ANNOTATED AH RELATIONSHIPS SHEET

First derivative of displacement = velocity $$v = \frac{ds}{dt}$$ $$velocity\ (ms^{-1})\ =\ rate\ (s^{-1}) of\ change\ of\ displacement\ (m)$$
Second derivative of displacement = acceleration **First derivative of velocity = acceleration** $$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$ $$acceleration\ (ms^{-2})\ =\ rate\ (s^{-1}) of\ change\ of\ velocity\ (ms^{-1})$$
$$v = u + at$$ $$final\ velocity\ (ms^{-1}) = initial\ velocity\ (ms^{-1}) + acceleration\ (ms^{-2}) \times time\ (s)$$
$$s\ =\ ut\ + \frac{1}{2}\ at^2$$ $$displacement = initial\ velocity\ \times time + \frac{1}{2} \times acceleration\ (ms^{-2}) \times time^2\ (s^2)$$
$$v^2\ =\ u^2\ +\ 2as$$ $$final\ velocity\ ^2\ (ms^{-1})^2\ =\ initial\ velocity\ ^2 (ms^{-1})^2 + 2 \times acceleration\ (ms^{-2}) \times dispacement\ (m)$$
First derivative of angular displacement = angular velocity $$\omega = \frac{d\theta}{dt}$$ $$angular\ velocity\ (ms^{-1})\ =\ rate\ (s^{-1}) of\ change\ of\ angular\ displacement\ (rad)$$
Second derivative of angular displacement = angular acceleration **First derivative of angular velocity = angular acceleration** $$a = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$ $$angular\ acceleration\ (rad\ s^{-2})\ =\ rate\ (s^{-1}) of\ change\ of\ angular\ velocity\ (rad\ s^{-1})$$
Equation for motion for <u>uniform</u> angular acceleration $$\omega = \omega_o + at$$ $$\frac{final\ angular\ velocity}{(rad\ s^{-1})} = \frac{initial\ angular\ velocity}{(rad\ s^{-1})} + \frac{angular\ acceleration}{(rad\ s^{-2})} \times \frac{time}{(s)}$$

Equation for motion for <u>uniform</u> angular acceleration $$\omega^2 = \omega_o{}^2 + 2a\theta$$ $$\frac{final\ angular\ velocity^2}{(rad\ s^{-1})^2} = \frac{initial\ angular\ velocity^2}{(rad\ s^{-1})^2} + 2 \times \frac{angular\ acceleration}{(rad\ s^{-2})} \times \frac{angular\ dispacement}{(rad)}$$
Equation for motion for <u>uniform</u> angular acceleration $$\theta = \omega_o t + \frac{1}{2}at^2$$ $$\frac{angular\ displacement}{(rad)} = \frac{initial\ angular\ velocity}{(rad\ s^{-1})} \times \frac{time}{(s)} + \frac{1}{2} \times \frac{angular\ acceleration}{(rad\ s^{-2})} \times \frac{time^2}{(s)^2}$$
To convert from angular quantity to linear equivalent. Angles must be in radians $$s = r\theta$$ $$\frac{linear\ distance}{(m)} = \frac{radius}{(m)} \times \frac{angular\ displacement}{(rad)}$$
To convert from angular quantity to linear equivalent. $$v = r\omega$$ $$\frac{tangential\ velocity}{(m\ s^{-1})} = \frac{radius}{(m)} \times \frac{angular\ velocity}{(rad\ s^{-1})}$$
To convert from angular quantity to linear equivalent $$a_t = ra$$ $$\frac{tangential\ acceleration}{(m\ s^{-2})} = \frac{radius}{(m)} \times \frac{angular\ acceleration}{(rad\ s^{-2})}$$
Converts between angular velocity and period *NB 2π rad = 1 revolution* $$\omega = \frac{2\pi}{T}$$ $$angular\ velocity\ (rad\ s^{-1}) = \frac{2\pi}{Period\ (s)}$$ $$angular\ frequency\ (rad\ s^{-1}) = \frac{2\pi}{Period\ (s)}$$
Converts between angular velocity, frequency $$\omega = 2\pi f$$ $$angular\ velocity\ (rad\ s^{-1}) = 2\pi \times frequency\ (Hz)$$ $$angular\ frequency\ (rad\ s^{-1}) = 2\pi \times frequency\ (Hz)$$

