is 3.0×10^8 m s^{-1}.
An object which is travelling at a speed close to the speed of light would appear (to a stationary observer) to have a mass greater than the mass possessed by the same object if it were stationary.

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m_0 = rest mass (i.e. mass when object's speed is zero)
v = speed of object
c = speed of light, (namely 3.0×10^8 m s^{-1}).

Thus no object can increase its speed indefinitely because its mass would become infinite if it were to travel at the speed of light - (the denominator in the above expression would be zero).

Other consequences of relativity are given here for the **interested** student.

Length Contraction

Objects travelling at relativistic speeds appear **shorter** than they would do at rest.

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}}$$ l = relativistic length.

Time Dilation

Time intervals appear to **increase**. $$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$\Delta t'$ is the time interval measured by a person in an object travelling at speed v.
Δt is the time interval measured by a person who is at rest.

An unusual consequence of time dilation is the **twin paradox**.

Consider twins of exactly the same age performing the following experiment.
On their 29th. birthday twin A sets off on a space journey at a speed of 0.995 c while twin B stays at home on Earth. Twin A returns 10 Earth years later and claims to be still 29 (just!).

Using $\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}}$ $\Delta t_B = \frac{\Delta t_A}{\sqrt{1 - \frac{(0.995c)^2}{c^2}}}$

Δt_B = 10 years, $\Delta t_A = 10 \text{ yrs} \times \sqrt{1 - (0.995)^2}$
$= 0.999$ yrs
$= 364.6$ days

Thus twin A comes back 0.4 days short of his 30th. birthday and the other twin is 40!

Relativistic Energy (total energy)

In non-relativistic motion, where mass does not change with speed, increasing the unbalanced force produces an increase in the body's kinetic energy. In relativistic situations the mass increases and we have to increase the accelerating force more and more until the theoretical point is reached of infinite mass requiring an even greater than infinite force to take the speed up to c. Such a force is not possible - thus Special Relativity predicts that no piece of matter can move faster than the speed of light.

The total (relativistic) energy of an object is given as

$$E = mc^2$$

This is made up of two parts:
- the rest mass energy m_0c^2 and
- the energy due to its motion (E_k)

It can be shown that the kinetic energy of mass m moving with speed v is given by:

$$E_k = mc^2 - m_0c^2$$

$$E_k = m_0c^2\left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right]$$

If we expand the right hand side of the above equation using the binomial theorem and assuming that v « c we obtain:

$$E_k = \frac{1}{2} m_o v^2 \quad \text{(approximately).}$$

[The binomial theorem states: $(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!}$ …..

Expanding $(1 - \frac{v^2}{c^2})^{-½}$ the first two terms are $1 + \{(-\frac{1}{2})(-\frac{v^2}{c^2})\}$ giving $1 + \frac{v^2}{2c^2}$.

For v « c the other terms will be very small and can be neglected.

$E_k = m_0c^2 (1 + \frac{v^2}{2c^2} - 1)$ giving $E_k = \frac{1}{2} m_o v^2$]

This is in agreement with the formula for the kinetic energy of the object using Newtonian mechanics.

Notice from above that when v is close to c, $v \approx c$, the relationship $E_k = mc^2 - m_0c^2$ must be used for the kinetic energy.

Note: the relativistic equation for mass will be met again later in the course when the motion of fast moving electrons is investigated.

ANGULAR MOTION

The angular velocity of a rotating body is defined as the rate of change of angular displacement.

$$\boxed{\omega = \frac{d\theta}{dt}}$$

where ω (rad s^{-1}) is the angular velocity
θ (rad) is the angular displacement

The radian (rad) is a unit of angle: $180^\circ = \pi$ rad

Angular acceleration, $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ unit of α: rad s^{-2}

We assume for this course that α is **constant**.

The derivation of the equations for angular motion are very similar to those for linear motion seen earlier. The derivations of these equations are **not required** for examination purposes.

Angular Motion Relationships

$$\boxed{\omega = \omega_o + \alpha t \quad ..... \quad 1}$$

$$\boxed{\theta = \omega_o t + \frac{1}{2}\alpha t^2 \quad ..... \quad 2}$$

$$\boxed{\omega^2 = \omega_o^2 + 2\alpha\theta \quad ..... \quad 3}$$

You will note that these **angular** equations have exactly the same form as the **linear** equations.
Remember that these equations only apply for **uniform** angular accelerations.

Uniform Motion in a Circle

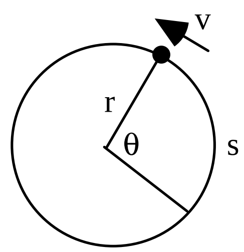
Consider a particle moving with uniform speed in a circular path as shown opposite.

$$\omega = \frac{d\theta}{dt}$$

The rotational speed v is constant, ω is also constant.
T is the period of the motion and is the time taken to cover 2π radians.

$$\omega = \frac{2\pi}{T} \quad \text{but} \quad v = \frac{2\pi r}{T}$$

$$\boxed{v = r\omega}$$



(Note: s is the arc swept out by the particle and $s = r\theta$)

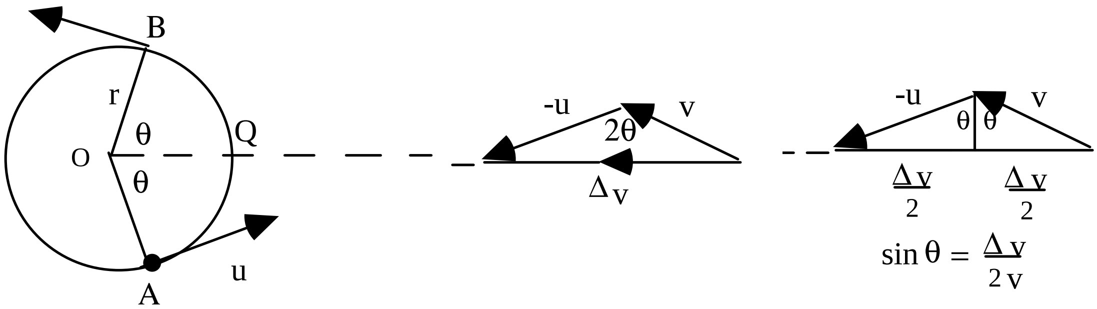
Radial and angular acceleration

The angular acceleration $\alpha = \frac{d\omega}{dt}$ and the radial acceleration $a = \frac{dv}{dt}$.

Therefore since $v = r\omega$, then at any instant $\frac{dv}{dt} = r\frac{d\omega}{dt}$ giving

$$a = r\alpha$$

Radial Acceleration



$$\sin\theta = \frac{\Delta v}{2v}$$

The particle travels from A to B in time Δt and with speed v, thus $|u| = |v|$ and $\Delta v = v + (-u)$ which is $\Delta v = v - u$

$$\Delta t = \frac{\text{arc AB}}{v} = \frac{r(2\theta)}{v}$$

$$\text{average acceleration, } a_{av} = \frac{\Delta v}{\Delta t} = \frac{2v\sin\theta}{\Delta t}$$

$$= \frac{2v\sin\theta}{r\,2\theta / v} = \frac{v^2}{r} \cdot \frac{\sin\theta}{\theta}$$

As $\theta \to 0$, $a_{av} \to$ instantaneous acceleration at point Q:

$$a = \frac{v^2}{r} \cdot \left[\lim_{\theta \to 0} \frac{\sin\theta}{\theta}\right] \quad \text{but} \left[\lim_{\theta \to 0} \frac{\sin\theta}{\theta}\right] = 1$$

when θ is small and is measured in radians $\sin\theta = \theta$.

$$a = \frac{v^2}{r} = \omega^2 r \quad \text{since } v = r\omega$$

The **direction** of this acceleration is always towards the **centre** of the circle.

Note: This is **not** a uniform acceleration. Radial acceleration is fixed only in size. Compare this with the angular acceleration which is constant for problems in this course.

This motion is typical of many **central force** type motions e.g. Planetary Motion, electrons ‘orbiting’ nuclei and electrons injected at right angles to a uniform magnetic field which will be covered later in the course.

Thus any object performing circular orbits at uniform speed must have a **centre-seeking** or **central** force responsible for the motion.

Central Force

Does a rotating body really have an inward acceleration (and hence an inward force)?

Argument Most people have experienced the sensation of being in a car or a bus which is turning a corner at high speed. The feeling of being 'thrown to the outside of the curve' is very strong, especially if you slide along the seat. What happens here is that the friction between yourself and the seat is insufficient to provide the central force needed to deviate you from the straight line path you were following before the turn. In fact, instead of being thrown outwards, you are, in reality, continuing in a straight line while the car moves inwards. Eventually you are moved from the straight line path by the inward (central) force provided by the door.

Magnitude of the Force

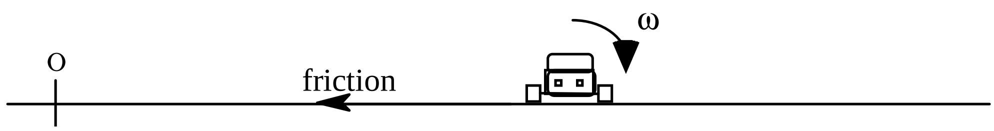
$F = m a$ but $a = \frac{v^2}{r}$ or $a = \omega^2 r$

Thus central force, $\boxed{F = m\frac{v^2}{r} \quad \text{or} \quad F = m r \omega^2}$ since $v = r\omega$

Examples

1. A Car on a Flat Track

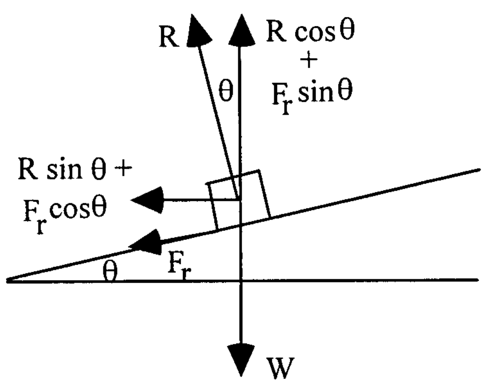
If the car goes too fast, the car 'breaks away' at a tangent. The force of friction is not enough to supply an adequate central force.



2. A Car on a Banked Track

For tracks of similar surface properties, a car will be able to go faster on a banked track before going off at a tangent because there is a component of the normal reaction as well as a component of friction, F_r, supplying the central force.

The central force is $R \sin\theta + F_r \cos\theta$ which reduces to $R \sin\theta$ when the friction is zero. The analysis on the right hand side is for the friction F_r equal to **zero**.



R is the 'normal reaction' force of the track on the car.

In the vertical direction there is no acceleration:

$R \cos\theta = mg$ 1

In the radial direction there is a central acceleration:

$R \sin\theta = \frac{mv^2}{r}$ 2

Divide Eq. 2 by Eq. 1:

$\tan\theta = \frac{v^2}{gr}$ (assumes friction is zero)

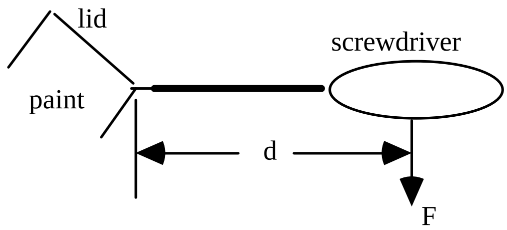
(This equation applies to all cases of 'banking' including aircraft turning in horizontal circles)

ROTATIONAL DYNAMICS

Moment of a force

The **moment** of a force is the **turning effect** it can produce.
Examples of moments are:

using a long handled screwdriver to 'lever off' the lid of a paint tin, see below
using a claw hammer to remove a nail from a block of wood or levering off a cap from a bottle.

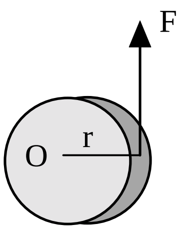


The magnitude of the moment of the force (or the turning effect) is F x d.

Where F is the force and d is the perpendicular distance from the direction of the force to the turning point.

Torque

For cases where a force is applied and this causes rotation about an axis, the moment of the force can be termed the **torque**.



Consider a force F applied tangentially to the rim of a disc which can rotate about an axis O through its centre. The radius of the circle is r.
The torque T associated with this force F is defined to be the force multiplied by the radius r.

$T = F \times r$ unit of T: newton metre (N m)

Torque is a **vector** quantity. The direction of the torque vector is at right angles to the plane containing both r and F and lies along the axis of rotation. (For interest only, in the example shown in the diagram torque, T, points out of the page).

A force acting on the rim of an object will cause the object to rotate; e.g. applying a push or a pull force to a door to open and close , providing it creates a non-zero resulting torque. The distance from the axis of rotation is an important measurement when calculating torque. It is instructive to measure the relative forces required to open a door by pulling with a spring balance firstly at the handle and then pulling in the middle of the door. Another example would be a **torque wrench** which is used to rotate the wheel nuts on a car to a certain 'tightness' as specified by the manufacturer.

An **unbalanced torque** will produce an **angular acceleration**. In the above diagram if there are no other forces then the force F will cause the object to rotate.

Inertia

In linear dynamics an unbalanced force produces a linear acceleration. The magnitude of the linear acceleration produced by a given unbalanced force will depend on the mass of the object, that is on its inertia. The word inertia can be loosely described as 'resistance to change in motion of an object' Objects with a large mass are difficult to start moving and once moving are difficult to stop.

Moment of Inertia

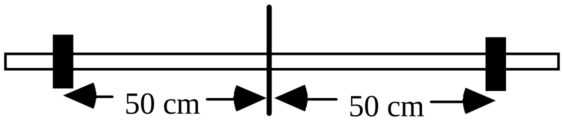
The moment of inertia I of an object can be described as its resistance to change in its angular motion. The moment of inertia I for rotational motion is analogous to the mass m for linear motion

The moment of inertia I of an object depends on the mass **and** the distribution of the mass about the axis of rotation.

For a mass m at a distance r from the axis of rotation the moment of inertia of this mass is given by the mass m multiplied by r^2.

$$\boxed{I = m r^2} \quad \text{unit of I: kg m}^2$$

For example, a very light rod has two 0.8 kg masses each at a distance of 50 cm from the axis of rotation.



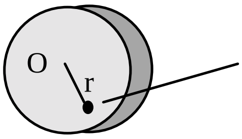
The moment of inertia of each mass is $m r^2 = 0.8 \times 0.5^2 = 0.2$ kg m^2 giving a total moment of inertia $I = 0.4$ kg m^2. Notice that we assume that all the mass is at the 50 cm distance. The small moment of inertia of the light rod has been ignored.

Another example is a hoop, with very light spokes connecting the hoop to an axis of rotation through the centre of the hoop and perpendicular to the plane of the hoop, e.g. a bicycle wheel. Almost all the mass of the hoop is at a distance R, where R is the radius of the hoop. Hence $I = M R^2$ where M is the total mass of the hoop.

For objects where all the mass can be considered to be at the **same** distance from the axis of rotation this equation $I = m r^2$ can be used directly.

However most objects do **not** have all their mass at a single distance from the axis of rotation and we must consider the distribution of the mass.

Moment of inertia and mass distribution



Consider a small particle of the disc as shown. This particle of mass m is at a distance r from the axis of rotation 0.

The contribution of this mass to the moment of inertia of the whole object (in this case a disc) is given by the mass m multiplied by r^2. To obtain the moment of inertia of the disc we need to consider all the particles of the disc, each at their different distances.

Any object can be considered to be made of n particles each of mass m. Each particle is at a particular radius r from the axis of rotation. The moment of inertia of the object is determined by the summation of all these n particles e.g. $\sum (m\, r^2)$. Calculus methods are used to determine the moments of inertia of extended objects. In this course, moments of inertia of extended objects, about specific axes, will be given.

