Angular Motion

AH Physics

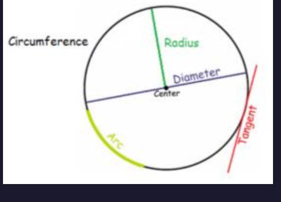
All you've done in Higher Physics but in circles.

If you need additional help try these….

Angular Motion

- https://youtu.be/mn6NKAs6tbk
- https://www.youtube.com/watch?v=jNc2SflUl9U
- https://isaacphysics.org/concepts/cp_ang_eq_of_motion
- https://www.youtube.com/watch?v=RHmrcxQCbwg
- https://www.youtube.com/watch?v=Vfy3bChhBnk

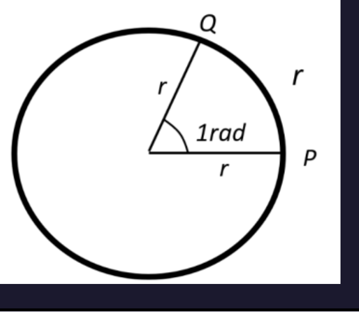
Basic terms

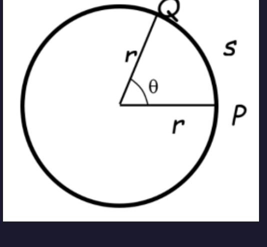


- Circumference - The Length of the Perimeter of a Circle
- Arc - The length of a section of the Circumference of a Circle
- Diameter - The Length of the Line connecting opposite sides of a Circle through its centre
- Radius - The Length of the Line connecting the Edge of a Circle to its centre
- Tangent - A Line at a right angle to the Radius that 'touches' the edge of a Circle.

Measuring Angles

Angles are measured in radians (rad) on the SI system





How far!

- *1 radian is equal to the angle subtended at the centre of a circle when an object moves around the circle a distance equal to the radius of the circle.*
- *If we move round the circle a distance, r, we subtend 1 rad.*
- *If we move round the circle a distance, 2r, we subtend 2 rad.*
- *If we move round the circle a distance, 1.76r, we subtend 1.76 rad.*
- *If we move round the circle a distance, $2\pi r$, we subtend 2π rad.*

If we move round the circle a distance, $2\pi r$, we subtend 2π rad.

But…..

But $2\pi r$ = circumference of a circle

$$2\pi r = 360^\circ$$

$$1^\circ = \frac{2\pi}{360}\ \text{rad}$$

If we move a distance, s, we subtend $\frac{s}{r}$ rads.

- $i.e.\ \theta = \frac{s}{r} \quad \therefore s = r\theta$

So what are the units of s, r and θ?

Compendium Questions

Convert the following from degrees to radians:	30°, 60°, 90°, 180°, 270°, 360°, 720°.
Convert the following from degrees into radians	30°, 14°, 5°
Convert the following from degrees into radians:	45°, 14°, 1°
Convert the following from radians to degrees:	1 rad, 10 rad, 0.1 rad, π rad, 2π rad, $\frac{\pi}{6}$ rad.
Convert the following from radians to degrees:	½ π rad, 5 rad, 0.1 rad, 0.01 rad
Convert the following from revolutions per minute to radians per second:	33 rpm, 45 rpm, 78 rpm, 300 rpm.

Answers

Angular Velocity

- *To describe how quickly an object moves around a circle we measure the angular velocity.*
- *The angular velocity is the rate at which an object turns through an angle. If, in t seconds the object subtends θ rads then:*

Angular velocity, ω

$$\omega = \frac{\theta}{t}$$

$$\{\text{rad s}^{-1}\} = \left\{\frac{rad}{s}\right\}$$

NB Watch the unit!

An electric drill rotates at 800 rpm, find its angular velocity.

In 1 rev there are 2π radians.
∴ in 800 revs there are 2π x 800 radians.

$$\therefore \omega = \frac{\theta}{t}$$

$$\omega = \frac{800 \times 2\pi}{60} = 83.8\,rad\,s^{-1}$$

Determine the angular velocity of the Earth.