Radial or centripetal acceleration for uniform speed in a circle or radius r $$a_r = \frac{v^2}{r} = r\omega^2$$ $$\underset{(m\,s^{-2})}{radial\ acceleration} = \frac{tangential\ speed^2 (m\,s^{-1})^2}{radius\ of\ the\ circular\ motion\ (m)} = \underset{(m)}{radius\ of\ the\ circular\ motion} \times \underset{(rad\,s^{-1})^2}{angular\ velocity^2}$$
Centripetal (central) force is the unbalanced force acting towards the centre of a circle $$F = \frac{mv^2}{r} = mr\omega^2$$ $$\underset{(N)}{Centripetal\ Force} = \frac{mass\ (kg) \times tangential\ speed^2 (m\,s^{-1})^2}{radius\ of\ the\ circular\ motion\ (m)} = \underset{(kg)}{mass} \times \underset{(m)}{radius\ of\ the\ circular\ motion} \times \underset{(rad\,s^{-1})^2}{angular\ velocity^2}$$
Moment of inertia $$I = \Sigma mr^2$$ $$Moment\ of\ inertia\ (kg\,m^2) = Sum\ of\ (the\ mass\ of\ each\ particle\ (kg) \times (distance\ from\ the\ axis\ of\ rotation)^2\ (m)^2)$$ ***NB the equation for individual shapes of rigid bodies around a point of rotation is given in the relationships sheet.***
Torque is also known as the moment of force (or the turning effect of a force) $$\tau = Fr$$ $$\underset{(Nm)}{torque} = \underset{(N)}{Force} \times \underset{(m)}{distance\ between\ the\ axis\ of\ rotation\ and\ force}$$ **For this equation the force perpendicular to the axis of rotation, you might need to find the component of force acting perpendicular to the axis of rotation.**
Torque, moment of inertia and angular acceleration. $$\tau = I\alpha$$ $$torque\ (Nm)\ or\ (kg\,m^2 s^{-2}) = Moment\ of\ inertia\ (kg\,m^2) \times angular\ acceleration\ (rad\,s^{-2})$$
Angular momentum of a particle $$L = mvr = mr^2\omega$$ $$\underset{(kg\,m^2 s^{-1})}{angular\ momentum\ of\ a\ particle} = \underset{(kg)}{mass} \times \underset{(m\,s^{-1})}{linear\ velocity} \times \underset{(m)}{distance\ to\ the\ turning\ point}$$ $$\underset{(kg\,m^2 s^{-1})}{angular\ momentum} = \underset{(kg)}{mass} \times \underset{(m)^2}{distance\ to\ turning\ point^2} \times \underset{(rad\,s^{-1})}{angular\ velocity}$$
Angular momentum of a rigid body $$L = I\omega$$ $$\underset{(kg\,m^2 s^{-1})}{angular\ momentum} = \underset{(kg\,m^2)}{Moment\ of\ inertia} \times \underset{(rad\,s^{-1})}{angular\ velocity}$$

Rotational Kinetic Energy of a rigid body

$$E_{k(rotational)} = \frac{1}{2} I\omega^2$$

$$\underset{(J)}{\text{Rotational kinetic energy}} = \frac{1}{2} \times \underset{(kg\ m^2)}{\text{Moment of inertia}} \times \underset{(rad\ s^{-1})^2}{(\text{angular velocity})^2}$$

For objects rolling down a slope

$$E_p = E_{k(transitional)} + E_{k(rotational)}$$

$$\underset{(J)}{\text{Loss in gravitational potential energy}} = \underset{(J)}{\text{gain in translational kinetic energy}} + \underset{(J)}{\text{gain in rotational kinetic energy}}$$

NB this assumes no slipping and no energy losses due to friction

Universal Law of Gravitation

$$F = \frac{GMm}{r^2}$$

$$\text{Gravitational Force } (N) = \frac{\text{Universal gravitational Constant } (m^3 kg^{-1} s^{-2}) \times Mass_1 (kg) \times Mass_2 (kg)}{\text{separation distance}^2\ (m^2)}$$

NB The Universal Gravitational Constant $= 6.67 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$ The separation distance is from the centre to centre of the masses.

For equating the gravitational force providing the central force to keep objects in orbit

$$F = \frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

$$\text{gravitational Force } (N) = \frac{\text{Universal gravitational Constant } (m^3 kg^{-1} s^{-2}) \times Mass_1 (kg) \times Mass_2 (kg)}{\text{separation distance}^2\ (m^2)} =$$

$$= \underset{(N)}{\text{Centripetal Force}} = \frac{\text{mass } (kg) \times \text{tangential speed}^2 (m\ s^{-1})^2}{\text{radius of the circular motion } (m)}$$

$$= \text{centripetal force} = \underset{(kg)}{\text{mass}} \times \underset{(m)}{\text{radius of the circular motion}} \times \underset{(rad\ s^{-1})^2}{\text{angular velocity}^2}$$

$$= \underset{(N)}{\text{Centripetal Force}} = \underset{(kg)}{\text{mass}} \times \underset{(m)}{\text{radius of the circular motion}} \times \left(\frac{2\pi}{\text{orbital period } (s)}\right)^2$$

Gravitational potential is the work done (energy transferred) per unit mass needed to move an object from infinity to that location. As 0 J is at infinity all gravitational potentials are negative

$$V = -\frac{GM}{r}$$

$$gravitational\ potential\ (J\ kg^{-1}) = -\frac{Universal\ Gravitational\ Constant\ (m^3kg^{-1}s^{-2})\ \times mass\ (kg)}{separation\ distance(m)}$$

Gravitational potential energy is the work done (energy transferred) needed to move an object from infinity to that location. As 0 J is at infinity all gravitational potential energies are negative

$$E_p = Vm = -\frac{GMm}{r}$$

$$gravitational\ potential\ energy\ (J) = gravitational\ potential\ (J\ kg^{-1}) \times mass\ (kg)$$

As the gravitational potential is negative the equation is Vm and not -Vm!

$$gravitational\ potential\ energy\ (J) = -\frac{Universal\ gravitational\ Constant\ (m^3kg^{-1}s^{-2})\ \times Mass_1(kg) \times Mass_2(kg)}{separation\ distance(m)}$$

Escape velocity is the minimum speed required for a free, non-propelled object to escape from the gravitational influence of a massive body, to eventually reach an infinite distance from it. As this has the number 2 in the equation it is derived from energy and not from forces.