It can be shown that the moment of inertia of a uniform rod of length L and total mass M through its centre is $\frac{ML^2}{12}$, but the moment of inertia of the same rod through its end is $\frac{ML^2}{3}$, i.e. four times bigger. This is because it is harder to make the rod rotate about an axis at the end than an axis through its middle because there are now more particles at a greater distance from the axis of rotation.

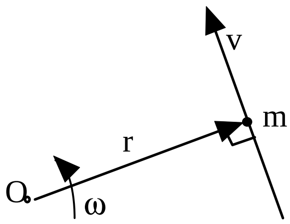
Torque and Moment of Inertia

An **unbalanced torque** will produce an **angular acceleration**. As discussed above, the moment of inertia of an object is the opposition to a change in its angular motion. Thus the angular acceleration α produced by a given torque T will depend on the moment of inertia I of that object.

$$\boxed{T = I\alpha}$$

Angular Momentum

The angular momentum L of a particle about an axis is defined as the **moment** of momentum.



A particle of mass m rotates at ω rad s^{-1} about the point O.
The linear momentum p = m v.
The moment of p = m v r (r is perpendicular to v).

Thus the angular momentum of this particle = m v r = m r^2 ω, since v = r ω.

For a rigid object about a fixed axis the angular momentum L will be the summation of all the individual angular momenta. Thus the angular momentum L of an object is given by $\sum (m\, r^2\, \omega)$. This can be written as $\omega \sum (m\, r^2)$ since all the individual parts of the object will have the same angular velocity ω. Also we have $I = \sum (m\, r^2)$.

Thus the angular momentum of a rigid body is:

$$\boxed{L = I\,\omega} \qquad \text{unit of L: kg m}^2\ \text{s}^{-1}.$$

Notice that the angular momentum of a rigid object about a fixed axis **depends** on the moment of inertia.

Angular momentum is a **vector** quantity. The **direction** of this vector is at right angles to the plane containing v (since p = m v and mass is scalar) and r and lies along the axis of rotation. For interest only, in the above example L is out of the page. (Consideration of the vector nature of T and L will not be required for assessment purposes.)

Conservation of angular momentum

The **total** angular momentum before an impact will equal the **total** angular momentum after impact providing no external torques are acting.

You will meet a variety of problems which involve use of the conservation of angular momentum during collisions for their solution.

Rotational Kinetic Energy

The rotational kinetic energy of a rigid object also depends on the moment of inertia. For an object of moment of inertia I rotating uniformly at ω rad s^{-1} the rotational kinetic energy is given by:

$$\boxed{E_k = \frac{1}{2} I\,\omega^2}$$

Energy and work done

If a torque T is applied through an angular displacement θ, then the work done = T θ. Doing work produces a transfer of energy, $T\,\theta = I\,\omega^2 - I\,\omega_0^2$ (work done = ΔE_k).

Summary and Comparison of Linear and Angular Equations

Quantity	***Linear Motion***		***Angular Motion***
acceleration	a	$(a = r\alpha)$	α
velocity	$v = u + at$	$(v = r\omega)$	$\omega = \omega_o + \alpha t$
displacement	$s = ut + \frac{1}{2}at^2$		$\theta = \omega_o t + \frac{1}{2}\alpha t^2$
momentum	$p = mv$		$L = I\omega$
kinetic energy	$\frac{1}{2}mv^2$		$\frac{1}{2}I\omega^2$
Newton's 2nd law	$F = m\frac{dv}{dt} = ma$		$T = I\frac{d\omega}{dt} = I\alpha$

Laws

Conservation of momentum	$m_Au_A + m_Bu_B = m_Av_A + m_Bv_B$	$I_A\omega_{oA} + I_B\omega_{oB} = I_A\omega_A + I_B\omega_B$
Conservation of energy	$F \cdot s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	$T\theta = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_o^2$

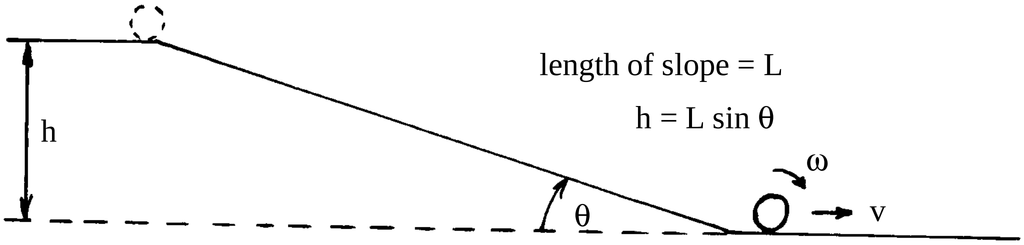
Some Moments of Inertia (for reference)

Thin disc about an axis through its centre and perpendicular to the disc.	$I = \frac{1}{2}MR^2$	R = radius of disc
Thin rod about its centre	$I = \frac{1}{12}ML^2$	L = length of rod
Thin hoop about its centre	$I = MR^2$	R = radius of hoop
Sphere about its centre	$I = \frac{2}{5}MR^2$	R = radius of sphere

Where M is the total mass of the object in each case.

Objects Rolling down an Inclined Plane

When an object such as a sphere or cylinder is allowed to run down a slope, the E_p at the top, (m g h), will be converted to both **linear** ($\frac{1}{2}$ m v^2) and **angular** ($\frac{1}{2}$ I ω^2) kinetic energy.



An equation for the energy of the motion (assume no slipping) is given below.

$$m g h = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$$

The above formula can be used in an experimental determination of the moment of inertia of a circular object.

Example

A solid cylinder is allowed to roll from rest down a shallow slope of length 2.0 m. When the height of the slope is 0.02 m, the time taken to roll down the slope is 7.8 s. The mass of the cylinder is 10 kg and its radius is 0.10 m.

Using this information about the motion of the cylinder and the equation above, calculate the moment of inertia of the cylinder.

Solution

$$E_P = m g h = 10 \times 9.8 \times 0.02 = 1.96 \text{ J}$$

Change in gravitational E_p = change in linear E_k + change in rotational E_k

$$m g h = (\frac{1}{2} m v^2 - 0) + (\frac{1}{2} I \omega^2 - 0)$$

$$s = \frac{(u + v)}{2} t$$

$$2.0 = \frac{(0 + v)}{2} \times 7.8$$

$$v = \frac{4.0}{7.8} = 0.513 \text{ m s}^{-1}$$

$$\omega = \frac{v}{r} = \frac{0.513}{0.10} = 5.13 \text{ rad s}^{-1}$$

$$E_{k(lin)} = \frac{1}{2} m v^2 = \frac{1}{2} \times 10 (0.513)^2 = 1.32 \text{ J}$$

$$E_{k(rot)} = E_P - E_{k(lin)}$$

$$\frac{1}{2} I \omega^2 = 1.96 - 1.32 = 0.64 \text{ J}$$

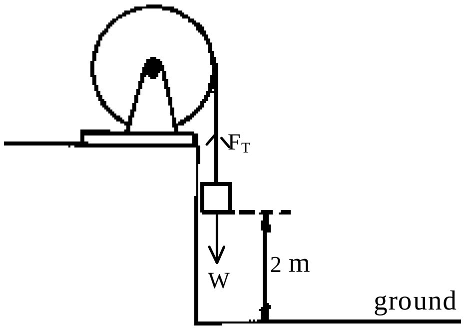
$$I = \frac{2 \times 0.64}{\omega^2} = \frac{2 \times 0.64}{(5.13)^2}$$

$$I = 0.049 \text{ kg m}^2$$

The Flywheel

Example

The flywheel shown below comprises a solid cylinder mounted through its centre and free to rotate in the vertical plane.



Flywheel: mass = 25 kg
radius = 0.30 m.

Mass of hanging weight = 2.5 kg

The hanging weight is released. This results in an angular acceleration of the flywheel. Assume that the effects of friction are negligible.

(a) Calculate the angular acceleration of the flywheel.

(b) Calculate the angular velocity of the flywheel just as the weight reaches ground level.

Solution

(a) We need to know I, the moment of inertia of the flywheel: $I = \frac{1}{2} M R^2$

$$I = \frac{1}{2} \times 25 \times (0.30)^2 = 1.125 \text{ kg m}^2$$

Consider the forces acting on the flywheel: $W - F_T = m a$ where $m = 2.5$ kg

$$24.5 - F_T = 2.5 \times 0.30\, \alpha \quad (a = r\alpha)$$

$$F_T = 24.5 - 0.75\,\alpha$$

$$\text{Torque, } T = F_T \times r = (24.5 - 0.75\,\alpha) \times 0.30$$

$$\text{and} \quad T = I\alpha = 1.125\,\alpha$$

Thus $1.125\,\alpha = 7.35 - 0.225\,\alpha$

$$\alpha = \frac{7.35}{1.35} = 5.44 \text{ rad s}^{-2}$$

(b) To calculate the angular velocity we will need to know θ, the angular displacement for a length of rope 2.0 m long being unwound.

$$\text{circumference} = 2\pi r = 2\pi \times 0.30$$

$$\text{no. of revs} = \frac{\text{length unwound}}{\text{circumference}} = \frac{2.0 \text{ m}}{2\pi \times 0.30 \text{ m}}$$

$$\theta = 2\pi \times \text{no. of revs} = 2\pi \times \frac{2.0}{2\pi \times 0.30} = 6.67 \text{ rad}$$

$\omega_o = 0$

$\omega = ?$

$\alpha = 5.44 \text{ rad s}^{-2}$

$\theta = 6.67$ rad

apply $\omega^2 = \omega_o^2 + 2\alpha\theta$

$\omega^2 = 0 + 2 \times 5.44 \times 6.67$

$\omega^2 = 72.57$

$\omega = 8.52 \text{ rad s}^{-1}$

Frictional Torque

Example

The friction acting at the axle of a bicycle wheel can be investigated as follows. The wheel, of mass 1.2 kg and radius 0.50 m, is mounted so that it is free to rotate in the vertical plane. A driving torque is applied and when the wheel is rotating at 5.0 revs per second the driving torque is removed. The wheel then takes 2.0 minutes to stop.

(a) Assuming that all the spokes of the wheel are very light and the radius of the wheel is 0.50 m, calculate the moment of inertia of the wheel.

(b) Calculate the frictional torque which causes the wheel to come to rest.

(c) The effective radius of the axle is 1.5 cm. Calculate the force of friction acting at the axle.

(d) Calculate the kinetic energy lost by the wheel. Where has this energy gone?

Solution

(a) In this case I for wheel $= MR^2$

$I = 1.2 \times (0.50)^2$ $(M = 1.2\text{ kg},\quad R = 0.50\text{ m})$

$I = 0.30\text{ kg m}^2$

(b) To find frictional torque we need the angular acceleration (α), because $T = I\alpha$

$\omega = 0, t = 120\text{ s}$ $\qquad \alpha = \dfrac{\omega - \omega_o}{t}$

$\omega_o = 5.0\text{ r.p.s.}$ $\qquad = \dfrac{0 - 31.4}{120}$

$= 31.4\text{ rad s}^{-1}$ $\qquad \alpha = -0.262\text{ rad s}^{-2}$

Now use $\qquad T = I\alpha$

$= 0.3 \times (-0.262)$

$T = -0.0786\text{ N m}$

(c) Also $\qquad T = rF \qquad (r = 1.5\text{ cm} = 0.015\text{ m})$

$$F = \frac{T}{r} = -\frac{0.0786}{0.015} = -5.24\text{ N}$$

i.e. negative value indicates force *opposing* motion.

(d)

$$E_{k(rot)} = \frac{1}{2} I \omega_o^2$$

$$= \frac{1}{2} \times 0.30 \times (31.4)^2$$

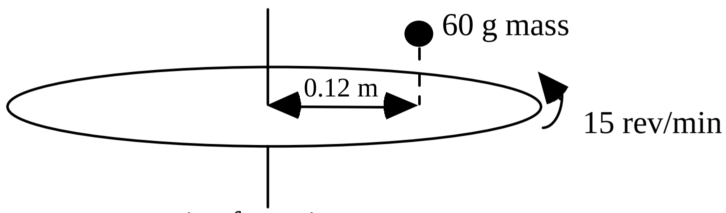
$$= 148\text{ J}$$

When the wheel stops $E_{k(rot)} = 0$. This 148 J will have changed to heat in the axle due to the work done by the force of friction.

Conservation of Angular Momentum

Example

A turntable, which is rotating on frictionless bearings, rotates at an angular speed of 15 revolutions per minute. A mass of 60 g is dropped from rest just above the disc at a distance of 0.12 m from the axis of rotation through its centre.



As a result of this impact, it is observed that the rate of rotation of the disc is reduced to 10 revolutions per minute.

(a) Use this information and the principle of conservation of angular momentum to calculate the moment of inertia of the disc.

(b) Show by calculation whether this is an elastic or inelastic collision.

Solution

(a) Moment of inertia of disc $= I$

Moment of inertia of 60 g mass $= m r^2$ (treat as 'particle' at radius r)

$= 0.06 \times (0.12)^2$

$I_{mass} = 8.64 \times 10^{-4}$ kg m^2

initial angular velocity $= \omega_o = 15$ rev min^{-1} $= \dfrac{15 \times 2\pi}{60}$

$\omega_o = 1.57$ rad s^{-1}

final angular velocity $= \omega = 10$ rev min^{-1}

$= 1.05$ rad s^{-1}

Total angular momentum before impact = total angular momentum after impact

$$I\,\omega_o = (I + I_{mass})\,\omega$$

$$I \times 1.57 = (I + 8.64 \times 10^{-4}) \times 1.05$$

$$0.52\, I = 9.072 \times 10^{-4}$$

$$I = \frac{9.072 \times 10^{-4}}{0.52} = 1.74 \times 10^{-3} \text{ kg m}^2$$

(b) E_k before impact $= \frac{1}{2} I \omega_o^2 = \frac{1}{2} \times 1.74 \times 10^{-3} \times (1.57)^2 = 2.14 \times 10^{-3}$ J

E_k after impact $= \frac{1}{2}(I + I_{mass})\,\omega^2 = \frac{1}{2} \times 2.60 \times 10^{-3} \times (1.05)^2 = 1.43 \times 10^{-3}$ J

E_k difference $= 7.1 \times 10^{-4}$ J

Thus the collision is **inelastic**. The energy difference will be changed to heat.

GRAVITATION

Historical Introduction

The development of what we know about the Earth, Solar System and Universe is a fascinating study in its own right. From earliest times Man has wondered at and speculated over the 'Nature of the Heavens'. It is hardly surprising that most people (until around 1500 A.D.) thought that the Sun revolved around the Earth because that is what it seems to do! Similarly most people were sure that the Earth was flat until there was definite proof from sailors who had ventured round the world and not fallen off!

It may prove useful therefore to give a brief historical introduction so that we may set this topic in perspective. For the interested student, you are referred to a most readable account of Gravitation which appears in "Physics for the Inquiring Mind" by Eric M Rogers - chapters 12 to 23 (pages 207 to 340) published by Princeton University Press (1960). These pages include astronomy, evidence for a round Earth, evidence for a spinning earth, explanations for many gravitational effects like tides, non-spherical shape of the Earth/precession, variation of 'g' over the Earth's surface. There is also a lot of information on the major contributors over the centuries to our knowledge of gravitation. A brief historical note on these people follows.

Claudius Ptolemy (A.D. 120) assumed the Earth was immovable and tried to explain the strange motion of various stars and planets on that basis. In an enormous book, the "Almagest", he attempted to explain in complex terms the motion of the 'five wandering stars' - the planets.

