Differentiation and Integration

AH Physics

1

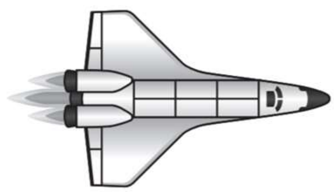
2019

Differentiation or integration?

v-a -> gradient -> Differentiation

$v = 4 \cdot 2t^2 + 1 \cdot 6t$	
$a\left(= \dfrac{dv}{dt}\right) = 8 \cdot 4t + 1 \cdot 6$	**(1)**
$24 = 8 \cdot 4t + 1 \cdot 6$	**(1)**
$t = 2 \cdot 7\ s$	**(1)**

1. A spacecraft accelerates from rest at time $t = 0$.



The velocity v of the spacecraft at time t is given by the relationship

$$v = 4 \cdot 2t^2 + 1 \cdot 6t$$

where v is measured in $m\,s^{-1}$ and t is measured in s.

Using calculus methods

(a) determine the time at which the acceleration of the spacecraft is $24\,m\,s^{-2}$ **3**

Space for working and answer

2

2019

Differentiation or integration?

v-s -> area -> integration

The velocity v of the spacecraft at time t is given by the relationship

$$v = 4{\cdot}2t^2 + 1{\cdot}6t$$

where v is measured in $\mathrm{m\,s^{-1}}$ and t is measured in s.

(b) determine the distance travelled by the spacecraft in this time.

Space for working and answer

$$s = \int (4{\cdot}2t^2 + 1{\cdot}6t).dt$$

$$s = \frac{4{\cdot}2t^3}{3} + \frac{1{\cdot}6t^2}{2} (+c) \qquad \textbf{(1)}$$

(s=0 when t=0, so c=0)

$$s = \frac{4{\cdot}2 \times 2{\cdot}7^3}{3} + \frac{1{\cdot}6 \times 2{\cdot}7^2}{2} \qquad \textbf{(1)}$$

$$s = 33\ \mathrm{m} \qquad \textbf{(1)}$$

Solution with limits also acceptable

$$s = \int_0^{2{\cdot}7} \left(4{\cdot}2t^2 + 1{\cdot}6t\right).\,dt$$

$$s = \left[\frac{4{\cdot}2 \times t^3}{3} + \frac{1{\cdot}6 \times t^2}{2}\right]_0^{2{\cdot}7} \qquad \textbf{(1)}$$

$$s = \left(\frac{4{\cdot}2 \times 2{\cdot}7^3}{3} + \frac{1{\cdot}6 \times 2{\cdot}7^2}{2}\right)(-0) \qquad \textbf{(1)}$$

$$s = 33\ \mathrm{m} \qquad \textbf{(1)}$$

3

2018

Differentiation or integration?

v-a -> gradient -> Differentiation

1. Energy is stored in a clockwork toy car by winding-up an internal spring using a key. The car is shown in Figure 1A.

Figure 1A

The car is released on a horizontal surface and moves forward in a straight line. It eventually comes to rest.

The velocity v of the car, at time t after its release, is given by the relationship

$$v = 0{\cdot}0071t - 0{\cdot}00025t^2$$

where v is measured in $\mathrm{m\,s^{-1}}$ and t is measured in s.

Using calculus methods:

(a) determine the acceleration of the car 20·0 s after its release;

Space for working and answer

$$v = 0{\cdot}0071t - 0{\cdot}00025t^2$$

$$a\left(=\frac{dv}{dt}\right) = 0{\cdot}0071 - 0{\cdot}0005t$$

$$a = 0{\cdot}0071 - (0{\cdot}0005 \times 20{\cdot}0)$$

$$a = -0{\cdot}0029\ \mathrm{ms^{-2}}$$

4

2018

Differentiation or integration?

v-d -> area -> integration

$$v = 0 \cdot 0071t - 0 \cdot 00025t^2$$
$$s\left(=\int_0^{20\cdot 0} v.dt\right) = \left[\frac{0 \cdot 0071}{2}t^2 - \frac{0 \cdot 00025}{3}t^3\right]_0^{20\cdot 0}$$
$$s = \left(\frac{0 \cdot 0071}{2} \times 20 \cdot 0^2\right) - \left(\frac{0 \cdot 00025}{3} \times 20 \cdot 0^3\right) - 0$$
$$s = 0 \cdot 75 \text{ m}$$

1. Energy is stored in a clockwork toy car by winding-up an internal spring using a key. The car is shown in Figure 1A.

Figure 1A

The car is released on a horizontal surface and moves forward in a straight line. It eventually comes to rest.