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

$$escape\ velocity\ (ms^{-1}) = \sqrt{\frac{2 \times Universal\ Gravitational\ Constant\ (m^3kg^{-1}s^{-2})\ \times Mass\ being\ escaped\ from\ (kg)}{radius\ of\ the\ object\ being\ escaped\ from\ (m)}}$$

The Schwarzchild radius- the radius of the event horizon of a black hole

$$r_{schwarzschild} = \frac{2GM}{c^2}$$

$$Schwarzschild\ radius\ (m) = \frac{2 \times Universal\ Gravitational\ Constant(m^3kg^{-1}s^{-2}) \times Mass\ of\ the\ black\ hole(kg)}{speed\ of\ light^2\ (ms^{-1})^2}$$

Apparent brightness how bright the star appears to a detector here on Earth

$$b = \frac{L}{4\pi d^2}$$

$$apparent\ brightness\ (Wm^{-2}) = \frac{Luminosity\ (W)}{4\pi \times distance^2(m)^2}$$

Power radiated per unit surface area from a black body

$$\frac{P}{A} = \sigma T^4$$

$$\frac{\text{Power per unit area}}{(Wm^{-2})} = \frac{\text{Stefan Boltzmann Constant}}{(5.67 \times 10^{-8}\ Wm^{-2}\ K^4)} \times \frac{(\text{temperature})^4}{(K)^4}$$

NB The Stefan Boltzmann Constant can be found on page 2 of the exam paper on the Data Sheet and is equal to $5.67 \times 10^{-8}\ Wm^{-2}\ K^4$

Equation for the luminosity of a star

$$L = 4\pi r^2 \sigma T^4$$

$$\frac{\text{Luminosity}}{(Wm^{-2})} = 4\pi \times \frac{\text{radius of the star}^2}{(m)^2} \times \frac{\text{Stefan Boltzmann Constant}}{(5.67 \times 10^{-8}\ Wm^{-2}\ K^4)} \times \frac{\text{Temperature}^4}{(K)^4}$$

NB The Stefan Boltzmann Constant can be found on page 2 of the exam paper on the Data Sheet and is equal to $5.67 \times 10^{-8}\ Wm^{-2}\ K^4$

$$E = hf$$

$$\text{energy } (J) = \text{Planck's Constant } (Js) \times \text{frequency } (Hz)$$

NB Planck's constant = 6.63 x 10^{-34} J s

Bohr's Quantisation of Angular Momentum

$$mvr = \frac{nh}{2\pi}$$

$$\frac{\text{mass}}{(kg)} \times \frac{\text{velocity}}{(ms^{-1})} \times \frac{\text{radius}}{(m)} = \frac{\text{no. of shell} \times \text{Planck's Constant } (J\ s)}{2\pi}$$

The 2π is an indication of the circular nature equalling 1 full circle. *NB Planck's constant = 6.63 x 10^{-34} J s*

The de Broglie wavelength, when a particle behaves like a wave and this is the wavelength associated with that.

$$\lambda = \frac{h}{p}$$

$$\text{de Broglie wavelength } (m) = \frac{\text{Planck's constant } (J\ s)}{\text{momentum} (kg\ ms^{-1})}$$

Uncertainty principle as it refers to position and momentum

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

$$\frac{\text{the uncertainty in position in x direction}}{(m)} \times \frac{\text{uncertainty in the x component of momentum}}{(kg\ ms^{-1})} \geq \frac{\text{Planck's Constant}}{4\pi}$$

NB In some old resources this was incorrectly marked as divided by 2π as h was given as ħ

$$\text{For the minimum uncertainty the uncertainty the product in the two quantities will} = \frac{h}{4\pi}$$