Nicolaus Copernicus (1510) insisted that the Sun and not the Earth was the centre of the solar system. First to really challenge Ptolemy. He was the first to suggest that the Earth was just another planet. His great work published in 1543, "On the Revolutions of the Heavenly Spheres", had far reaching effects on others working in gravitation.

Tycho Brahe (1580) made very precise and accurate observations of astronomical motions. He did not accept Copernicus' ideas. His excellent data were interpreted by his student Kepler.

Johannes Kepler (1610) Using Tycho Brahe's data he derived three general rules (or laws) for the motion of the planets. He could not explain the rules.

Galileo Galilei (1610) was a great experimenter. He invented the telescope and with it made observations which agreed with Copernicus' ideas. His work caused the first big clash with religious doctrine regarding Earth-centred biblical teaching. His work "Dialogue" was banned and he was imprisoned. (His experiments and scientific method laid the foundations for the study of Mechanics).

Isaac Newton (1680) brought all this together under his theory of Universal Gravitation explaining the moon's motion, the laws of Kepler and the tides, etc. In his mathematical analysis he required calculus - so he invented it as a mathematical tool!

Consideration of Newton's Hypothesis

It is useful to put yourself in Newton's position and examine the hypothesis he put forward for the variation of gravitational force with distance from the Earth. For this you will need the following data on the Earth/moon system (all available to Newton).

Data on the Earth

"g" at the Earth's surface = 9.8 m s^{-2}
radius of the Earth, R_E = 6.4×10^6 m
radius of moon's orbit, r_M = 3.84×10^8 m
period, T, of moon's circular orbit = 27.3 days = 2.36×10^6 s.

take $\frac{R_E}{r_M} = \left[\frac{1}{60}\right]$

Assumptions made by Newton

- All the mass of the Earth may be considered to be concentrated at the centre of the Earth.

- The gravitational attraction of the Earth is what is responsible for the moon's circular motion round the Earth. Thus the observed central acceleration can be calculated from measurements of the moon's motion: $a = \frac{v^2}{r}$.

Hypothesis

Newton asserted that the acceleration due to gravity "g" would quarter if the distance from the centre of the Earth doubles i.e. an inverse square law.

$$\text{"g"} \propto \frac{1}{r^2}$$

- Calculate the central acceleration for the Moon: use $a = \frac{v^2}{R}$ or $a = \frac{4\pi^2 R}{T^2}$ m s^{-2}.
- Compare with the "diluted" gravity at the radius of the Moon's orbit according to the hypothesis, viz. $\frac{1}{(60)^2} \times 9.8 \text{ m s}^{-2}$.

Conclusion

The inverse square law applies to gravitation.

General Data

Planet or satellite	Mass/ kg	Density/ kg m^{-3}	Radius/ m	Grav. accel./ m s^{-2}	Escape velocity/ m s^{-1}	Mean dist from Sun/ m	Mean dist from Earth/ m
Sun	1.99×10^{30}	1.41×10^3	7.0×10^8	274	6.2×10^5	--	1.5×10^{11}
Earth	6.0×10^{24}	5.5×10^3	6.4×10^6	9.8	11.3×10^3	1.5×10^{11}	--
Moon	7.3×10^{22}	3.3×10^3	1.7×10^6	1.6	2.4×10^3	--	3.84×10^8
Mars	6.4×10^{23}	3.9×10^3	3.4×10^6	3.7	5.0×10^3	2.3×10^{11}	--
Venus	4.9×10^{24}	5.3×10^3	6.05×10^6	8.9	10.4×10^3	1.1×10^{11}	--

Inverse Square Law of Gravitation

Newton deduced that this can only be explained if there existed a universal gravitational constant, given the symbol G.

We have already seen that Newton's "hunch" of an inverse square law was correct. It also seems reasonable to assume that the force of gravitation will vary with the masses involved.

$$F \propto m, \quad F \propto M, \quad F \propto \frac{1}{r^2} \qquad \text{giving} \quad F \propto \frac{Mm}{r^2}$$

$$\boxed{F = \frac{GMm}{r^2}} \qquad \text{where } G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$$

Consider the Solar System

$$M \dashrightarrow M_s \qquad \text{and} \qquad m \dashrightarrow m_p$$

Force of attraction on a planet is: $F = \dfrac{GM_s m_p}{r^2}$ (r = distance from Sun to planet)

Now consider the central force if we take the motion of the planet to be circular.

Central force $F = \dfrac{GM_s m_p}{r^2}$

Also $F = \dfrac{m_p v^2}{r}$ force of gravity supplies the central force.

Thus $\dfrac{GM_s m_p}{r^2} = \dfrac{m_p v^2}{r}$ and $v = \dfrac{2\pi r}{T}$

$$\frac{GM_s m_p}{r^2} = \frac{m_p}{r} \cdot \frac{4\pi^2 r^2}{T^2}$$

rearranging $\dfrac{r^3}{T^2} = \dfrac{GM_s}{4\pi^2}$

Kepler had already shown that $\dfrac{r^3}{T^2}$ = a constant, and M_s is a constant, hence it follows that G must be a constant for **all** the planets in the solar system (i.e. a **universal** constant).

Notes:
- We have assumed circular orbits. In reality, orbits are elliptical.
- Remember that Newton's Third Law always applies. The force of gravity is an action-reaction pair. Thus if your weight is 600 N on the Earth; as well as the Earth pulling you down with a force of 600 N, you also pull the Earth up with a force of 600 N.
- Gravitational forces are very weak compared to the electromagnetic force (around 10^{39} times smaller). Electromagnetic forces only come into play when objects are charged or when charges move. These conditions only tend to occur on a relatively small scale. Large objects like the Earth are taken to be electrically neutral.

"Weighing" the Earth

Obtaining a value for "G" allows us to "weigh" the Earth i.e. we can find its mass. Consider the Earth, mass M_e, and an object of mass m on its surface. The gravitational force of attraction can be given by **two** equations:

$$F = mg \qquad \text{and} \qquad F = \frac{GmM_e}{R_e^2}$$

where R_e is the separation of the two masses, i.e. the radius of the earth.

$$\text{Thus} \quad m\,g = \frac{GmM_e}{R_e^2} \qquad M_e = \frac{gR_e^2}{G} = \frac{9.8 \times (6.40 \times 10^6)^2}{6.67 \times 10^{-11}}$$

Thus the mass of the Earth $= 6.02 \times 10^{24}$ kg

The Gravitational Field

In earlier work on gravity we restricted the study of gravity to small height variations near the earth's surface where the force of gravity could be considered constant.

Thus $F_{grav} = m\,g$

Also $E_p = m\,g\,h$ where g = constant ($9.8\ N\ kg^{-1}$)

When considering the Earth-Moon System or the Solar System we cannot restrict our discussions to small distance variations. When we consider force and energy changes on a large scale we have to take into account the variation of force with distance.

Definition of Gravitational Field at a point.

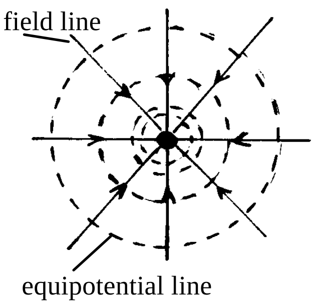
This is defined to be the force per unit mass at the point. i.e. $g = \frac{F}{m}$

The concept of a field was not used in Newton's time. Fields were introduced by Faraday in his work on electromagnetism and only later applied to gravity.

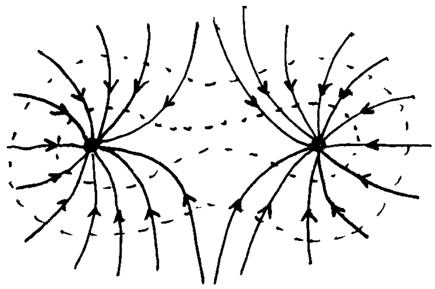
Note that g and F above are both vectors and whenever forces or fields are added this must be done vectorially.

Field Patterns (and Equipotential Lines)

(i) An Isolated 'Point' Mass



(ii) Two Equal 'Point' Masses



(equipotential lines shown broken)

Note that equipotential lines are always at **right angles** to field lines.

Variation of g with Height above the Earth (and inside the Earth)

An object of mass m is on the surface of the Earth (mass M). We now know that the weight of the mass can be expressed using Universal Gravitation.

Thus $mg = \frac{GMm}{r^2}$ (r = radius of Earth in this case)

$g = \frac{GM}{r^2}$ (note that $g \propto \frac{1}{r^2}$ above the Earth's surface)

However the density of the Earth is **not** uniform and this causes an unusual variation of g with radii **inside** the Earth.

Gravitational Potential

We define the gravitational potential (V_p) at a point in a gravitational field to be the work done by external forces in moving unit mass m from infinity to that point.

$$V_p = \frac{\text{work done}}{\text{mass}}$$

We define the theoretical zero of gravitational potential for an isolated point mass to be at **infinity**. (Sometimes it is convenient to treat the surface of the Earth as the practical zero of potential. This is valid when we are dealing with **differences** in potential.)

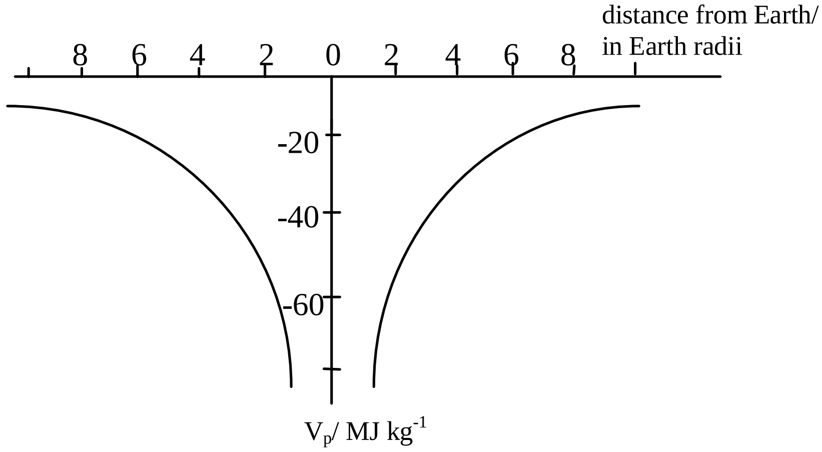
Gravitational Potential at a distance r from mass m

This is given by the equation below.

$$\boxed{\text{gravitational potential } V_P = -\frac{GM}{r}}$$

unit of V_P: J kg^{-1}

The Gravitational Potential 'Well' of the Earth

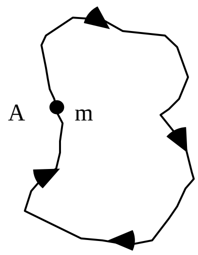


This graph gives an indication of how masses are 'trapped' in the Earth's field.

Conservative field

The force of gravity is known as a conservative force because the work done by the force on a particle that moves through any round trip is zero i.e. energy is conserved. For example if a ball is thrown vertically upwards, it will, if we assume air resistance to be negligible, return to the thrower's hand with the same kinetic energy that it had when it left the hand.

An unusual consequence of this situation can be illustrated by considering the following path taken in moving mass m on a round trip from point A in the Earth's gravitational field. If we assume that the only force acting is the force of gravity and that this acts vertically downward, work is done **only** when the mass is moving vertically, i.e. only vertical components of the displacement need be considered. Thus for the path shown below the work done is zero.



By this argument a non conservative force is one which causes the energy of the system to change e.g. friction causes a decrease in the kinetic energy. Air resistance or surface friction can become significant and friction is therefore labelled as a non conservative force.

Escape Velocity

The escape velocity for a mass m escaping to infinity from a point in a gravitational field is the **minimum velocity** the mass must have which would allow it to escape the gravitational field.

At the surface of a planet the gravitational potential is given by: $V = -\frac{GM}{r}$.

The **potential energy** of mass m is given by **V x m** (from the definition of gravitational potential).

$$E_p = -\frac{GMm}{r}$$

The potential energy of the mass at infinity is zero. Therefore to escape completely from the sphere the mass must be given energy equivalent in size to $\frac{GMm}{r}$.

To escape completely, the mass must just reach infinity with its $E_k = 0$

(Note that the condition for this is that at all points; $E_k + E_p = 0$).

at the surface of the planet $\frac{1}{2} m v_e^2 - \frac{GMm}{r} = 0$ m cancels

$$v_e^2 = \frac{2GM}{r}$$

$$\boxed{v_e = \sqrt{\frac{2GM}{r}}}$$ or greater

Atmospheric Consequences:

$v_{r.m.s.}$ of H_2 molecules = 1.9 km s^{-1} (at 0°C)
$v_{r.m.s.}$ of O_2 molecules = 0.5 km s^{-1} (at 0°C)

When we consider the range of molecular speeds for hydrogen molecules it is not surprising to find that the rate of loss to outer space is considerable. In fact there is very little hydrogen remaining in the atmosphere. Oxygen molecules on the other hand simply have too small a velocity to escape the pull of the Earth.
The Moon has no atmosphere because the escape velocity (2.4 km s^{-1}) is so small that any gaseous molecules will have enough energy to escape from the moon.

Black Holes and Photons in a Gravitational Field

A dense star with a sufficiently large mass/small radius could have an escape velocity greater than 3 x 10^8 m s^{-1}. This means that light emitted from its surface could not escape - hence the name **black hole**.

The physics of the black hole cannot be explained using Newton's Theory. The correct theory was described by Einstein in his General Theory of Relativity (1915). Another physicist called Schwarzschild calculated the radius of a spherical mass from which light cannot escape. It is given here for interest only $r = \frac{2GM}{c^2}$.

Photons are affected by a gravitational field. There is gravitational force of attraction on the photon. Thus photons passing a massive star are **deflected** by that star and stellar objects 'behind' the star appear at a very slightly different position because of the bending of the photon's path.

Further Discussion on Black Holes - for interest only

If a small rocket is fired vertically upwards from the surface of a planet, the velocity of the rocket decreases as the initial kinetic energy is changed to gravitational potential energy. Eventually the rocket comes to rest, retraces its path downwards and reaches an observer near to the launch pad.

Now consider what happens when a photon is emitted from the surface of a star of radius r and mass M. The energy of the photon, hf, decreases as it travels to positions of greater gravitational potential energy but **the velocity of the photon remains the same.** Observers at different heights will observe the frequency and hence the wavelength of the photon changing, i.e. blue light emitted from the surface would be observed as red light at a distance from a sufficiently massive, high density star. (N.B. this is known as the gravitational redshift **not** the well known Doppler redshift caused by the expanding universe).

If the mass and density of the body are greater than certain critical values, the frequency of the photon will decrease to zero at a finite distance from the surface and the photon will not be observed at greater distances.

It may be of interest to you to know that the Sun is not massive enough to become a black hole. The critical mass is around 3 times the mass of our Sun.

Satellites in Circular Orbit

This is a very important application of gravitation.
The central force required to keep the satellite in orbit is provided by the force of gravity.

Thus: $$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}} \qquad \text{but} \qquad v = \frac{2\pi r}{T}$$

$$T = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

Thus a satellite orbiting the Earth at radius, r, has an orbit period, $T = 2\pi\sqrt{\frac{r^3}{GM}}$

Energy and Satellite Motion

Consider a satellite of mass m a distance r from the centre of the parent planet of mass M where M >> m.

Since $$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

Re-arranging, we get $$\frac{1}{2}mv^2 = \frac{GMm}{2r} \; ; \qquad \text{thus} \qquad E_k = \frac{GMm}{2r}$$

Note that E_k is always positive.