1 rev takes 24 hours

∴ 1 rev takes 24 x 60 x 60 seconds = 86400 seconds

1 rev is 2π radians

$$\therefore \omega = \frac{\theta}{t}$$

$$\omega = \frac{2\pi}{86400} = 7.3 \times 10^{-5}\ rad\ s^{-1}$$

Converting rpm to rad s^{-1}

- **1 revolution is 2π radians**
- $\mathbf{1\ rev\ per\ min} = \frac{2\pi}{60}\ \mathbf{rad\ s^{-1}}$
- $\mathbf{N\ rev\ per\ min} = N \times \frac{2\pi}{60}\ \mathbf{rad\ s^{-1}}$

Find the angular velocity of a washing machine which spins at 1300 rpm.

$$\omega = \frac{\theta}{t} = \frac{1300 \times 2\pi}{60} = 136\ rad\ s^{-1}$$

- Determine the angular velocity of a geostationary satellite.

NB. radians have no dimensions.

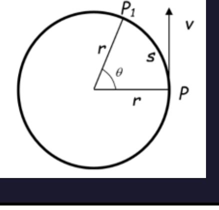
Compendium Questions

Calculate the angular velocity of each of the following:

(a) A bicycle spoke turning through 5.8 rad in 3.6 s.
(b) A playground roundabout rotating once every 4 s.
(c) An electric drill bit rotating at 3000 revolutions per minute (rpm).
(d) An electric drill bit rotating at 40 revolutions per second.
(e) The second hand of an analogue watch.
(f) The Moon orbiting the Earth with a period of 27.3 days.
(g) The Earth spinning about its polar axis.
(h) A rotating object whose angular displacement, θ, is given by θ = 5t + 4 radians, where t is the time in seconds.

To relate linear speed, v, to angular speed

- ***To relate linear speed, v, to angular speed take an object, P, moving around a circle or radius, r.***
- ***In time, t, P moves a distance, s, and reaches point P_1:***



$$\therefore speed = \frac{s}{t}$$

but we know $s = r\theta$

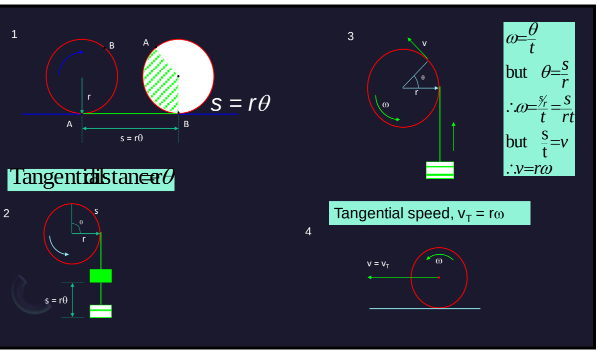
$$\therefore speed = v = \frac{r\theta}{t}$$

$$\boxed{\therefore v = r\omega}$$

Look back at the three questions from before calculate the linear speed on the outer circumference of each object:

	ω (rad s^{-1})	radius (m)
Drill	83.8	0.005
Earth	7.3 x 10^{-5}	6.4 x 10^{6}
Washing machine	136	0.2
Geostationary satellite	7.3 x 10^{-5}	36 000 km above the Earth!!

(467 m s^{-1} represents the speed at the Earth's equator, and is equivalent to 1045 mph!),



Compendium Questions

A propeller rotates at 95 rpm.
(a) Calculate the angular velocity of the propeller.
(b) Each propeller blade has a length of 0.35 m.
Calculate the linear speed of the tip of a propeller.

A CD of diameter 120 mm rotates inside a CD player.

The linear speed of point A on the circumference of the CD is 1.4 m s^{-1}.
Calculate the angular velocity of the CD:
(a) in rad s^{-1}
(b) in rpm.

Angular Motion

Concept	Translational	Rotational	Comments
Displacement	s	θ	$s = r\theta$
Velocity	$v = \frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$	$v = r\omega$
Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$	$a_t = r\alpha$
Equations of motion for constant acceleration.	$v = u + at$	$\omega = \omega_0 + \alpha t$	
	$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
	$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$	

Where
θ angular displacement, rad
ω angular velocity, rad s^{-1}
ω_0 initial angular velocity rad s^{-1}
ω final angular velocity, rad s^{-1}
α angular acceleration rad s^{-2}
t time s

NB a_t in $a_t = r\omega$ is the tangential acceleration

Angular Acceleration

Something to be aware of!

Acceleration is a vector quantity-
change the direction you change the acceleration

ANGULAR ACCELERATION

α

(Rate of change of angular velocity)

Angular acceleration $\alpha = \frac{d\omega}{dt}$.

This is the rate of change of angular velocity.
This may have the effect of increasing the velocity of the orbit.

$$\text{Angular acceleration } \alpha = \frac{\omega - \omega_0}{t}$$

where $\omega - \omega_0$ is the change in angular velocity during the time interval t.