Uncertainty principle as it refers to energy and time $$\Delta E \Delta t \geq \frac{h}{4\pi}$$ $$\frac{\text{the uncertainty in energy}}{(J)} \times \frac{\text{uncertainty in the time}}{(s)} \geq \frac{\text{Planck's Constant}}{4\pi}$$ NB In some old resources this was incorrectly marked as divided by 2π as h was given as ħ $$\text{For the minimum uncertainty the uncertainty in theproduct of the two quantities will} = \frac{h}{4\pi}$$
Magnetic Force $$F = qvB$$ $$\text{Magnetic Force } (N) = \text{Charge}(C) \times \text{velocity } (ms^{-1}) \times \text{magnetic induction } (T)$$ This occurs where F and B are perpendicular, if they are not some trigonometry is required. $F = qvB \sin \theta$
Centripetal Force $$F = \frac{mv^2}{r}$$ $$\frac{\text{Centripetal Force}}{(N)} = \frac{\text{mass } (kg) \times \text{tangential speed}^2 (m\ s^{-1})^2}{\text{radius of the circular motion } (m)}$$
Proof for an object moving in SHM, and equation for force on a spring. $$F = -ky$$ $$\text{Force } (N) = -\text{ constant } (Nm^{-1}) \times \text{distance } (m)$$ The negative sign indicates the Force (and hence acceleration) is in the opposite direction to the displacement. Other units are possible but remember these quantities do have units
Converts between angular velocity, frequency and period $$\omega = 2\pi f = \frac{2\pi}{T}$$ $$\text{angular frequency } (rad\ s^{-1}) = 2\pi \times \text{frequency } (Hz) = \frac{2\pi}{\text{Period } (s)}$$ *NB 2π rad = 1 revolution*
Definition of S.H.M $$a = \frac{d^2y}{dt^2} = -\omega^2 y$$ $$\frac{\text{acceleration}}{(ms^{-2})} = \text{second derivative of displacement} = \frac{-\text{angular frequency}^2}{(rad\ s^{-1})^2} \times \frac{\text{displacementfrom rest position}}{(m)}$$

Solution to SHM equation *Sine function occurs when y=0 and t=0, cosine function occurs when y=A at t=0* $$y = A\cos\omega t \text{ or } y = A\sin\omega t$$ $$displacement\ (m) = Amplitude(m) \times \cos\ angular\ frequency\ (rad\ s^{-1}) \times time\ (s)$$ or $$displacement\ (m) = Amplitude(m) \times \sin\ angular\ frequency\ (rad\ s^{-1}) \times time\ (s)$$
Velocity of a particle undergoing SHM $$v = \pm\omega\sqrt{(A^2 - y^2)}$$ $$velocity\ (ms^{-1}) = \pm angularfrequency(rad\ s^{-1}) \times \sqrt{(Amplitude^2(m)^2 - displacment^2(m)^2)}$$ $$NB\ v_{max}\ occurs\ when\ y = 0\ \therefore\ v_{max} = \pm\omega A$$
Kinetic energy of a particle undergoing SHM $$E_k = \frac{1}{2}m\omega^2(A^2 - y^2)$$ $$kinetic\ energy\ (J) = \frac{1}{2} \times mass(kg) \times angular\ frequency^2\ (rad\ s^{-1})^2 \times (Amplitude^2(m)^2 - displacment^2(m)^2))$$ NB Ek is at a maximum when y = 0 and zero when y = A $$E_{k\ (max)} = \frac{1}{2}m\omega^2A^2$$
Potential energy of a particle undergoing SHM $$E_p = \frac{1}{2}m\omega^2y^2$$ $$potential\ energy\ (J) = \frac{1}{2} \times mass(kg) \times angular\ frequency^2\ (rad\ s^{-1})^2 \times displacment^2(m)^2$$ NB The sum of Ep and Ek remains constant Ep is maximum when y = A and zero when y = 0
Energy of a wave $$E = kA^2$$ $$Energy\ of\ a\ wave\ (J) = constant(\ Jm^{-2}) \times\ amplitude^2(m)^2$$ Other equivalent units are possible, but the quantities do have units

Travelling Wave Equation

$$y = A\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$$

$$displacement\ (m) = Amplitude\ (m) \times \text{sine} \times 2 \times \pi\left(frequency\ (Hz) \times time\ (s) - \frac{distance\ (m)}{wavelength\ (m)}\right)$$

Lengths could be in units other than m but would need to be stated, the default position is metres

A wave travelling in the positive direction (left to right) has a negative sign in the equation, a wave travelling in the negative direction (right to left) has a positive in the equation.

Check out the various combinations of this equation where 2π could be multiplied out, a number can appear on the top which would equal $1/\lambda$, and $2\pi f$ can be replaced with ω.

It must be a travelling wave as the value of y changes with time.

$$y = A\sin 2\pi f\left(t - \frac{x}{f\lambda}\right)$$

$$y = Asin2\pi f(t - \frac{x}{v})$$

$$y = A\sin\omega\left(t - \frac{x}{v}\right)$$

$$y = Acos\omega(t - \frac{x}{v})$$

Phase difference or phase angle between two positions on a travelling wave.

$$\emptyset = \frac{2\pi x}{\lambda}$$

$$phase\ angle\ (rad) = \frac{2 \times \pi \times distance\ between\ the\ two\ positions\ (m)}{wavelength\ (m)}$$

An equation which governs whether waves in materials travelling different routes will be in phase or out of phase.

$$opd = n \times gpd$$

$$optical\ path\ difference\ (m) = refractive\ index\ \times geometric\ path\ difference\ (m)$$