But the gravitational potential energy of the system, $E_p = -\frac{GMm}{r}$

Note that E_p is always negative.

Thus the total energy is $$E_{tot} = E_k + E_p$$

$$= \frac{GMm}{2r} + \left[-\frac{GMm}{r}\right]$$

$$E_{tot} = -\frac{GMm}{2r}$$

Care has to be taken when calculating the energy required to move satellites from one orbit to another to remember to include **both** changes in gravitational potential energy and changes in kinetic energy.

Some Consequences of Gravitational Fields

The notes which follow are included as **illustrations** of the previous theory.

Kepler's Laws

Applied to the Solar System these laws are as follows:

- The planets move in elliptical orbits with the Sun at one focus,
- The radius vector drawn from the sun to a planet sweeps out equal areas in equal times.
- The square of the orbital period of a planet is proportional to the cube of the semi-major axis of the orbit.

Tides

The two tides per day that we observe are caused by the **unequal attractions** of the Moon (and Sun) for masses at different sides of the Earth. In addition the rotation of the Earth and Moon also has an effect on tidal patterns.

The Sun causes two tides per day and the Moon causes two tides every 25 hours. When these tides are in phase (i.e. acting together) **spring** tides are produced. When these tides are out of phase **neap** tides are produced. Spring tides are therefore larger than neap tides. The tidal humps are held 'stationary' by the attraction of the Moon and the earth rotates beneath them. Note that, due to tidal friction and inertia, there is a time lag for tides i.e. the tide is not directly 'below' the Moon. In most places tides arrive around 6 hours late.

Variation of "g" over the Earth's Surface

The greatest value for "g" at sea level is found at the poles and the least value is found at the equator. This is caused by the rotation of the earth.

Masses at the equator experience the maximum spin of the earth. These masses are in circular motion with a period of 24 hours at a radius of 6400 km. Thus, part of a mass's weight has to be used to supply the small central force due to this circular motion. This causes the measured value of "g" to be smaller.

Calculation of central acceleration at the equator:

$$a = \frac{v^2}{r} \quad \text{and} \quad v = \frac{2\pi r}{T} \quad \text{giving} \quad a = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 \times 6.4 \times 10^6}{(24 \times 60 \times 60)^2} = 0.034 \text{ m s}^{-2}$$

Observed values for "g": at poles = 9.832 m s^{-2} and at equator = 9.780 m s^{-2}

difference is 0.052 m s^{-2}

Most of the difference has been accounted for. The remaining 0.018 m s^{-2} is due to the non-spherical shape of the Earth. The equatorial radius exceeds the polar radius by 21 km. This flattening at the poles has been caused by the centrifuge effect on the liquid Earth as it cools. The Earth is 4600 million years old and is still cooling down. The poles nearer the centre of the Earth than the equator experience a greater pull.

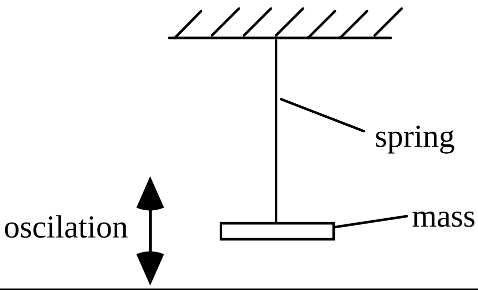
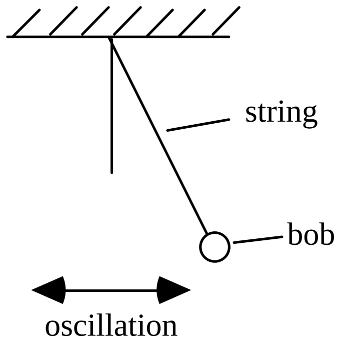
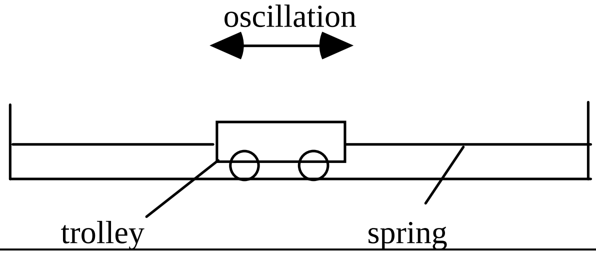
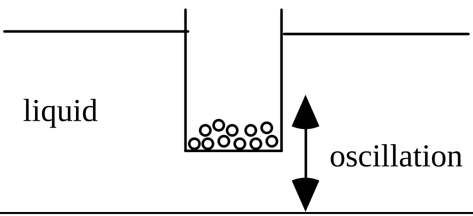
In Scotland "g" lies between these two extremes at around 9.81 or 9.82 m s^{-2}. Locally "g" varies depending on the underlying rocks/sediments. Geologists use this fact to take *gravimetric surveys* before drilling. The shape of underlying strata can often be deduced from the variation of "g" over the area being surveyed. Obviously very accurate means of measuring "g" are required.

SIMPLE HARMONIC MOTION (SHM)

If an object is subject to a linear restoring force, it performs an oscillatory motion termed ‘simple harmonic’. Before a system can perform oscillations it must have (1) a means of storing potential energy and (2) some mass which allows it to possess kinetic energy. In the oscillating process, energy is continuously transformed between potential and kinetic energy.

Note: any motion which is periodic and complex (i.e. not simple!) can be analysed into its simple harmonic components (Fourier Analysis). An example of a complex waveform would be a sound wave from a musical instrument.

Examples of SHM

Example and Diagram	E_p stored as:	E_k possesed by moving:
mass on a coil spring 	elastic energy of spring	mass on spring
simple pendulum 	potential energy (gravitational) of bob	mass of the bob
trolley tethered between springs 	elastic energy of the springs	mass of the trolley
weighted tube floating in a liquid 	potential energy (gravitational) of the tube	mass of the tube

Note that for the mass oscillating on the spring, there is always an **unbalanced** force acting on the mass and this force is always **opposite** to its direction of motion. The unbalanced force is momentarily zero as the mass passes through the rest position.

To see this, consider the following: when the mass is moving upwards past the rest position, the gravitational force (**downwards**) is greater than the spring force. Similarly when moving downwards past the rest position, the spring force (**upwards**) is greater than the gravitational force downwards.

This situation is common to all SHMs. The force which keeps the motion going is therefore called the **restoring** force.

Definition of Simple Harmonic Motion

When an object is displaced from its equilibrium or at rest position, and the unbalanced force is proportional to the displacement of the object and acts in the opposite direction, the motion is said to be simple harmonic.

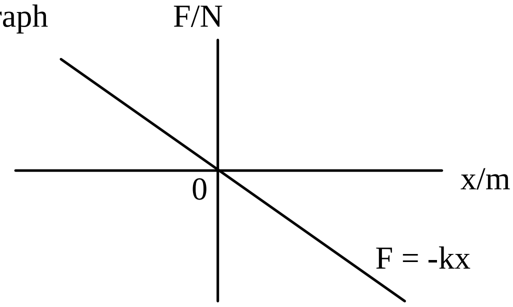
Graph of Force against displacement for SHM

$$F = -kx$$

F is the restoring force (N)
k is the force constant ($N\ m^{-1}$)
x is the displacement (m)

The negative sign shows the direction of vector F is always opposite to vector x.

graph



If we apply Newton's Second Law in this situation the following alternative definition in terms of acceleration as opposed to force is produced.

$$F = ma = m\frac{d^2x}{dt^2} = -kx$$

$$a = -\frac{k}{m}x \qquad \text{thus} \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Remember that k is a force constant which relates to the oscillating system.

The constant, $\frac{k}{m}$ is related to the period of the motion by $\omega^2 = \frac{k}{m}$, $\omega = \frac{2\pi}{T}$

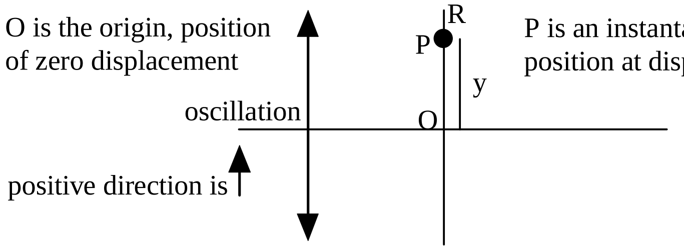
This analysis could equally well have been done using the y co-ordinate.

Thus an equivalent expression would be $\boxed{\frac{d^2y}{dt^2} = -\omega^2 y}$.

All future equations will use y for displacement

Kinematics of SHM

Point P is oscillating with SHM between two fixed points R and S. The amplitude of the oscillation is therefore ½ RS and this is given the symbol a. The displacement y is the vector OP.



The period, T, of the motion is the time taken to complete one oscillation, e.g. path O->R->O ->S->O.
The frequency, f, is the number of oscillations in one second.

$$\boxed{f = \frac{1}{T}} \qquad \text{and because } \omega = \frac{2\pi}{T} \qquad \boxed{\omega = 2\pi f}$$

Solutions of Equation for SHM

The equation $\frac{d^2y}{dt^2} = -\ \omega^2 y$ could be solved using integration to obtain equations for velocity v and displacement y of the particle at a particular time t. However the calculus involves integration which is not straightforward. We will therefore start with the solutions and use differentiation.

The possible solutions for the displacement y at time t depend on the **initial conditions** and are given by:

$$\boxed{y = a\cos\omega t \text{ if } y = 0 \text{ at } t = 0 \qquad \text{and} \qquad y = a\sin\omega t \text{ if } y = a \text{ at } t = 0}$$

Acceleration

Differentiating $\frac{dy}{dt} = \frac{d}{dt}(a\cos\omega t)$

$= -\ a\omega\sin\omega t$

Differentiating again $\frac{d^2y}{dt^2} = -\ a\omega^2\cos\omega t$

but $y = a\cos\omega t$ $\frac{d^2y}{dt^2} = -\ \omega^2 y$

$\frac{dy}{dt} = \frac{d}{dt}(a\sin\omega t)$

$= a\omega\cos\omega t$

$\frac{d^2y}{dt^2} = -\ a\omega^2\sin\omega t$

$\frac{d^2y}{dt^2} = -\ \omega^2 y$ ($y = a\sin\omega t$)

Velocity

$v = \frac{dy}{dt} = -\ a\omega\sin\omega t$

$v^2 = a^2\omega^2\sin^2\omega t$ and $y^2 = a^2\cos^2\omega t$

$\sin^2\omega t + \cos^2\omega t = 1$

Thus $\frac{v^2}{a^2\omega^2} + \frac{y^2}{a^2} = 1$

$v^2 = \omega^2(a^2 - y^2)$

Thus $\boxed{v = \pm\,\omega\sqrt{a^2 - y^2}}$

$v = \frac{dy}{dt} = a\omega\cos\omega t$

$v^2 = a^2\omega^2\cos^2\omega t$ and $y^2 = a^2\sin^2\omega t$

$\cos^2\omega t + \sin^2\omega t = 1$

Thus $\frac{v^2}{a^2\omega^2} + \frac{y^2}{a^2} = 1$

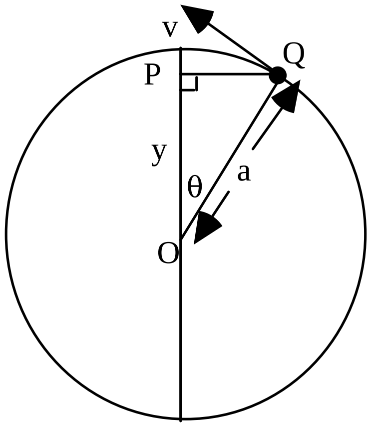
$v^2 = \omega^2(a^2 - y^2)$

Thus $\boxed{v = \pm\,\omega\sqrt{a^2 - y^2}}$

Linking SHM with Circular Motion

This allows us to examine the mathematics of the motion and is provided for interest.

If the point Q is moving at constant speed, v, in a circle, its projection point P on the y axis will have **displacement** $y = a \cos \theta$



positive direction of y is upwards

note that $\sin \theta = \frac{QP}{OQ}$

$\sin \theta = \frac{\sqrt{a^2 - y^2}}{a}$

radius OQ sweeps out ω rad s^{-1}

The **velocity** of point P is: $v_p = \frac{d(y)}{dt} = \frac{d}{dt}(a \cos \theta)$ and $\theta = \omega t$

$v_p = \frac{d}{dt}(a \cos \omega t)$ $\boxed{v_p = - a\omega \sin \omega t}$ (negative sign: assume P moving down)

Special cases: when y = 0, $\theta = \frac{\pi}{2}$ and $\sin \theta = 1$

$\boxed{v_{max} = \pm a\omega}$ and occurs as P goes through the origin in either direction.

when $y = \pm a$, $\theta = 0$ or π and $\sin \theta = 0$

$\boxed{v_{min} = 0}$ and occurs as P reaches the extremities of the motion.

The **acceleration** of point P is: $acc_p = \frac{dv_p}{dt} = \frac{d}{dt}(-a\omega \sin \omega t)$

$\boxed{acc_p = - a\omega^2 \cos \omega t}$

Special cases: when y = 0, $\theta = \frac{\pi}{2}$ and $\cos \theta = 0$

$\boxed{acc_{min} = 0}$ and occurs as P goes through the origin in either direction.

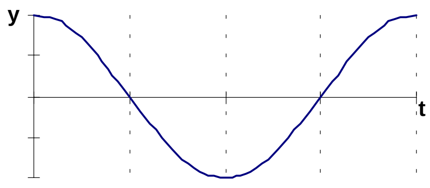
when $y = \pm a$, $\theta = 0$ or π and $\cos \theta = 1$

$\boxed{acc_{max} = \pm a\omega^2}$ and occurs as P reaches the extremities of the motion.

Note: the acceleration is negative when the displacement, y, is positive and vice versa; i.e. they are out of phase, see graphs of motion below. Knowledge of the positions where the particle has maximum and minimum acceleration and velocity **is required**

To understand these graphs it is helpful if you see such graphs being generated using a motion sensor. In particular, pay close attention to the phases of the graphs of the motion and note that the basic shape is that of the sine/cosine graphs.

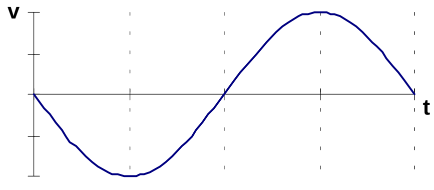
Displacement-time



Summary of Equations

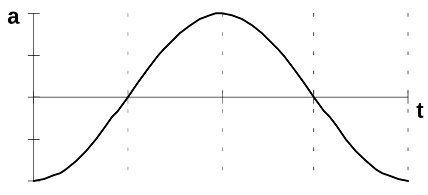
$y = a \cos \omega t$

Velocity-time



$v = \pm a\omega \sin \omega t = \pm \omega\sqrt{a^2 - y^2}$

Acceleration-time



$acc = - a\omega^2 \cos \omega t$

substitute for y:

$acc = - \omega^2 y$

Note that this form, acceleration = $- \omega^2 y$, is consistent with our definition of SHM ω^2 is a positive constant. This implies that the sine and cosine equations must be solutions of the motion.

Compare this constantly **changing** acceleration with situation where only **uniform** acceleration was considered.

The equation used in a particular situation **depends on the initial conditions.**

Thus: if $y = 0$ at time $t = 0$ use $y = a \sin\omega t$

if $y = a$ at time $t = 0$ use $y = a \cos\omega t$

Another possible solution for SHM is: $y = a \sin(\omega t + \phi)$ where ϕ is known as the phase angle.