TANGENTIAL ACCELERATION

$a_\perp$

(only experienced by objects whose angular velocity is changing)

a measure of how the tangential velocity of a point at a certain radius changes with time

Tangential acceleration

angular acceleration $\boldsymbol{\alpha} = \frac{d\omega}{dt}$ but $v = r\omega$

$$\frac{dv}{dt} = r \cdot \frac{d\omega}{dt}$$

tangential acceleration $a_\perp = r\alpha$

$$a_\perp = r\alpha$$

As an aside
Linking angular acceleration and tangential acceleration

From the formula for tangential acceleration it can be seen that angular acceleration and tangential acceleration are related by the radius of the circle of motion.

$$\text{But } \omega = \frac{v}{r}$$

$$\therefore \alpha = \frac{\frac{v}{r} - \frac{u}{r}}{t} = \frac{v-u}{rt}$$

$$\alpha r = \frac{v-u}{t}$$

$$\text{but } \frac{v-u}{t} = a$$

$$\therefore r\alpha = a$$

$$a_\perp = r\alpha$$

ACCELERATION IN CIRCLES

Acceleration is a vector quantity-
change the direction you change the acceleration

- All particles travelling in a circular motion experience **centripetal** (**radial acceleration**). Only those whose **angular velocity** is changing experience **tangential acceleration**.
- Any particle moving in a circular path has radial acceleration (although the linear velocity maybe constant). A particle may have angular acceleration which is described as the rate of change of angular velocity.

RADIAL OR CENTRIPETAL/ CENTRAL ACCELERATION

a

(IF OBJECTS ARE MOVING IN CIRCLES THEIR DIRECTION IS CHANGING AND HENCE THEY ARE ACCELERATING)

Radial (centripetal acceleration) a= v^2/r . This is the rate of change of the direction of motion.

$$a = \frac{v^2}{r} = \omega^2 r$$

$$a = \frac{v^2}{r} = \omega^2 r$$

- **Radial Acceleration**
- **a= v^2 / r** but v=rω
- therefore $v^2 = r^2\ \omega^2$
- so a = $r^2\ \omega^2$ /r = **rω^2**

A 150g ball at the end of a string is swinging in a horizontal circle of radius 0.60m. The ball makes exactly 2.00 revs per second. What is its centripetal acceleration?

- $a = \frac{v^2}{r}$ therefore first find v
- $v = \frac{2\pi r}{t} = \frac{2 \times 3.14 \times 0.6}{0.5}\ or \frac{2 \times 2 \times 3.14 \times 0.6}{1} = 7.5\ ms^{-1}$
- $a = \frac{v^2}{r} = \frac{(7.5)^2}{0.6} = 95\ ms^{-2}$ (note the units of centripetal acceleration are ms^{-2})

The moon's nearly circular orbit about the earth has a radius of about 385000 km and a period T of 27.3 days. Determine the acceleration of the moon towards the earth.

- In orbit the moon travels a distance of $2\pi r$ where $r = 3.85 \times 10^8$ m
- $v = 2\pi r / T = 1.02 \times 10^3$ ms^{-1}
- $a = v^2/r = (1.02 \times 10^3)^2 / 3.85 \times 10^8 = 2.73 \times 10^{-3}$ ms^{-2}
- or $\omega = 2\pi / t$ rad s^{-1} = 2.66×10^{-6}
- $a = r\omega^2 = 3.85 \times 10^8 \times (2.66 \times 10^{-6})^2 = 2.73 \times 10^{-3}$ ms^{-2}
-

Central Force

- A centripetal force is a force that makes a body follow a curved path. Its direction is always towards the centre of curvature of the path.
- To calculate the size of a central acceleration

Do these with the turntable or https://physicsflashrepo.cyou/ah-physics-experiments/

- **AH Exp – Mechanics 01: Measurement of angular velocity**

Aim – To measure the angular velocity of a rotating turntable

- **AH Exp – Mechanics 02: Measurement of angular acceleration**

Aim – To measure the angular acceleration of a rotating turntable

- **AH Exp – Mechanics 03A: Variation of central force with angular velocity**

Aim – To investigate the relationship between the central force required to maintain circular motion of a rubber stopper and the angular velocity of the stopper.

- **AH Exp – Mechanics 03B: Central force and angular velocity**

Aim – To investigate the relationship between the central force maintaining the circular motion of a ball and the angular velocity of the ball.