Conditions for constructive and destructive interference

$$opd = m\lambda\ or\left(m + \frac{1}{2}\right)\lambda\ where\ m = 0, 1, 2, \dots$$

$$\frac{optical\ path\ difference}{(m)} = \frac{a\ whole\ number\ of wavelength}{(m)}\ or\ \frac{a\ whole\ number\ of wavelength + \frac{1}{2}wavelength}{(m)}$$

$$\left(m + \frac{1}{2}\right)\lambda = destructive\ interference$$

$$m\lambda = constructive\ interference$$

Fringe separation for a thin wedge $$\Delta x = \frac{\lambda l}{2d}$$ $$fringe\ separation\ (m) = \frac{wavelength\ (m) \times wedge\ length\ (m)}{2\ \times wedge\ thickness\ (m)}$$ **Beware, <u>in a few questions only</u> the distance between a certain number of fringes is given and this is (n-1) for the fringe separation, eg the distance between 11 fringes is 2.0 × 10^{-4} m. This is 10 fringe separations so Δx = 2.0× 10^{-5} m**
Non- reflection lens coating thickness $$d = \frac{\lambda}{4n}$$ $$coating\ thickness\ (m) = \frac{wavelength\ of\ light\ to\ be\ reduced\ (m)}{4\ \times refractive\ index\ of\ the\ lens\ coating}$$
Fringe spacing for Young's Double Slit which only applies when D>>Δx $$\Delta x = \frac{\lambda D}{d}$$ $$fringe\ separation\ (m) = \frac{wavelength\ (m) \times distance\ from\ the\ slits\ to\ screen\ (m)}{slit\ spacing\ (m)}$$ **Beware, sometimes the distance between a certain number of fringes is given and this is n-1 for the fringe separation**
Brewster Angle or polarising angle formula $$n = \tan i_p$$ $$refractive\ index = \tan of\ the\ polarising\ angle\ (°or\ rad)$$
Coulomb's Inverse Square Law $$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$$ $$Electrostatic\ Force\ (N) = \frac{Charge\ on\ point\ charge\ 1\ (C)\ \times charge\ on\ point\ charge\ 2\ (C)}{4\ \times\ \pi\ \times permitivity\ of\ free\ space\ (Fm^{-1}) \times separation\ of\ the\ two\ charges^2\ (m)^2}$$ **Like charges will repel, opposite charges will attract** $$\varepsilon_o\ is\ the\ permitivity\ of\ free\ space\ and\ is\ 8.85\ \times 10^{-12}\ Fm^{-1}$$ $$\frac{1}{4\pi\varepsilon_o} \approx 9 \times 10^9$$

Electric Potential

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

$$Electric\ potential\ (V) = \frac{Magnitude\ of\ the\ charge\ \ (C)}{4\ \times\ \pi\ \times permitivity\ of\ free\ space\ (Fm^{-1}) \times distance\ from\ the\ charge\ (m)}$$

ε_o *is the permitivity of free space and is* $8.85\ \times 10^{-12}\ Fm^{-1}$ $\qquad \frac{1}{4\pi\varepsilon_o} \approx 9 \times 10^9$

Electric field strength

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$Electric\ field\ strength\ (NC^{-1}) = \frac{Magnitude\ of\ the\ charge\ \ (C)}{4\ \times\ \pi\ \times permitivity\ of\ free\ space\ (Fm^{-1}) \times separation\ of\ the\ two\ charges^2\ (m)^2}$$

ε_o *is the permitivity of free space and is* $8.85\ \times 10^{-12}\ Fm^{-1}$ $\qquad \frac{1}{4\pi\varepsilon_o} \approx 9 \times 10^9$

Use for the definition of Electric field strength

$$F = QE$$

$$Electric\ Force\ (N) = magnitudeof\ the\ charge\ (C)\ \times Electric\ field\ strength\ (NC^{-1})$$

Relationship for a uniform electric field / parallel plates

$$V = Ed$$

$$Electric\ potential\ (V) = Electric\ field\ strength\ (NC^{-1}) \times distance\ (m)$$

Work done moving a charge through a potential difference.

$$W = QV$$

$$work\ done\ (J) = magnitudeof\ the\ charge\ (C)\ \times potential\ difference\ (V)$$

$$E_k = \frac{1}{2}mv^2$$

$$kinetic\ energy\ (J) = \frac{1}{2} \times mass\ (kg) \times speed^2\ \ (m\ s^{-1})^2$$

Magnetic induction at a perpendicular distance from an "infinite" straight current carrying conductor. $$B = \frac{\mu_o I}{2\pi r}$$ $$Magnetic\ Induction\ (T) = \frac{Permeability\ of\ free\ space\ (Hm^{-1}) \times current\ (A)}{2 \times \pi \times distance\ (m)}$$ $\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$
Force on a current carrying conductor in a magnetic field $$F = IlB \sin\theta$$ $$\frac{Magnetic\ Force}{(N)} = \frac{Current}{(A)} \times \frac{length}{(m)} \times \frac{magnetic\ induction}{(T)} \times \sin\left(\frac{angle\ between\ I\ and\ B}{(°\ or\ rad)}\right)$$
Derive from $F = IlB\sin\theta$ $$F = qvB$$ $$\frac{Electric\ Force}{(N)} = \frac{magnitudeof\ the\ charge}{(C)} \times \frac{velocity\ of\ the\ charge}{(m\ s^{-1})} \times \frac{magnetic\ induction}{(T)}$$
The time constant is the time required to charge the capacitor, through the resistor, from an initial charge voltage of zero to approximately 63.2% of the value of an applied DC voltage, or to discharge the capacitor through the resistor to approximately 36.8% of its initial charge voltage. $$\tau = RC$$ $$time\ constant\ (s) = Resistanceof\ series\ resistor\ (\Omega) \times Capacitance\ (F)$$
Reactance (opposition to A.C.) of a capacitor. V and I may either <u>both</u> be peak or r.m.s. $$X_c = \frac{V}{I}$$ $$Capacitive\ reactance\ (\Omega) = \frac{voltage\ (V)}{current\ (A)}$$
Reactance (opposition to A.C.) of a capacitor. $$X_c = \frac{1}{2\pi f C}$$ $$Capacitive\ reactance\ (\Omega) = \frac{1}{2 \times \pi \times frequency\ (Hz) \times Capacitance\ (F)}$$