Example

An object is vibrating with simple harmonic motion of amplitude 0.02 m and frequency 5.0 Hz. Assume that the displacement of the object, y = 0 at time, t = 0 and that it starts moving in the positive y-direction.

(a) Calculate the maximum values of velocity and acceleration of the object.

(b) Calculate the velocity and acceleration of the object when the displacement is 0.008 m.

(c) Find the time taken for the object to move from the equilibrium position to a displacement of 0.012 m.

Solution

Initial conditions require; $y = a \sin \omega t$; $v = a\omega \cos \omega t$; and $acc = -\omega^2 x$

$f = 5$ Hz $\quad \omega = 2\pi f = 31.4$ rad s^{-1}

(a) $v_{max} = \omega a = 31.4 \times 0.02 = 0.63$ m s^{-1}

$acc_{max} = -\omega^2 a = -(31.4)^2 \times 0.02 = -19.7$ m s^{-2}

(b) $v = \pm \omega\sqrt{a^2 - y^2} = \pm 31.4\sqrt{0.02^2 - 0.008^2} = \pm 0.58$ m s^{-1}

$acc = -\omega^2 y = -31.4^2 \times 0.008 = -7.9$ m s^{-2}

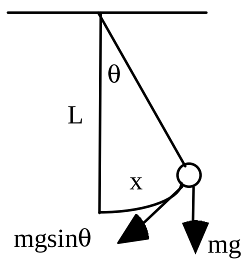
(c) use $y = a \sin \omega t$; $0.012 = 0.02 \sin 31.4t$ (when $y = 0.012$ m)

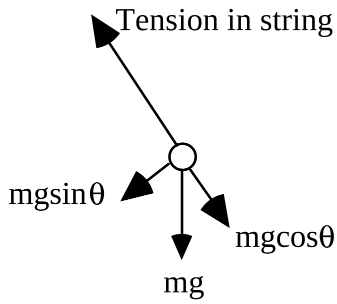
$\sin 31.4\,t = \dfrac{0.012}{0.02} = 0.6$ giving $31.4\,t = 0.644$ and $t = \dfrac{0.644}{31.4}$

Thus $t = 0.0205$ s (Remember that angles are in radians)

Optional Extra *Proof that the Motion of a Simple Pendulum approximates to SHM*

The sketches below show a simple pendulum comprising a point mass, m, at the end of an inextensible string of length L. The string has negligible mass.





The restoring force F on the bob is $F = -mg\sin\theta$

If the ***angle*** θ ***is small*** *(less than about* 10^0*) then* $\sin\theta = \theta$ *in radians and* $\theta = \dfrac{x}{L}$

Then $F = -mg\theta = -mg\dfrac{x}{L}$ *Thus* $F = -\dfrac{mg}{L}x$

The restoring force therefore satisfies the conditions for SHM for small displacements.

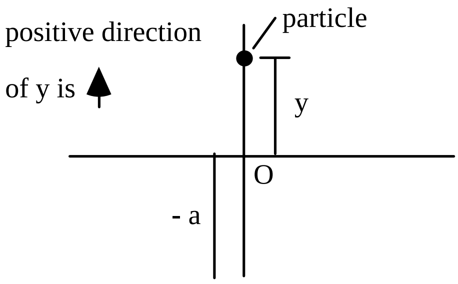
Then acceleration is $a = -\dfrac{g}{L}x$ *which if compared with* $a = -\omega^2 x$ *gives* $\omega^2 = \dfrac{g}{L}$ $(\omega = 2\pi f)$

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$ *and the period of the pendulum* $\boxed{T = 2\pi\sqrt{\dfrac{L}{g}}}$

Energy Equations for SHM.

Consider the particle moving with simple harmonic motion below.

The particle has maximum amplitude a and period $T = \frac{2\pi}{\omega}$



Kinetic energy equation for the particle

$$E_k = \frac{1}{2} m v^2 \qquad = \frac{1}{2} m [\pm \omega \sqrt{a^2 - y^2}\ ]^2$$

$$\boxed{E_k = \frac{1}{2} m \omega^2 (a^2 - y^2)}$$

Potential energy equation for the particle

When at position O the potential energy is zero, (with reference to the equilibrium position) and the **kinetic energy is a maximum**.

The kinetic energy is a maximum when y = 0: $E_{kmax} = \frac{1}{2} m \omega^2 a^2$

At point O total energy $E = E_k + E_p = \frac{1}{2} m \omega^2 a^2 + 0$

$$E = \frac{1}{2} m \omega^2 a^2 \quad \text{or} \quad E = \frac{1}{2} k a^2 \quad \text{because } \omega^2 = \frac{k}{m}$$

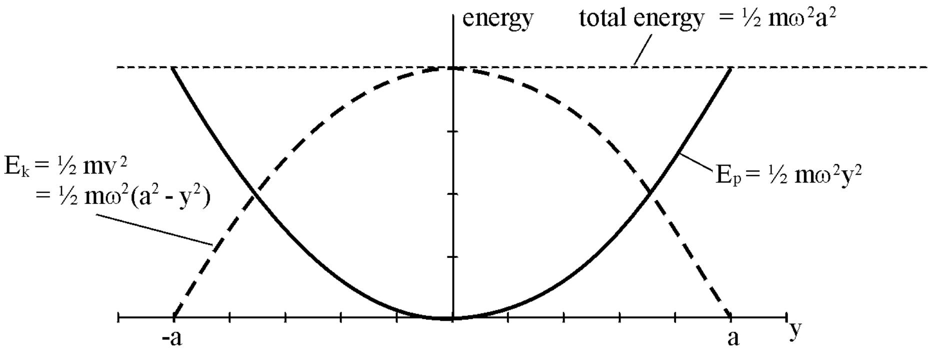
The **total energy E** is the **same at all points** in the motion.

Thus for any point on the swing: as above $E = E_k + E_p$

$$\frac{1}{2} m \omega^2 a^2 = \frac{1}{2} m \omega^2 (a^2 - y^2) + E_p$$

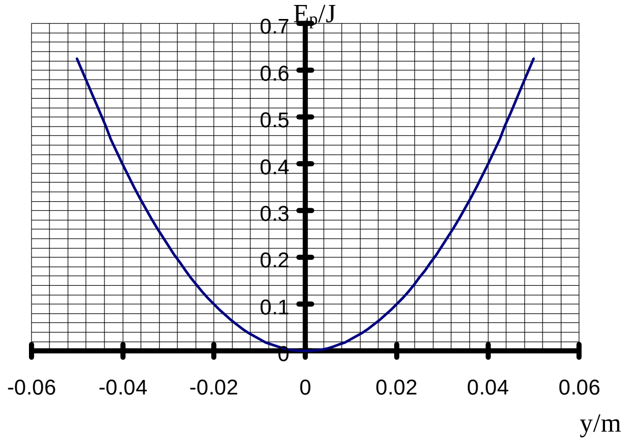
$$\boxed{E_p = \frac{1}{2} m \omega^2 y^2}$$

The graph below shows the relation between potential energy, E_p, kinetic energy E_k, and the total energy of a particle during SHM as amplitude y changes from - a to + a.



Example on energy and SHM

The graph below shows how the potential energy, E_p, of an object undergoing SHM, varies with its displacement, y. The object has mass 0.40 kg and a maximum amplitude of 0.05 m.



(a) (i) Find the potential energy of the object when it has a displacement of 0.02 m.

(ii) Calculate the force constant, k for the oscillating system.

(k should have unit N m^{-1}).

(b) Find the amplitude at which the potential energy equals the kinetic energy.

Solution

(a) (i) *From graph* $E_p = 0.10$ J

(ii) $E_p = \frac{1}{2} k y^2$

$0.1 = \frac{1}{2} k (0.02)^2$

$k = \frac{0.2}{(0.02)^2} = 500 \text{ N m}^{-1}$

(b)

$E_p = E_k$

$\frac{1}{2} k y^2 = \frac{1}{2} m \omega^2 (a^2 - y^2)$

$= \frac{1}{2} k (a^2 - y^2)$ since $\omega^2 = \frac{k}{m}$

$y^2 = a^2 - y^2$ or $2y^2 = a^2$

$y = \frac{a}{\sqrt{2}}$ when $E_p = E_k$

$y = \frac{0.05}{\sqrt{2}} = 0.035$ m

Damping of Oscillations

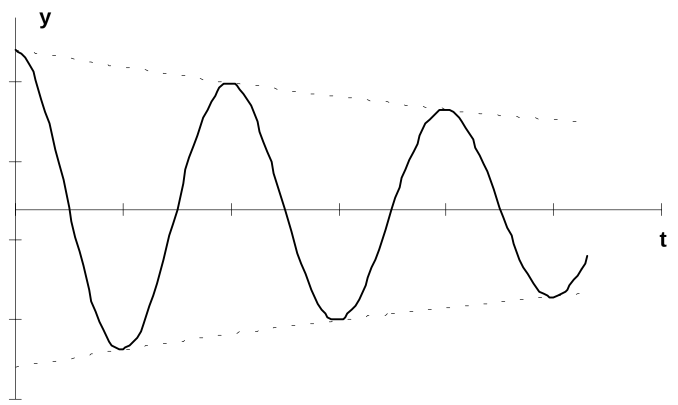
Oscillating systems, a mass on a spring, a simple pendulum, a bobbing mass in water, all come to rest eventually. We say that their motion is **damped**. This means that the amplitude of the motion decreases to zero because energy is transformed from the system. A simple pendulum takes a long time to come to rest because the frictional effect supplied by air resistance is small - we say that the pendulum is lightly damped. A tube oscillating in water comes to rest very quickly because the friction between the container and the water is much greater - we say that the tube is heavily damped.

If the damping of a system is increased there will be a value of the frictional resistance which is just sufficient to prevent any oscillation past the rest position - we say the system is **critically** damped. Systems which have a very large resistance, produce no oscillations and take a long time to come to rest are said to be **overdamped**. In some systems overdamping could mean that a system takes longer to come to rest than if underdamped and allowed to oscillate a few times.

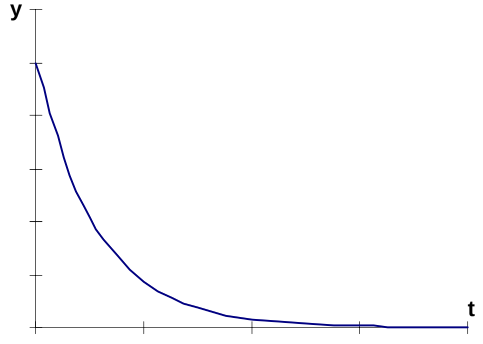
An example of damped oscillations is a car shock absorber which has a very thick oil in the dampers. When the car goes over a bump, the car does not continue to bounce for long. Ideally the system should be critically damped. As the shock absorbers get worn out the bouncing may persist for longer.

The graphs below give a graphical representation of these different types of damping.

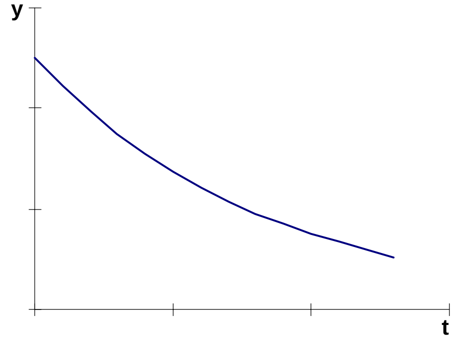
Damped oscillations



Critically damped



Overdamped



WAVE-PARTICLE DUALITY

Introduction

You have already seen in the Higher Physics course that the phenomenon of interference of light, as demonstrated in grating experiments, is evidence for the wave model of light. In contrast the photoelectric effect can be explained in terms of the particle model of light. In this model light is made up of photons each having an energy hf where h is Planck's constant and f is the frequency of the light. This wave-particle duality applies to all forms of electromagnetic radiation.

Radiation as particles

For a photon the energy E is given by E = hf. In 1923 Compton investigated the scattering of X-rays by matter. It appeared that the photons collided elastically with electrons in the material. It was noted that the frequency of the X-rays was reduced.

The momentum, p, of the photon depends on the energy, E, of the photon as $p = \frac{E}{c}$.

Combining the two equations E = hf

and $p = \frac{E}{c}$ gives $p = \frac{hf}{c} = \frac{h}{\lambda}$ and

$$\text{wavelength } \lambda = \frac{h}{p}$$

Particles as Waves

Electrons have been regarded as showing particle like properties e.g. the deflection path of a stream of electrons is just like the path of a projectile. Electrons do however show wave properties as for example in **electron diffraction**.

The French physicist de Broglie in 1924 proposed that an electron of mass m moving with a speed v would have a wavelength given by

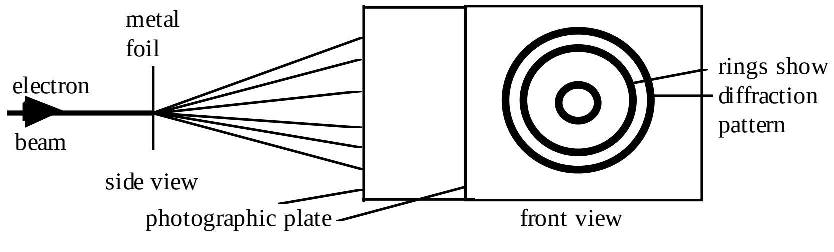
$$\lambda = \frac{h}{p}$$

This equation gives the wavelength of any material particle given its momentum.

De Broglie's hypothesis was verified experimentally by Davisson and Germer (USA) in 1927 and in 1928 by G P Thomson and A Reid in Aberdeen. Davisson and G.P.Thomson shared the Nobel prize in 1937 for their discovery. It is interesting to note that J J Thomson, the father of G P Thomson was awarded the Nobel prize in 1907 for demonstrating the particle nature of electrons.

Electron Diffraction

In the Thomson-Reid experiment a narrow beam of electrons, accelerated through a potential difference of several tens of kV, was "scattered" as it passed through a thin metal foil. The metal consists of a random array of microscopic crystals.



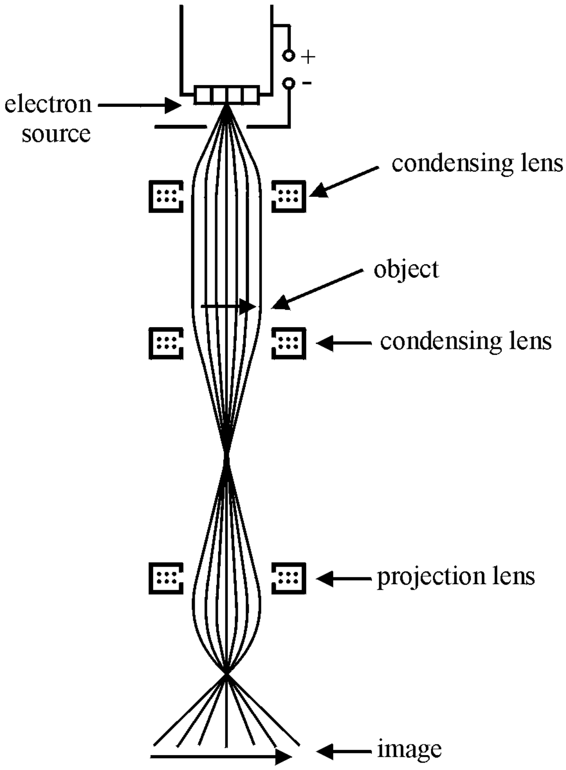
A ring system was recorded on a photographic plate. This ring system was in accordance with the Bragg theory for the behaviour of X-rays when diffracted.

The Electron Microscope

Interference of light waves is readily demonstrated when the dimensions of the "scattering" object are less than about 10 wavelengths. Extending this to particles, the dimensions of a "scattering object" should be less than 10 "de Broglie" wavelengths.