The velocity v of the car, at time t after its release, is given by the relationship

$$v = 0 \cdot 0071t - 0 \cdot 00025t^2$$

where v is measured in m s^{-1} and t is measured in s.

(b) determine the distance travelled by the car 20·0 s after its release.

Space for working and answer

5

2017

$$v = 0{\cdot}4t^2 + 2t$$
$$v = (0{\cdot}4 \times 3{\cdot}10^2) + (2 \times 3{\cdot}10)$$
$$v = 10{\cdot}0 \text{ m s}^{-1}$$

Accept: 10, 10·04, 10·044

An athlete competes in a one hundred metre race on a flat track, as shown in Figure 1A.

Figure 1A

Starting from rest, the athlete's speed for the first 3·10 seconds of the race can be modelled using the relationship

$$v = 0 \cdot 4t^2 + 2t$$

where the symbols have their usual meaning.

According to this model:

(a) determine the speed of the athlete at $t = 3{\cdot}10$ s;

6

2017

Differentiation or integration?

v-d -> area -> integration

Starting from rest, the athlete's speed for the first 3·10 seconds of the race can be modelled using the relationship

$$v = 0\cdot 4t^2 + 2t$$

where the symbols have their usual meaning.

(b) determine, using **calculus** methods, the distance travelled by the athlete in this time.

Space for working and answer

$$s = \int (0\cdot 4t^2 + 2t).dt$$

$$s = \frac{0\cdot 4}{3}t^3 + t^2 (+c)$$

$s = 0$ when $t = 0$, $c = 0$

$$s = \frac{0\cdot 4}{3} \times (3\cdot 10)^3 + 3\cdot 10^2$$

$$s = 13\cdot 6\,\text{m}$$

Accept: 14, 13·58, 13·582

$s = \int_0^{3.10} (0\cdot 4t^2 + 2t).dt$	
$s = \left[\frac{0\cdot 4 \times t^3}{3} + t^2\right]_{(0)}^{(3.10)}$	1
$s = \left(\frac{0\cdot 4 \times 3\cdot 10^3}{3} + 3\cdot 10^2\right) - 0$	1
$s = 13\cdot 6\,\text{m}$	1

7

2016

Differentiation or integration?

v-a -> gradient -> Differentiation

$$v = 0\cdot 135t^2 + 1\cdot 26\text{t}$$

$$a = \frac{dv}{dt} = 0\cdot 135 \times 2t + 1\cdot 26$$

$$a = (0\cdot 135 \times 2 \times 15\cdot 0) + 1\cdot 26$$

$$a = 5{\cdot}31\ \text{m s}^{-2}$$

1.

Calvin Chan/shutterstock.com

A car on a long straight track accelerates from rest. The car's run begins at time $t = 0$.

Its velocity v at time t is given by the equation

$$v = 0\cdot 135t^2 + 1\cdot 26t$$

where v is measured in m s^{-1} and t is measured in s.

Using **calculus** methods:

(a) determine the acceleration of the car at $t = 15\cdot 0$ s;

8

2016

A car on a long straight track accelerates from rest. The car's run begins at time $t = 0$.

Its velocity v at time t is given by the equation

$$v = 0{\cdot}135t^2 + 1{\cdot}26t$$

where v is measured in m s^{-1} and t is measured in s.

(b) determine the displacement of the car from its original position at this time. **3**

Differentiation or integration?

v-d -> area -> integration

$$v = 0{\cdot}135t^2 + 1{\cdot}26\text{t}$$

$$s = \int_0^{15\cdot0} v.dt = \left[0{\cdot}045t^3 + 0{\cdot}63t^2\right]_0^{15\cdot0} \quad \mathbf{1}$$

$$s = (0{\cdot}045 \times 15{\cdot}0^3) + (0{\cdot}63 \times 15{\cdot}0^2) \quad \mathbf{1}$$

$$s = 294 \text{ m} \quad \mathbf{1}$$

9

2020/2021

Differentiation or integration?

v-a -> gradient -> Differentiation

During a rollercoaster ride, a train is moving along a track as shown in **Figure 1A**.

Figure 1A

At time $t = 0$, the train reaches a straight section of track. It takes 4·0 seconds to move over this section of track.