$$\varepsilon = -L\frac{dI}{dt}$$

$$induced\ e.m.f\ (V) = -inductance(H) \times rate\ of\ change\ of\ current\ (As^{-1})$$

The negative sign indicates the induced e.m.f opposes the change causing it. ε is also a negative and the two negatives cancel.

Energy stored in the magnetic field of an inductor

$$E = \frac{1}{2}LI^2$$

$$energy\ stored\ in\ the\ magnetic\ field\ of\ an\ inductor\ (J) = \frac{1}{2} \times inductance(H) \times\ current\ (A)^2$$

Reactance (opposition to A.C.) of an inductor. V and I may either both be peak or r.m.s.

$$X_L = \frac{V}{I}$$

$$inductive\ reactance\ (\Omega) = \frac{voltage\ (V)}{current\ (A)}$$

Reactance (opposition to A.C.) of an inductor.

$$X_L = 2\pi f L$$

$$inductive\ reactance\ (\Omega) = 2 \times \pi \times frequency\ (Hz) \times inductance\ (H)$$

Equation to calculate the speed of light and all electromagnetic radiations in a vacuum.

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

$$speed\ of\ light\ in\ a\ vacuum\ (ms^{-1}) = \frac{1}{\sqrt{permittivity\ of\ free\ space\ (Fm^{-1}) \times permeability\ of\ free\ space\ (Hm^{-1})}}$$

$\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$, ε_o *is the permitivity of free space and is* $8.85 \times 10^{-12}\ Fm^{-1}$

A method for combining uncertainties in a single measurement

$$\Delta W = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$$

Total uncertainty in W is equal to the square root of the square of the uncertainties in X, Y and Z

NB the units of X,Y and Z are the units of the measurement

A method of combining fractional (or percentage) uncertainties in different variables.

$$\frac{\Delta W}{W} = \sqrt{\left(\frac{\Delta X}{X}\right)^2 + \left(\frac{\Delta Y}{Y}\right)^2 + \left(\frac{\Delta Z}{Z}\right)^2}$$

Fractional uncertainty in W is equal to the square root of the square of the fractional uncertainties in X, Y and Z

A method of calculating an uncertainty raised to a power $$\left(\frac{\Delta W^n}{W^n}\right) = n\left(\frac{\Delta W}{W}\right)$$ *The fractional uncertainty in W raised to power n is n times the fractional uncertainty in W*
$$d = \bar{v}t$$ $$distance\ (m) = average\ speed\ (ms^{-1}) \times time\ (s)$$
$$s = \bar{v}t$$ $$displacement\ (m) = average\ velocity\ (ms^{-1}) \times time\ (s)$$
$$v = u + at$$ $$final\ velocity\ (ms^{-1}) = initial\ velocity\ (ms^{-1}) + acceleration\ (ms^{-2}) \times time\ (s)$$
$$s = ut + \frac{1}{2}at^2$$ $$displacement = initial\ velocity \times time + \frac{1}{2} \times acceleration\ (ms^{-2}) \times time^2\ (s^2)$$
$$v^2 = u^2 + 2as$$ $$final\ velocity^2\ (ms^{-1})^2 = initial\ velocity^2 (ms^{-1})^2 + 2 \times acceleration\ (ms^{-2}) \times dispacement\ (m)$$
$$s = \frac{1}{2}(v + u)t$$ $$displacement\ (m) = \frac{1}{2} \times (final\ velocity\ (ms^{-1}) + initial\ velocity\ (ms^{-1})) \times time\ (s)$$
$$F = ma$$ $$force\ (N) = mass\ (kg) \times acceleration\ (ms^{-2})$$
$$W = mg$$ $$weight\ (N) = mass\ (kg) \times gravitational\ field\ strength\ (N\ kg^{-1})$$
$$E_w = Fd$$ $$work\ done\ (J) = force\ (N) \times distance\ (m)$$
$$E_p = mgh$$ $$gravitational\ potential\ energy\ (J) = mass\ (kg) \times gravitational\ field\ strength\ (N\ kg^{-1}) \times vertical\ height\ (m)$$
$$E_k = \frac{1}{2}mv^2$$ $$kinetic\ energy\ (J) = \frac{1}{2} \times mass\ (kg) \times speed^2 (ms^{-1})^2$$
$$P = \frac{E}{t}$$ $$power\ (W) = \frac{energy\ (J)}{time\ (s)}$$