For a macroscopic particle, even something as small as a speck of dust of mass 1 mg and moving at 1 mm s^{-1} the de Broglie wavelength is so small that the particle's motion is adequately described in terms of the laws of classical mechanics. In this example of the dust particle, $\lambda = 6.63 \times 10^{-22}$ m which is many orders of magnitude smaller than even nuclear dimensions.

In an electron microscope a beam of electrons is accelerated through a p.d. of 100 kV.



A relativistic calculation shows that the de Broglie wavelength is about 0.004 nm. Such a beam is therefore suitable for examining matter on an atomic scale. In practice non-uniformities in the "optics" of the electron microscope limit the resolution to about 0.4 nm which is still 1000 times better than an optical microscope.

The Bohr Model of the Atom

The exerimental results of Geiger and Marsden on the scattering of α - particles by gold foil were interpreted by Rutherford in terms of a new model of the atom, one in which the positive charges in an atom are concentrated in what came to be called the nucleus.
Niels Bohr then suggested a model of the atom in which electrons revolve in dynamic equilibrium round the nucleus. The **electrons occupy only certain allowed orbits.** This solved the problem of the existence of spectra composed of well defined frequencies, because radiation occurs only when an electron 'jumps' from one orbit to another of lower energy. The energy of the radiation emitted is given by:

$$hf = E_n - E_m$$

Quantisation of Angular Momentum

Bohr postulated that the angular momentum of an electron was quantised in units of $\frac{h}{2\pi}$

$$\boxed{mvr = \frac{nh}{2\pi}}$$ where n is an integer.

Example

An electron in the ground state orbits around the hydrogen nucleus. The speed of the electron in the orbit is 2.2×10^6 m s^{-1}

(a) Find the de Broglie wavelength of the electron.

(b) Calculate the radius of the ground state orbit.

Solution

(a) $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.2 \times 10^6} = 3.3 \times 10^{-10}$ m

(b)

$$mvr = \frac{nh}{2\pi}$$

$$r = \frac{nh}{2\pi mv} = \frac{1 \times 6.63 \times 10^{-34}}{2\pi \times 9.1 \times 10^{-31} \times 2.2 \times 10^6}$$

$$= 5.3 \times 10^{-11} \text{ m}$$

One of the problems with this model is that classical physics predicted that charged electrons moving in orbit in this way are accelerating and should therefore lose energy by radiating electromagnetic waves. The atom would therefore be unstable and the electrons would spiral inwards towards the nucleus.

The **stability** of the atom is now explained in terms of quantum theory and the Rutherford-Bohr model has been superseded by the work of Schrodinger and Dirac. The Bohr model is however still a useful one.

Quantum Mechanics and Probability

The Bohr theory was developed in 1913 and has been superseded by much more advanced theories. It remains a useful introduction to atomic theory and still gives some valid results.

In modern theory the wave particle-duality has been superseded by describing all matter in terms of wave fuctions**.** These waves have no physical significance but are useful as a computational device. The square of the amplitude of the wave function can be observed and is interpreted as a **probability.** This far-reaching model of atomic and nuclear structure interprets waves in terms of probabilities.

In the old Physics the attempt was to predict exactly what will happen in certain circumstances. The predictions of the new Physics are in terms of probabilities. As applied to atoms, the electrons are no longer thought of as point objects travelling in orbits. There are just regions in which the probability of the electron being found is high. The positions of the maximum probabilities for the hydrogen atom correspond to the radii of the Bohr orbitals and the expression for the allowed energy values is exactly as predicted by Bohr.

Quantum Mechanics therefore provides methods to allow us to calculate probabilities, for example the probability of finding an electron at a particular point in the space around an atom.

Heisenberg Uncertainty Principle: optional extra

We must remember that at the level of the atom, we are dealing with matter on a very small scale. To arrive at measurements relating to an electron in an atom we need to probe that atom with a photon. Now the photon, in probing into the atom, will affect the electron it is trying to locate!

We have experience of this on a rather larger scale where a glass thermometer is used to measure the temperature of a small amount of hot liquid. The thermometer itself will affect the measured result - in this case give a lower reading than it should.

In 1928 Heisenberg showed that the energy of a particle in a small time interval Δt could not be measured exactly. Its value would be within a range ΔE such that:

$$\Delta E \times \Delta t \approx h.$$

On a large scale the uncertainty in energy measurement is not significant. However if we try to estimate the energy of a photon as it crosses an atom ($\Delta t = 1 \times 10^{-22}$ s) the uncertainty ΔE works out to be around 1×10^{-12} J which is significant.

This uncertainty principle also applies to attempts to measure the position, x, and the momentum, p, of a particle. In measuring one we alter the other. The uncertainty in the two measurements is given by:

$$\Delta x \, \Delta p_x \approx h$$

where p_x is the component of the momentum in the x direction.
If we wish to measure the position exactly, the momentum, and hence the velocity, would be completely unknown. The more precisely we measure one of these quantities the more ‘uncertain’ we must be of the other quantity.

TUTORIAL 1

You will find tutorials on each topic. The fully worked out answers are available. The idea is that you check your work yourself as far as possible and consult your teacher/lecturer when you have problems.

Equations of Motion and Relativistic Dynamics

1 A projectile is launched from the top of a building with an initial speed of 20 m s^{-1} at an angle of 30° to the horizontal. The height of the building is 30 m.

(a) Calculate how long it takes the projectile to reach the ground.
(b) Calculate the velocity of the projectile on impact with the ground, (magnitude and direction).

2 Calculate the mass of an electron which has a velocity of 1 x 10^8 m s^{-1}.

3 Protons can be accelerated to very high speeds in particle accelerators. Calculate the speed required to increase the proton's rest mass by 50%.

4 A jet aircraft of mass 8000 kg is travelling at Mach 2. (Mach 1 = speed of sound in air).

(a) Using the relation $m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$, calculate the relativistic mass of the aircraft.
(b) Is your answer significantly different from the rest mass?

5 When a particle in an accelerator reaches a certain speed the ratio of $\dfrac{m}{m_0}$ is 4.0.

(a) Calculate the velocity of the particle for this condition.
(b) The velocity now increases until $\dfrac{m}{m_0}$ rises to 8.0.
Show by calculation whether this means the velocity also doubles.

6 An electron is injected into an accelerator at an initial speed of 1 x 10^7 m s^{-1}. It is subsequently accelerated to 2.5 x 10^8 m s^{-1}.
Calculate the momentum of the electron at the start and end of the process.

Data

speed of light in vacuum	c	3 x 10^8 m s^{-1}
mass of electron at rest	m_e	9.11 x 10^{-31} kg
speed of sound in air	v	3.4 x 10^2 m s^{-1}

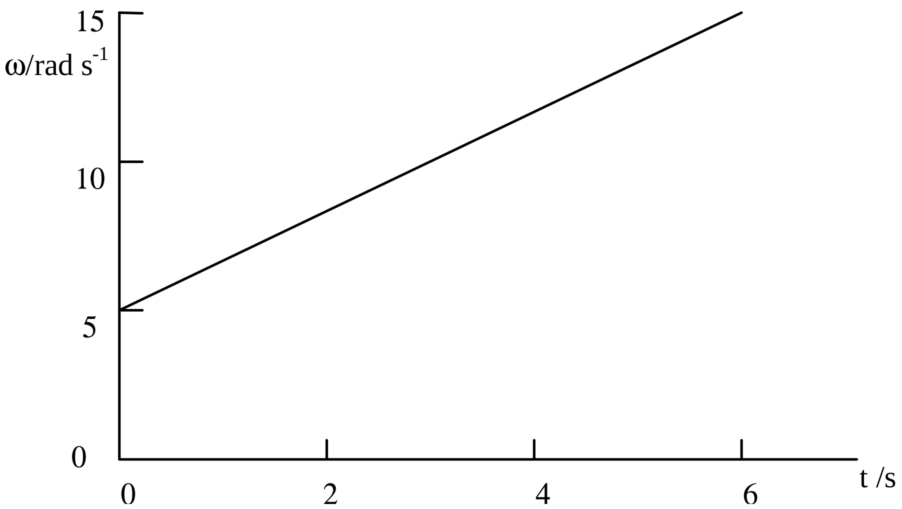
TUTORIAL 2

Angular Motion

1 If 2π radians equals $360°$, calculate the number of degrees in one radian.

2 Calculate the angular velocity in rad s^{-1} of the second hand of an analogue watch.

3 The graph below shows the variation of angular velocity with time for a rotating body.



(a) Find the angular displacement θ covered in the first 3 seconds.

(b) Find the total angular displacement for the 6 seconds.

(c) Calculate the angular acceleration of the rotating body.

4 A wheel accelerates uniformly from rest. After 12 s the wheel is completing 100 revolutions per minute (r.p.m.)

(a) Convert 100 r.p.m. to its equivalent value in rad s^{-1}.

(b) Calculate the average angular acceleration of the wheel.

5 The angular velocity of a car engine's drive shaft is increased from 100 rad s^{-1} to 300 rad s^{-1} in 10 s.

(a) Calculate the angular acceleration of the drive shaft.

(b) Calculate the angular displacement during this time.

(c) A point on the rim of the drive shaft is at a radius of 0.12 m.

Calculate the distance covered by this point in the 10 s time interval.

6 Use calculus methods to derive the equations for angular motion. The method is very similar to that for linear motion seen previously (page 2).

Note: in the unit or course assessment you may be asked to derive the linear motion equations but **not** the angular motion equations.

TUTORIAL 3

Circular Motion

1 An Earth satellite is required to be in a circular orbit at a distance of 7.5×10^6 m from the centre of the Earth. The central force is due to the gravitational force. The acceleration due to the Earth's gravity at this point is 7.0 m s^{-2}
Find:
(a) the required satellite speed
(b) the period of revolution of the satellite.

2 What would be the period of rotation of the Earth about its axis if its speed of rotation increased to such an extent that an object at the equator became 'weightless'?
(Hint: equate mg to $\frac{mv^2}{r}$).

3 A sphere of mass 0.20 kg is rotating in a circular path at the end of a string 0.80 m long. The other end of the string is fixed. The period of the motion is 0.25 s.

(a) Calculate the tension in the string, which you may assume to be horizontal.
(b) In practice the string is not horizontal. Explain why this is so.
(c) Draw a force diagram for the sphere.
From this calculate the angle the string would make with the horizontal.

4 The moon takes 27.3 days (2.0×10^6 s) to complete one orbit of the Earth.
The distance between the centres of the Earth and Moon is 4.0×10^8 m.
Calculate the magnitude of the Moon's acceleration towards the Earth.

5 A ball of mass 2.0 kg is attached to a string 1.6 m long and is made to travel in a vertical circle. The ball passes its highest point with a speed of 5.0 m s^{-1}.
(a) What is the kinetic energy of the ball at its highest point?
(b) What is its potential energy when it is at the highest point (with reference to its lowest point)?
(c) What is its kinetic energy at the lowest point?
(d) What is its speed at the lowest point?
(e) What is the tension in the string at the highest and lowest points?
(f) What is the **least** speed the ball could have at the highest point in order to be able to complete a vertical circle at all?

6 An old humpback bridge has a radius of curvature of 20 m. What is the maximum speed at which a car can pass over this bridge if the car is not to leave the road surface?

7 (a) A pail of water is swinging in a vertical circle of radius 1.2 m, so that the water does not fall out. What is the minimum linear speed required for the pail of water.

(b) Convert this speed into an angular velocity.

8 An object of mass 0.20 kg is connected by a string to an object of half its mass. The smaller mass is rotating at a radius of 0.15 m on a table which has a frictionless surface. The larger mass is suspended through a hole in the middle of the table.

Calculate the number of revolutions per minute the smaller mass must make so that the larger mass is stationary.

Banking of a Track

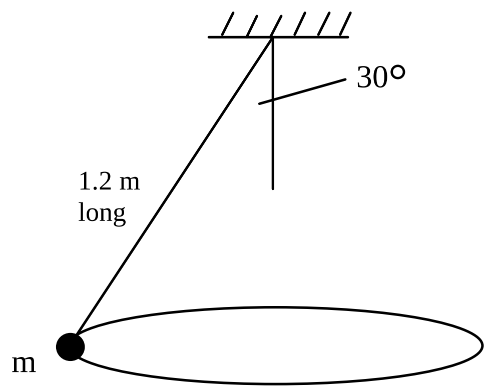
9 A circular track of radius 60 m is banked at angle θ. A car is driven round the track at 20 m s^{-1}.

(a) Draw a diagram showing the forces acting on the car.

(b) Calculate the angle of banking required so that the car can travel round the track without relying on frictional forces (i.e. no side thrust supplied by friction on the track surface).

Conical Pendulum

10 A small object of mass m revolves in a horizontal circle at constant speed at the end of a string of length 1.2 m. As the object revolves, the string sweeps out the surface of a right circular cone.



The cone has semi-angle 30°.
Calculate:
(a) the period of the motion
(b) the speed of the object.

[Hint: try resolving the tension in the string into horizontal and vertical components.]

TUTORIAL 4

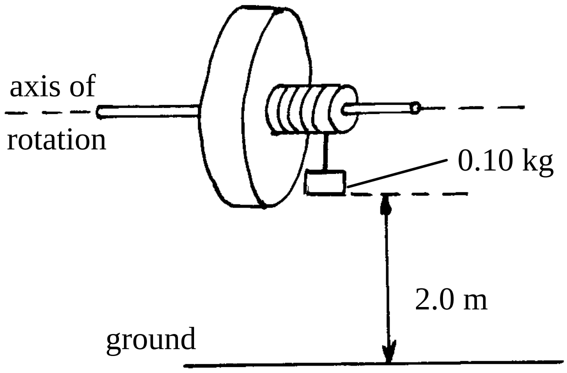
Torque, Moments of Inertia and Angular Momentum

1 A flywheel has a moment of inertia of 1.2 kg m^2. The flywheel is acted on by a torque of magnitude 0.80 N m.

(a) Calculate the angular acceleration produced.

(b) The torque acts for 5.0 s and the flywheel starts from rest. Calculate the angular velocity at the end of the 5.0 s.

2 A mass of 0.10 kg is hung from the axle of a flywheel as shown below. The mass is released from a height of 2.0 m above ground level.



The following results were obtained in the experiment:

time for mass to fall to the ground t = 8.0 s

radius of axle R = 0.10 m.

(a) By energy considerations, show that the final speed of the flywheel is given by $v = \sqrt{\dfrac{3.92}{0.1 + 100I}}$ where *I* is the moment of inertia of the flywheel. Friction effects are ignored.

(b) Calculate the moment of inertia of the flywheel.

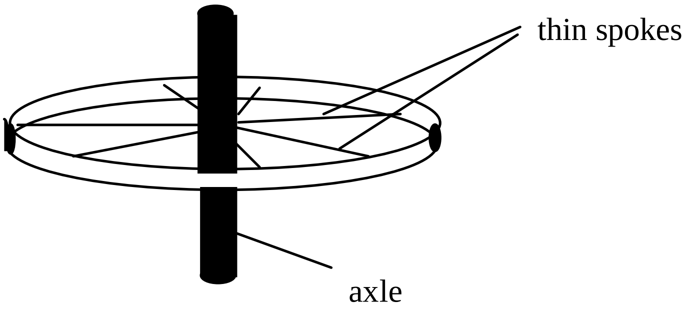
3 A heavy drum has a moment of inertia of 2.0 kg m^2. It is rotating freely at 10 rev s^{-1} and has a radius of 0.50 m. A constant frictional force of 5.0 N is then exerted at the rim of the drum.