The horizontal velocity v_h of the train, over this section of track, is given by the relationship

$$v_h = 8 + 4t^2 - \frac{2}{3}t^3$$

where v_h is in m s^{-1} and t is in s.

Using calculus methods

(a) determine the horizontal acceleration of the train at $t = 4{\cdot}0$ s

$$a \left(= \frac{dv}{dt} \right) = 8t - 2t^2$$

$$a = 8 \times 4{\cdot}0 - 2 \times 4{\cdot}0^2$$

$$a = 0{\cdot}0 \text{ m s}^{-2}$$

10

2020/2021

The horizontal velocity v_h of the train, over this section of track, is given by the relationship

$$v_h = 8 + 4t^2 - \frac{2}{3}t^3$$

where v_h is in m s^{-1} and t is in s.

(b) determine the horizontal displacement of the train at $t = 4{\cdot}0$ s.

$$s\left(=\int v.dt\right) = 8t + \frac{4}{3}t^3 - \frac{2}{3\times 4}t^4 \ (+\ c)$$

$$s = 8\times 4\cdot 0 + \frac{4}{3}\times 4\cdot 0^3 - \frac{2}{3\times 4}\times 4\cdot 0^4$$

$$s = 75 \text{ m}$$

11

Deriving the Equations- still an outcome! 2002

1. (*a*) An object moves with constant acceleration *a*.

At time $t = 0$ its displacement s is zero.

The velocity v of the object is given by $v = u + at$.

Derive the equation

$$s = ut + \tfrac{1}{2}at^2$$

where the symbols have their usual meanings.

1. (*a*)

$$\frac{ds}{dt} = v \therefore ds = v.dt$$

$$\int ds = \int (u + at).dt$$

$$s = ut + \tfrac{1}{2}at^2 + c$$

at $t = 0$, $c = 0$

$$\therefore s = ut + \tfrac{1}{2}at^2$$

12

2012

2. (*a*) The acceleration of a particle moving in a straight line is given by

$$a = \frac{dv}{dt}$$

where the symbols have their usual meaning.

(i) Show, by integration, that when a is constant

$$v = u + at.$$

(ii) Show that when a is constant

$$v^2 = u^2 + 2as.$$

13

$$a = \frac{dv}{dt}$$

$\int dv = \int a.dt$ or $\int \frac{dv}{dt} dt = \int a.dt$

$v = at + c$

at $t = 0, c = u$

Must be specific with respect to time

$v = u + at$ SHOW ME

$$a = \frac{dv}{dt}$$

$\int_u^v dv = \int_0^t a.dt$ (½) + (½)

½ for integrals, ½ for limits need both before can progress

$[v]_u^v = [at]_0^t$ (½)

$v - u = at\,(-0)$ (½)

$v = u + at$

$v = u + at$

$v^2 = (u + at)^2$

$v^2 = (u + at)(u + at)$

$v^2 = u^2 + uat + uat + a^2t^2$

$v^2 = u^2 + 2uat + a^2t^2$

$v^2 = u^2 + 2uat + \frac{2}{2}a^2t^2$

$v^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$

BUT $s = (ut + \frac{1}{2}at^2)$

so by substitution

$v^2 = u^2 + 2as$

14

2007

1. (*a*) A particle has displacement $s = 0$ at time $t = 0$ and moves with constant acceleration a.

The velocity of the object is given by the equation $v = u + at$, where the symbols have their usual meanings.

Using calculus, derive an equation for the displacement s of the object as a function of time t.

$\frac{ds}{dt} = v$

$\int ds = \int (u + at) \cdot dt$

$s = ut + \frac{1}{2} at^2 + c$

at $t = 0$, $s = 0$, so $c = 0$

$\therefore s = ut + \frac{1}{2} at^2$

No c or limits $\Rightarrow$ max (½)
If first line is W.P. – (0)
eg WP = $\int s$, $\int \frac{ds}{dt}$, $\int v$

OR

$\int_o^s ds = \int_o^t (u + at)\, dt$

(½) equation
(½) limits

$[s]_o^s = \left[ut + \frac{1}{2} at^2\right]_o^t$ (½)

$s = ut + \frac{1}{2} at^2$ (½)

15

$\frac{ds}{dt} = v \therefore ds = v.dt$ (½)

$\int ds = \int (u + at).dt$ (½)

$s = ut + \frac{1}{2} at^2 + c$ (½)

at $t = 0$, $c = 0$ (½)

$\therefore s = ut + \frac{1}{2} at^2$

Learn It!

16