$$p = mv$$ $$momentum\ (kgms^{-1}) = mass(kg) \times velocity\ (ms^{-1})$$
$$Ft = mv - mu$$ $$Impulse\ (Ns) = mass\ (kg) \times final\ velocity\ (ms^{-1}) - mass\ (kg) \times initial\ velocity\ (ms^{-1})$$ $$Impulse\ (Ns) = change\ in\ momentum\ (kg\ ms^{-1})$$
$$F = G\frac{m_1 m_2}{r^2}$$ $$Force\ (N) = Universal\ gravitational\ Constant\ (m^3kg^{-1}s^{-2})\frac{Mass_1(kg) \times Mass_2(kg)}{separation\ distance^2\ (m^2)}$$ $$\text{NB The Universal Gravitational Constant} = 6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2}$$
$$t' = \frac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$ $$relativistic\ time\ (s) = \frac{time\ (s)}{\sqrt{1-\left(\frac{speed\ (ms^{-1})}{speed\ of\ light\ in\ vacuum\ (ms^{-1})}\right)^2}}$$ *NB time can be in other units as this is a ratio, but both times must be in the same unit.* $c = 3.0 \times 10^8\ ms^{-1}$
$$l' = l\sqrt{1-\left(\frac{v}{c}\right)^2}$$ $$relativistic\ length\ (m) = length\ (m) \times \sqrt{1-\left(\frac{speed\ (ms^{-1})}{speed\ of\ light\ in\ vacuum\ (ms^{-1})}\right)^2}$$ $c = 3.0 \times 10^8\ ms^{-1}$
$$f_o = f_s\left(\frac{v}{v \pm v_s}\right)$$ $$\frac{frequency\ observed}{(Hz)} = \frac{frequency\ of\ source}{(Hz)} \times \left(\frac{speed\ of\ sound\ (ms^{-1})}{speed\ of\ sound\ (ms^{-1}) \pm velocity\ of\ source\ (ms^{-1})}\right)$$ ADD when the object moves AWAY from the observer and TAKE AWAY (subtract) when the object comes TOWARDS the observer

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$ $$Redshift\ (no\ unit) = \frac{observed\ wavelength\ (m) - rest\ wavelength\ (m)}{rest\ wavelength\ (m)}$$
$$z = \frac{v}{c}$$ $$Redshift\ (no\ unit) = \frac{recessional\ velocity\ (ms^{-1})}{speed\ of\ light\ in\ vacuum\ (ms^{-1})}$$
$$v = H_o d$$ $$\underset{(ms^{-1})}{recessional\ velocity} = \underset{(s^{-1})}{Hubble's\ Constant} \times \underset{(m)}{distance\ from\ galaxy\ to\ observer}$$ NB for this course the Hubble Constant Ho is given as $2{\cdot}3 \times 10^{-18}\ s^{-1}$
$$W = QV$$ $$Work\ done\ moving\ a\ charge\ across\ a\ p.d.\ (J) = electrical\ charge\ (C) \times voltage\ (V)$$
$$E = mc^2$$ $$Energy\ (J) = mass\ (kg) \times speed\ of\ light\ squared\ (ms^{-1})^2$$ NB the speed of light squared is equal to $9.0 \times 10^{16}\ m^2s^{-2}$
$$E = hf$$ $$energy\ (J) = Planck's\ Constant\ (Js) \times frequency\ (Hz)$$ NB Planck's constant = 6.63×10^{-34} Js
$$E_k = hf - hf_o$$ $$\underset{(J)}{Kinetic\ Energy} = \left(\underset{(Js)}{Planck's\ Constant} \times \underset{(Hz)}{incident\ frequency}\right) - \left(\underset{(Js)}{Planck's\ Constant} \times \underset{(Hz)}{threshold\ frequency}\right)$$ NB Planck's constant = 6.63×10^{-34} Js hf_o is also known as the work function (J), hf is the energy of the incident photon (J)
$$E_2 - E_1 = hf$$ $$most\ excited\ energy (J) - least\ excited\ energy\ (J) = Planck's\ Constant\ (Js) \times frequency\ (Hz)$$
$$T = \frac{1}{f}$$ $$Period\ (s) = \frac{1}{Frequency\ (Hz)}$$
$$v = f\lambda$$ $$speed\ (ms^{-1}) = frequency\ (Hz) \times wavelength\ (m)$$