(a) Calculate the time taken for the drum to come to rest.

(b) Calculate the angular displacement in this time.

(c) Hence calculate the heat generated in the braking action.

4 A cycle wheel is mounted so that it can rotate horizontally as shown.
Data on wheel: radius of wheel = 0.50 m, mass of wheel = 2.0 kg.



(a) Calculate the moment of inertia of the wheel system. State any assumptions you make.

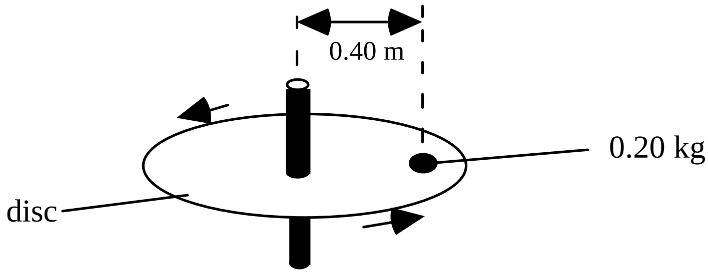
(b) A constant driving force of 20 N is applied to the rim of the wheel.

(i) Calculate the magnitude of the driving torque on the wheel.
(ii) Calculate the angular acceleration of the wheel.

(c) After a period of 5.0 s, calculate:

(i) the angular displacement,
(ii) the angular momentum of the wheel, and
(iii) the kinetic energy of the wheel.

5 A very light but strong disc is mounted on a free turning bearing as shown below.



A mass of 0.20 kg is placed at a radius of 0.40 m and the arrangement is set rotating at 1.0 rev s^{-1}.
(The moment of inertia of the disc can be considered to be negligible.)

(a) Calculate the angular momentum of the 0.20 kg mass.

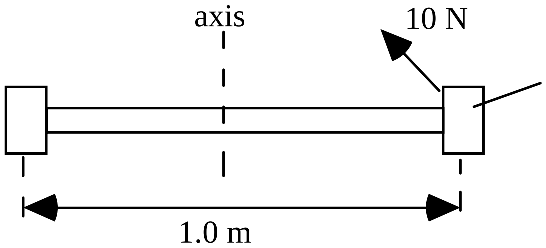
(b) Calculate the kinetic energy of the mass.

(c) The mass is pushed quickly into a radius of 0.20 m.

By applying the principle of conservation of angular momentum, calculate the new angular velocity of the mass in rad s^{-1}.

(d) Find the new kinetic energy of the mass and account for any difference.

6 A uniform metal rod has a mass, M, of 1.2 kg and a length, L, of 1.0 m. Clamped to each end of the rod is a mass of 0.50 kg as shown below.



(a) Calculate an approximate value for the moment of inertia of the complete arrangement about the central axis as shown. Assume that $I_{rod} = \frac{1}{12} ML^2$ about this axis.

(b) The arrangement is set rotating by a force of 10 N as shown in the diagram. The force acts at a tangent to the radius.

(i) Calculate the applied torque.

(ii) Hence find the maximum angular acceleration. You may assume that the force of friction is negligible.

(iii) Calculate the kinetic energy of the arrangement 4.0 s after it is set rotating.

7 An unloaded flywheel, which has a moment of inertia of 1.5 kg m^2, is driven by an electric motor . The flywheel is rotating with a constant angular velocity of 52 rad s^{-1}. The driving torque, of 7.7 Nm, supplied by the motor is now removed.

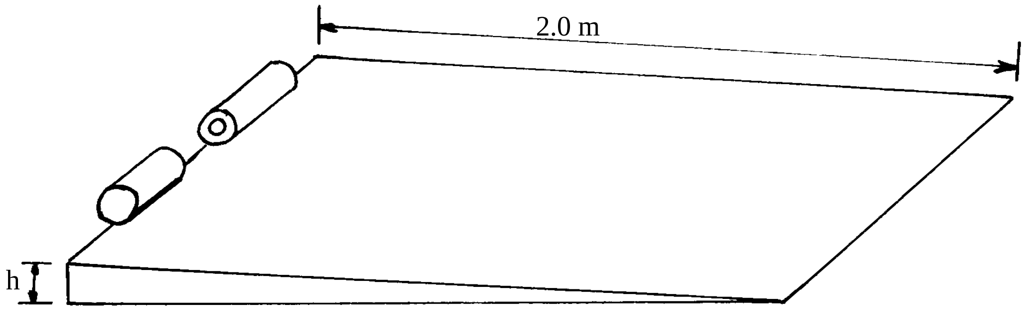
How long will it take for the flywheel to come to rest.
You may assume that the frictional torque remains constant?

8 A solid aluminium cylinder and a hollow steel cylinder have the same mass and radius. The two cylinders are released together at the top of a slope.

(a) Which of the two cylinders will reach the bottom first?

(b) Explain your answer to part (a).

9 A solid cylinder and a hollow cylinder each having the same mass M and same outer radius R, are released at the same instant at the top of a slope 2.0 m long as shown below.
The height of the slope is 0.04 m.



h = 0.04 m

solid cylinder hollow cylinder

end-on view of the cylinders

M = 10 kg, R = 0.10 m	M = 10 kg, R = 0.10 m, r = 0.05 m
$I = \frac{1}{2} M R^2$	$I = \frac{1}{2} M (R^2 + r^2)$

It is observed that one of the cylinders reaches the bottom of the slope before the other.

(a) Using the expressions given above, show that the moments of inertia for the cylinders are as follows:
(i) solid cylinder; $I = 0.05 \text{ kg m}^2$ (ii) hollow cylinder; $I = 0.0625 \text{ kg m}^2$.

(b) By energy considerations, show that the linear velocity of any cylinder at the bottom of the slope is given by:

$$v = \sqrt{\frac{2 g h}{\left[1 + \frac{I}{M R^2}\right]}}.$$

(c) Using the expression in (b) above, calculate the velocities of the two cylinders at the bottom of the slope and hence show that one of the cylinders arrives at the bottom of the slope 0.23 s ahead of the other.

TUTORIAL 5

Gravitation

1 Show that the force of attraction between two large ships of mass 50000 tonnes and separated by a distance of 20 m is 417 N. (1 tonne = 1000 kg)

2 Calculate the gravitational force of attraction between the proton and the electron in a hydrogen atom. Assume the electron is describing a circular orbit with a radius of 5.3×10^{-11} m.
(mass of proton = 1.67×10^{-27} kg; mass of electron = 9.11×10^{-31} kg).

3 A satellite, of mass 1500 kg, is moving at constant speed in a circular orbit 160 km above the Earth's surface.

(a) Calculate the period of rotation of the satellite.

(b) Calculate the total energy of the satellite in this orbit.

(c) Calculate the minimum amount of extra energy required to boost this satellite into a geostationary orbit which is at a distance of 36 000 km above the Earth's surface.

4 The planet Mars has a mean radius of 3.4×10^{6} m. The Earth's mean radius is 6.4×10^{6} m. The mass of Mars is 0.11 times the mass of the Earth.

(a) How does the mean density of Mars compare with that of the Earth?

(b) Calculate the value of "g" on the surface of Mars.

(c) Calculate the escape velocity on Mars.

5 Determine the potential energy between the planet Saturn and its rings.
Assume the rings have a mass of 3.5×10^{18} kg and are concentrated at an average distance of 1.1×10^{8} m from the centre of Saturn.
The mass of Saturn is 5.72×10^{26} kg.

6 During trial firing of Pioneer Moon rockets, one rocket reached an altitude of 125 000 km.

Neglecting the effect of the Moon, estimate the velocity with which this rocket struck the atmosphere of the Earth on its return. (Assume that the rocket's path is entirely radial and that the atmosphere extends to a height of 130 km above the Earth's surface).

7 (a) Sketch the gravitational field pattern between the Earth and Moon.

(b) Gravity only exerts attractive forces. There should therefore be a position between the Earth and Moon where there is no gravitational field - a so-called 'null' point.

By considering the forces acting on a mass m placed at this point, calculate how far this position is from the centre of the Earth.

8 Mars has two satellites named Phobos and Deimos. Phobos has an orbital radius of 9.4×10^6 m and an orbital period of 2.8×10^4 s.

Using Kepler's third law ($\frac{r^3}{T^2}$ = constant), calculate the orbital period of Deimos which has an orbital radius of 2.4×10^7 m.

9 When the Apollo 11 satellite took the first men to the Moon in 1969 its trajectory was very closely monitored.

The satellite had a velocity of 5374 m s^{-1} when 26306 km from the centre of the Earth and this had dropped to 3560 m s^{-1} when it was 54368 km from the centre of the Earth. The rocket motors had **not** been used during this period.

Calculate the gravitational potential difference between the two points. Remember that the unit of gravitational potential is J kg^{-1} .

10 Show that an alternative expression for the escape velocity from a planet may be given by:

$$\boxed{v_e = \sqrt{2gR}}$$ where g = the planet's surface gravitational attraction and R = the radius of the planet.

11 The Escape Velocity for the Earth $v_e = \sqrt{\frac{2GM_E}{r_E}}$ or $v_e = \sqrt{2gr_E}$

Using data on the Earth, show that the escape velocity equals 1.1×10^4 m s^{-1}, or 11 km s^{-1}.

12 Show that a satellite orbiting the Earth at a height of 400 km has an orbital period of 93 minutes. Note that a height of 400 km is equal to a radius of R_E + 400 km.

13 (a) A geostationary orbit has a period of approximately 24 hours. Find the orbital radius for a satellite in such an orbit.

(b) Hence find the height of this satellite above the Earth.

(c) Show on a sketch of the Earth the minimum number of geostationary satellites needed for **world-wide** communication.

TUTORIAL 6

Simple Harmonic Motion

1 The displacement, in cm, of a particle is given by the equation: $y = 4 \cos 4\pi t$.

(a) State the amplitude of the motion.

(b) Calculate the frequency, and hence the period, of the oscillation.

(c) Calculate the location of the particle, in relation to its rest position, when;

(i) $t = 0$
(ii) $t = 1.5$ s.

2 A body, which is moving with SHM, has an amplitude of 0.05 m and a frequency of 40 Hz.

(a) Find the period of the motion.

(b) State an appropriate equation describing the motion.

(c) (i) Calculate the acceleration at the mid-point of the motion **and** at the position of maximum amplitude.

(ii) Calculate the maximum speed of the body and state at which point in the motion this speed occurs.

3 An object of mass 0.50 kg moves with SHM. The amplitude and period of the motion are 0.12 m and 1.5 s respectively.
Assume that the motion starts with $a = + 0.12$ m.

From this information , calculate:

(a) the position of the object when $t = 0.40$ s

(b) the force (magnitude and direction) acting on this object when $t = 0.40$ s

(c) the minimum time needed for the object to travel from its starting point to a point where the displacement is - 0.06 m.

4 A prong of a tuning fork, which can be assumed to be moving with simple harmonic motion, has the following equation governing its motion:
$y = 2.0 \sin (3.22 \times 10^3 t)$ where y is in mm.

(a) Find the maximum amplitude and the frequency of the tuning fork's motion.

(b) Calculate the maximum acceleration of the prong on the tuning fork.

(c) On graph paper, draw the variation of displacement against time for the first two cycles of the motion. Assume that the motion starts from the equilibrium position.

(d) As the sound of a tuning fork dies away, the frequency of the note produced does not change.
What conclusion can we draw about the period of this, and indeed any object, moving with SHM?

5 A sheet of metal is clamped in the horizontal plane and made to vibrate with SHM in the vertical plane with a frequency of 40 Hz.

When some sand grains are sprinkled on to the plate, it is noted that the sand grains can lose contact with the sheet of metal. This occurs when the acceleration of the SHM is $\geq 10 \text{ m s}^{-2}$. Calculate the maximum amplitude of the motion for which the sand will always be in contact with the metal sheet.

6 A vertical spring stretches 0.10 m when 1.2 kg mass is allowed to hang from the end of the spring.

(a) Calculate the spring constant, k, given by these figures.

(b) The mass is now pulled down a distance of 0.08 m below the equilibrium position and released from rest.

(i) State the amplitude of the motion.

(ii) Calculate the period **and** the frequency of the motion.

(iii) Find the maximum speed of the mass **and** the total energy of the oscillating system.

7 A block of mass 5.0 kg is suspended from a spring which has a force constant of 450 N m^{-1}.

A dart which has a mass of 0.060 kg is fired into the block from below with a speed of 120 m s^{-1}, along the vertical axis of the spring. The dart embeds in the block.

(a) Find the amplitude of the resulting simple harmonic motion of the spring/block system.

(b) What percentage of the original kinetic energy of the dart appears as energy in the oscillating system?

8. Explain what is meant by the terms ‘damping’ and ‘critical damping’ when applied to oscillating systems.

TUTORIAL 7

Wave-Particle Duality

1 (a) An electron moves with a velocity of 3×10^6 m s^{-1}. What is its de Broglie wavelength?

(b) A proton moves with the same velocity. Determine its de Broglie wavelength.

2 An electron is accelerated from rest through a p.d. of 200 V .

(a) Calculate the non-relativistic velocity of this electron.

(b) What is the de Broglie wavelength of this electron?

(c) Would this electron show particle or wave like characteristics on meeting an obstacle of diameter 1 mm?

3 Calculate the de Broglie wavelength of an electron accelerated through a potential difference of 20 kV. (This is the p.d. typically used in a colour television tube.)

4 The wave and particle models in physics are related by the expression: $\lambda = \frac{h}{p}$.

(a) State what is meant by each of the symbols in the equation.

(b) Give an illustration in physics where:

(i) photons are said to behave like waves

(ii) photons are said to behave like particles

(iii) electrons are said to behave like waves

(iv) electrons are said to behave like particles.

5 (a) Calculate the wavelength of electrons in an electron microscope which have been accelerated through a p.d. of 100 kV.
Use a non-relativistic calculation

(b) Hence explain why such electrons can be used to examine objects on the atomic scale.

6 Explain why the wave nature of matter is not more evident in everyday life.

7 Calculate the de Broglie wavelength associated with an athlete of mass 70 kg running with a speed of 10 m s^{-1}.

8 (a) Using the non-relativistic equations for momentum and kinetic energy, show that the de Broglie wavelength of an electron accelerated through a p.d. of V volts can be written as: $\lambda = \frac{1.23 \times 10^{-9}}{\sqrt{V}}$ m.

(b) Using the equation in (a) above, calculate the wavelength of electrons which have been accelerated, from rest, through a potential difference of 1000 V.

(c) Electrons as described in (b) above, are fired at a sample of crystalline material. A diffraction pattern is observed for the electrons as they pass through the crystals. State a possible approximate value for the spacing of the molecules in the crystalline solid.

9 Describe what is meant by saying that the angular momentum of an electron about the nucleus is quantised.

ACTIVITY 1

Title **Measurement of angular velocity**

Apparatus Record turntable or motorised circular disc.

Instructions

Set the rotating object in motion and ensure that it has reached its final steady speed.
Count the number of rotations in one minute.
Convert your reading to rad s^{-1}.

Repeat this measurement and determine the uncertainty in this result using

$$\frac{\text{maximum value - minimum value}}{\text{number of readings}}$$

Write a brief evaluation of this experiment.

ACTIVITY 2

Title **Measurement of angular acceleration**

Apparatus Turntable, large and small pulley wheels, 50 g mass, stop watch or light gate.