$$d\sin\theta = m\lambda$$ $$Slit\ separation\ (m) \times sin\ of\ angle\ from\ centre\ to\ the\ spot\ =\ m\ \ a\ whole\ number\ of\ wavelengths\ (m)$$ NB This equation is for constructive interference
$$n = \frac{\sin\theta_1}{\sin\theta_2}$$ $$Refractive\ index\ = \frac{sine\ of\ the\ angle\ in\ vacuum/air}{sine\ of\ the\ angle\ in\ the\ material}$$
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$ $$Refractive\ index\ = \frac{sin\ of\ the\ angle\ in\ vacuum/air}{sin\ of\ the\ angle\ in\ the\ material} = \frac{wavelength\ (air)(m)}{wavelength\ (material)(m)} = \frac{speed\ \ (air)\ (ms^{-1})}{speed\ (material)(ms^{-1})}$$ $$refractive\ index = ratio\ of\ wavelengths\ in\ vacuum/air\ and\ material$$ $$refractive\ index = ratio\ of\ the\ speeds\ in \frac{vacuum}{air} and\ the\ material$$ *This formula really applies to material 1 being a vacuum, but there is not much difference between the refractive indexes of air and a vacuum ∴ we assume for Higher they have the same value.*
$$\sin\theta_c = \frac{1}{n}$$ $$Sine\ of\ the\ critical\ angle\ = \frac{1}{refractive\ index}$$ The critical angle is the angle in the material when the angle in air is 90°
$$I = \frac{k}{d^2}$$ $$irradiance\ (Wm^{-2}) = \frac{constant\ (W)}{distance^2(m^2)}$$ This is more easily understood as $$irradiance(Wm^{-2}) \times distance^2(m^2) = constant\ value\ (W)$$
$$I = \frac{P}{A}$$ $$irradiance\ (Wm^{-2}) = \frac{power\ (W)}{area\ (m^2)}$$
$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$ $$root\ mean\ square\ A.C. voltage\ (V)\ = \frac{peak\ voltage\ (V)}{1.414}$$

$I_{rms} = \frac{I_{peak}}{\sqrt{2}}$ $root\ mean\ square\ A.C.current\ (A) = \frac{peak\ current\ (A)}{1.414}$
$Q = It$ $Charge\ (C) = current\ (A) \times time\ (s)$ $This\ is\ better\ explained\ as\ current\ is\ the\ rate\ of\ flow\ of\ charge\ \left(I = \frac{Q}{t}\right)$
$V = IR$ $Voltage (V) = Current\ (A) \times Resistance (\Omega)$
$P = IV = I^2R = \frac{V^2}{R}$ $Power\ (W) = current\ (A) \times voltage (V) = current\ ^2 (A^2) \times Resistance\ (\Omega) = \frac{Voltage^2\ (V^2)}{Resistance\ (\Omega)}$
For resistors in series $R_T = R_1 + R_2 + \cdots$ $total\ resistance\ (\Omega) = resistance\ _1 (\Omega) + resistance\ _2 (\Omega) + \cdots$
For resistors in parallel $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$ $\frac{1}{total\ resistance\ (\Omega)} = \frac{1}{resistance\ _1 (\Omega)} + \frac{1}{resistance\ _2 (\Omega)} + \cdots$
$E = V + Ir$ $e.m.f\ (V) = terminial\ potential\ difference\ (V) + current\ (A) \times internal\ resistance\ (\Omega)$ This can also be written as $E = I(R + r) \quad or \quad E = IR + Ir$ I is the total current in the circuit, r is in series with the combined circuit resistance
$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_s$ $voltage\ across\ component\ 1\ in\ potential\ divider (V) = \left(\frac{resistance_1\ (\Omega)}{total\ resistance (\Omega)}\right) \times supply\ voltage\ (V)$

For resistances in series (potential divider circuits)

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

Ratio of the voltages in series = ratio of the resistance in series

$$\frac{Voltage\ across\ resistor 1\ (V)}{voltage across\ resistor\ 2\ (V)} = \frac{resistance\ of\ resistor 1\ (\Omega)}{resistance\ of\ resistor\ 2\ (\Omega)}$$

$$C = \frac{Q}{V}$$

$$Capacitance\ (F) = \frac{Charge\ (C)}{Voltage\ (V)}$$

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

$$\underset{(J)}{Energy\ stored\ in\ capacitor} = \frac{1}{2} \times \underset{(C)}{charge\ stored\ in\ capacitor} \times \underset{(V)}{voltage\ across\ capacitor}$$

$$\underset{(J)}{Energy\ stored\ in\ capacitor} = \frac{1}{2} \times \underset{(F)}{capacitance} \times \underset{(V)^2}{voltage\ across\ capacitor^2}$$

$$\underset{(J)}{Energy\ stored\ in\ capacitor} = \frac{1}{2} \times \frac{(charge\ stored\ in\ capacitor)^2\ (C^2)}{voltage\ across\ capacitor\ (V)}$$

$$Path\ difference = m\lambda\ or\ \left(m + \frac{1}{2}\right)\lambda, where\ m = 0, 1, 2 \ldots$$

$$\underset{(m)}{Path\ difference} = \underset{(constructive\ interference)}{whole\ number\ of\ wavelengths}$$

$$\underset{(m)}{path\ difference} = \underset{(destructive\ interference)}{whole\ number\ of\ wavelengths + \frac{1}{2}\ a\ wavelength}$$

$$Random\ Uncertainty = \frac{Max\ value - \min value}{number\ of\ values} \qquad or\ \Delta R = \frac{R_{max} - R_{min}}{n}$$

NB for the random uncertainty in a value the units of the random uncertainty are the same as for the quantity you are finding the uncertainty for.

$$Random\ Uncertainty (units\ of\ the\ quantity) = \frac{Max\ value - \min value}{number\ of\ values}$$