Instructions

Set up the apparatus as shown in the diagram.
Release the mass, which will cause the turntable to turn. The weight of the mass must exceed the friction of the system

Time a specific angular displacement θ.
Calculate the angular acceleration using $\theta = \frac{1}{2} \alpha t^2$ ($\omega_o = 0$)

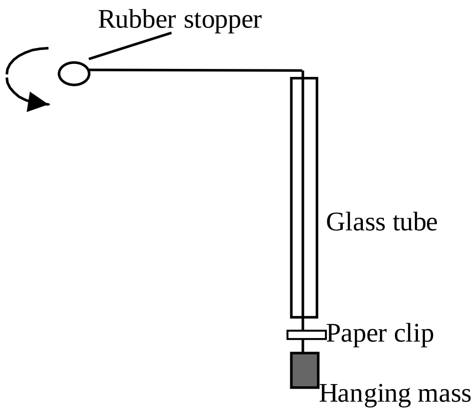
Repeat this measurement and determine the uncertainty in this result.

Write a brief evaluation of this experiment.

ACTIVITY 3A **(Outcome 3)**

Title **Variation of central force with angular velocity**

Apparatus Rubber stopper, string, weight carrier, masses, stop watch, glass tube, paper clip.



Instructions

Attach the string to the stopper.
Thread the string though the glass tube.
Attach the paper clip to the string on the opposite end of the string from the stopper.
Attach the weight carrier to the end of the string.

Hold the glass tube and whirl the stopper around your head as nearly as possible in a *horizontal* circle.
Note: take care when whirling the rubber stopper. Other students should stand well back.

Keep the angular velocity constant and at such a value that the paper clip just touches the bottom of the glass tube.

Determine the angular velocity of the stopper.

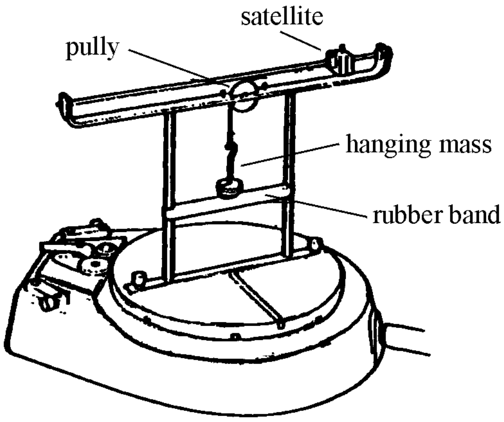
Repeat for different masses.

Use an appropriate graphical format to show the relationship between central force and angular velocity.

ACTIVITY 3B **(Outcome 3)**

Title **Central force and angular velocity**

Apparatus Central force frame, disc, air bearing, masses.



Instructions

Attach the central force frame to the disc as shown in the diagram.
Slip the larger sliding mass (satellite) onto the top rod and fix to the frame with the kurled nut.
Adjust the position of the collar so that the centre of the sliding mass is 20 cm from the axis of rotation, which is marked on the cross bar.
Tie the thread from the satellite, over the pulley wheel to the 10 g mass carrier so that the carrier is about 2 cm below the pulley wheel.
Adjust the position of the rubber band so that it is at the same level as the bottom of the mass carrier.

Engage the drive and slowly increase the speed until the sliding mass *just* begins to move outwards. Observe the gap between the mass hanger and rubber band.

Determine the angular velocity of the disc.

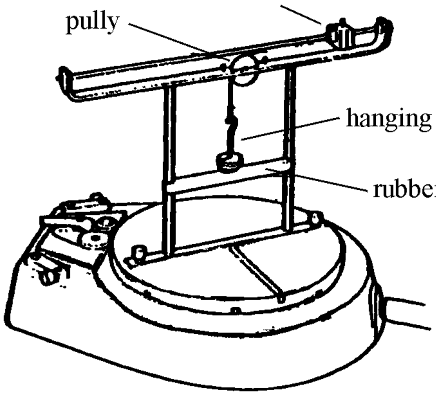
Repeat for different masses.

Use an appropriate graphical format to show the relationship between central force and angular velocity.

ACTIVITY 3C **(Outcome 3)**

Title **Central force: radius and period of rotation**

Apparatus Central force frame, disc, air bearing, masses.



Instructions

Attach the central force frame to the disc as shown in the diagram.
Slip the smaller sliding mass (satellite) onto the top rod and fix to the frame with the kurled nut.
Adjust the position of the collar so that the centre of the sliding mass is 20 cm from the axis of rotation, which is marked on the cross bar.
Tie the thread from the satellite, over the pulley wheel to the 10 g mass carrier so that the carrier is about 2 cm below the pulley wheel.
Adjust the position of the rubber band so that it is at the same level as the bottom of the mass carrier.

Engage the drive and slowly increase the speed until the sliding mass *just* begins to move outwards. Observe the gap between the mass hanger and rubber band.

Determine the period of rotation.

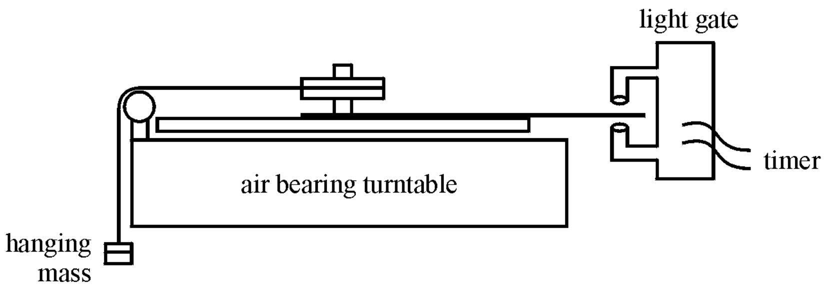
Repeat for different radii of satellite orbit.
It may be necessary to shorten the length of the thread for smaller orbits to prevent the hanging mass behaving like a conical pendulum.

Use an appropriate graphical format to show the relationship between period of rotation and the radius of orbit.

ACTIVITY 4 **(Outcome 3)**

Title **Variation of angular acceleration with torque**

Apparatus Air bearing turntable, air blower, disc, weight carrier, masses, pulley, light gate, card, twine, stop watch.



Instructions

Set up the apparatus as shown in the diagram above.
Use the weight carrier to apply a constant torque to the disc on the air table.
Determine the torque applied to the disc.

Release the weight carrier and start the stop watch.
Find the angular velocity of the disc after a known time.
Calculate the angular acceleration of the disc.

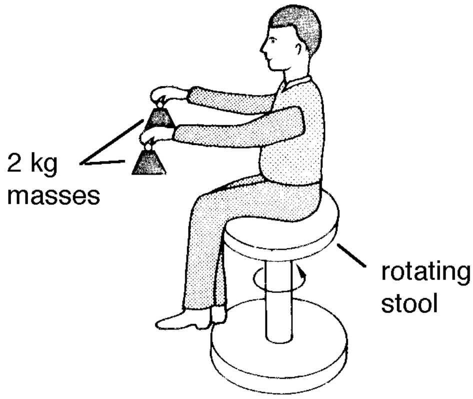
Repeat for other constant unbalanced torques.

Use an appropriate graphical format to show the relationship between angular acceleration and torque.

ACTIVITY 5

Title **Conservation of angular momentum**

Apparatus Chair which can be rotated very easily, two 2 kg masses.



Instructions

A person sits on the chair holding a 2 kg mass in each hand.
The person holds the masses close to the chest.
Set the chair spinning.

The arms of the person are thrust outwards, holding the masses at arms length.

Note any change in angular velocity.

The arms are drawn back inwards,
Note any change in angular velocity.

Consider the relationship for angular momentum ($L = mvr = m\omega r^2$) and explain the results.

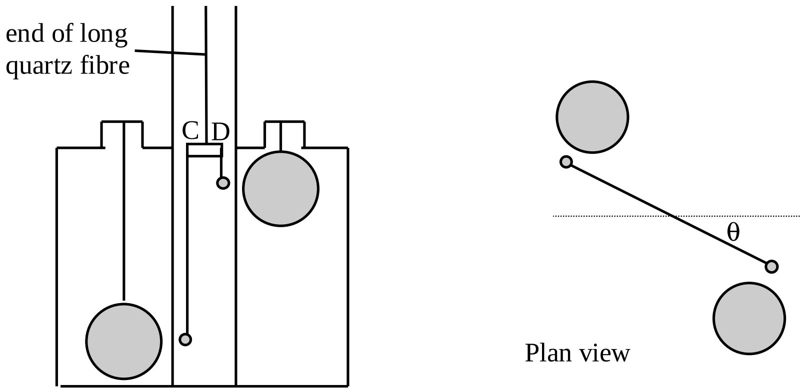
ACTIVITY 6

Title **Cavendish-Boys determination of G**

Description

In 1894 C V Boys carried out a celebrated experiment to measure G, using a method similar to that used by Cavendish in 1789. The discovery of quartz fibres, which can be fine and strong enough to support considerable weight, had taken place during the 19^{th} century.

Two identical 5 mm diameter gold spheres were hung at either end of a polished bar CD. This bar CD, of length 23 mm, was suspended by a long quartz fibre. Two large identical lead spheres, of diameter 115 mm were then brought into position near the small spheres as shown in the figure below.



As a result of the attraction between the masses the bar CD twists through an angle θ. This angle is measured by means of a beam of light reflected from the bar CD onto a scale. The small size of the apparatus allows it to be screened from air convection currents.

The torque T acting on the bar is given by

$$T = F_g \times \frac{1}{2} CD$$

where F_g is the force of attraction between a large sphere and small sphere. Measurement of the masses, their separation, the length CD and the torsional constant of the wire enables G to be determined from the angle of rotation. (The torsional constant of a wire k is the torque required to produce an angular displacement of one radian, $T = k\theta$.)

Instructions

Study the above description.

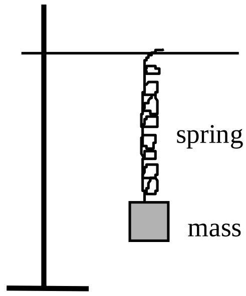
In a similar experiment, each small mass is 2.00 g and each large mass is 0.600 kg. The distance between the centres of the spheres is 70 mm. Show that the gravitational force between a large and small sphere is 1.63×10^{-11} N.

The torque producing the rotation of the fibre is given by $T = k\theta$ where k is the torsional constant. Explain whether k should be large or small in order to obtain an accurate value for G.

ACTIVITY 7 **(Outcome 3)**

Title **Variation of period of oscillation with mass for a spring**

Apparatus Spring, weight carrier, masses, stop watch, clamp stand.



Instructions

Suspend the spring vertically from the clamp stand.
Attach an known mass to the end of the spring.
Start the mass oscillating and determine the period of oscillation.

Repeat for other known masses.

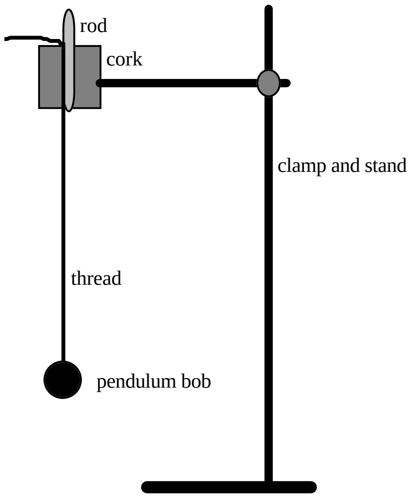
Use an appropriate graphical format to show the relationship between the period of oscillation and mass.

(A motion sensor could be used to determine the period of oscillation.)

ACTIVITY 8 **(Outcome 3)**

Title **Variation of period of swing with length for a simple pendulum**
Determination of 'g'

Apparatus Pendulum bob, thread, cork, stop watch, clamp and stand, metre rule.



Instructions

Set up the pendulum. Ensure that the thread is securely clamped in position.

Measure the length of the pendulum.

Start the pendulum swinging and determine the period of swing.

Repeat for different lengths of pendulum.

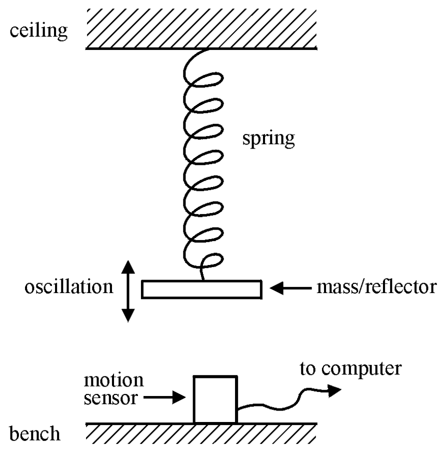
Use an appropriate graphical format to show the relationship between the period of swing and length of pendulum.

Use the graph to obtain a value for the gravitational field strength.

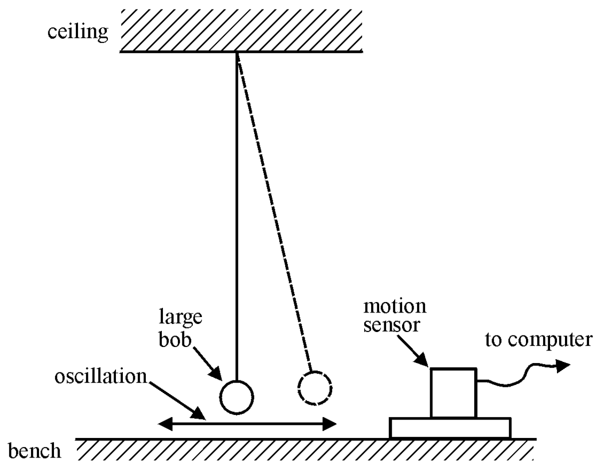
ACTIVITY 9

Title **Experiments showing SHM using a motion sensor**

Apparatus Spring with mass and reflector, long pendulum, motion sensor, computer.



Long Spring



Pendulum

Instructions

Long spring

Set up the apparatus as shown in the diagrams.
Ensure that the motion sensor's beam of ultrasound can reflect off a suitable moving object as shown.

Position the sensor appropriately.
Set the spring oscillating and capture a set of displacement readings.
Display the displacement-time and velocity-time graph on the same screen.

Compare these two graphs and comment on the phases.

If possible display the acceleration-time graph and compare with the other two.

Write brief comments on these graphs.

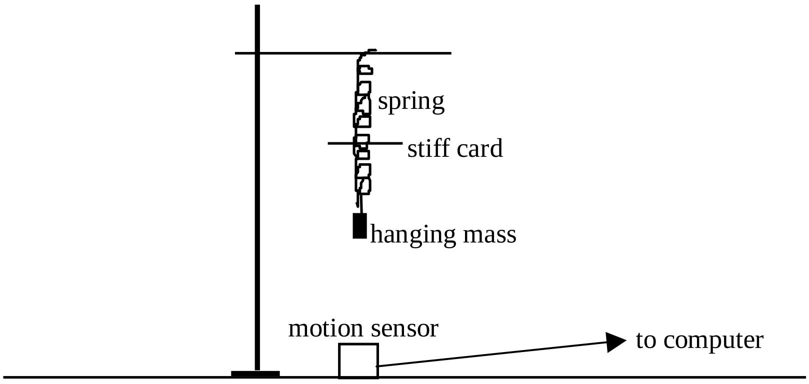
Pendulum

Repeat the above procedure for the pendulum.

ACTIVITY 10

Title **Experiments showing damped oscillations**

Apparatus Spring with hanging mass, stiff cards of different areas, motion sensor, computer.



Instructions

Set up the apparatus as shown in the diagram, but without the stiff card in position.
Ensure that the motion sensor's beam of ultrasound can reflect properly off the moving object.

Position the sensor appropriately.
Set the spring oscillating and capture a set of displacement readings.
Display the displacement-time graph on the screen.

Repeat the experiment with a piece of stiff card in position.

Repeat the experiment with another piece of stiff card of larger area.

Compare these displacement-time graph with the previous graph.

Write brief comments on these graphs.