4. Our Dynamic Universe Answers
ANSWERS FOR TUTORIAL 1

1. \( X = R \cos \phi \) \( Y = R \sin \phi \) gives \( X = 20.616 \cos 14.036^\circ = 20 \)
   \( Y = 20.616 \sin 14.036^\circ = 5.09 \)
   The rectangular coordinates are (20, 5.09)

2. \( R^2 = (X^2 + Y^2) \)
   \( \tan \phi = \frac{Y}{X} \) gives \( R^2 = 49.737^2 + 40.277^2 = 4096 \)
   \( R = 64 \)
   \( \tan \phi = \frac{40.277}{49.737} = 0.818 \)
   \( \phi = 39^\circ \)

   The distance from the origin is 64 units and it is at an angle of 39\(^\circ\) to the X-axis.

3. The term polar coordinates refers to the position of a point given as the distance from the origin and the direction from the X-axis. It is customary to measure the angular component in the anti-clockwise direction.

   e.g.
   \[
   \begin{align*}
   (a) & \quad (100, 36.87^\circ) \quad \text{gives} \quad X = 100 \cos 36.87^\circ = 80 \quad Y = 100 \sin 36.87^\circ = 60 \\
   & \text{The rectangular coordinates are (80, 60)}
   \\
   (b) & \quad (26.93, 68.2^\circ) = (10, 25)
   \\
   (c) & \quad (45.34, 41.42^\circ) = (34, 30)
   
   \text{N.B. Some calculators perform this transformation using buttons labelled Rec-Pol and Pol-Rec.}
   \]

4. \( R^2 = X^2 + Y^2 \), which gives us:
   \[
   R^2 = 5.64^2 + 2.05^2 \\
   = 31.81 + 4.20 \\
   = 36.01
   \Rightarrow \quad R = 6
   
   \tan \theta = \frac{Y}{X} \Rightarrow \quad \tan \theta = \frac{2.05}{5.64} \\
   = 0.36 \\
   \Rightarrow \quad \theta = 19.97
   \]

   The pipe is 6m long at an angle of 20\(^\circ\) to the longer wall.

5. Parallelogram.
ANSWERS FOR TUTORIAL 2

1. The only measurements on the list that are vector quantities are \textit{acceleration}, \textit{force} and \textit{momentum}, (\textit{don’t worry about momentum you haven’t been taught about it yet!})

2.

Adding coordinates:
We usually take the horizontal line across the page as our x-axis and, if it is convenient, as it is in this case, we line up one of the vectors with it.

\begin{align*}
\text{i)} & \quad \text{We start with the vectors arranged in no particular way.} \\
\text{ii)} & \quad \text{Our new diagram rotates the vectors to a more useful position.} \\
\text{iii)} & \quad \text{We can now put the vectors into their rectangular coordinates.}
\end{align*}

\begin{align*}
X &= 10 \cos 60 = 5 \\
Y &= 10 \sin 60 = 8.66 \\
\text{The sum of the coordinates is} \quad X &= 10 + 5 = 15N \\
\text{The sum of the Y coordinates is} \quad Y &= 0 + 8.66 = 8.66N \\
\text{The resultant is} \quad (15N, 8.66N)
\end{align*}
Converting this to polar coordinates gives us \((17.32\text{N}, 30^\circ)\) where the angular part is measured from the direction of whichever vector we chose as the X direction.

2. Scale diagrams

Follow the steps for question one.

Adding coordinates
Taking the bottom force direction as our X-axis, and finding the rectangular components of the top force gives us:

\[X = 14 \cos 70 = 4.8 \text{N}\]
\[Y = 14 \sin 70 = 13.2 \text{N}\]

The sum of the coordinates is \(X = 14 + 4.8 = 18.8 \text{N}\)
The sum of the Y coordinates is \(Y = 0 + 13.2 = 13.2 \text{N}\)

The resultant is \((18.8 \text{N}, 13.2 \text{N})\)
Converting this to polar coordinates gives us \((22.9 \text{N}, 35^\circ)\) where the angular part is measured from the direction of whichever vector we chose as the X direction.

4.

\[R \text{ measures } 9.4 \text{ms}^{-1} \quad \theta \text{ measures } 20^\circ\]

Adding coordinates:
Resultant \((0 + 5\cos 50, 5 + 5\sin 50) = (3.2, 8.8)\)
Polar coordinates = \((9.4 \text{ ms}^{-1}, 70^\circ)\)
5. (a) A scale drawing which looks like the one below, gives the resultant \( R \) as 4.6m at an angle of 27.2\(^\circ\)

![Diagram](image1)

(b) A similar drawing to the above gives us \( R \) as 18.5m at 27.2\(^\circ\)

![Diagram](image2)

(c) If one pair of vectors is a simple multiple of another pair, then their resultants will be the same multiple of each other.

6. (a) 4m from speed = distance \( \div \) time

\[
\text{distance} = \text{speed} \times \text{time} \quad d = 2 \times 2 = 4m
\]

(b) 16m from speed = distance \( \div \) time

\[
\text{distance} = \text{speed} \times \text{time} \quad d = 8 \times 2 = 16m
\]

(c) A scale diagram gives \( R \) as 16.5m at 14\(^\circ\) to the side of the deck.

(d) Since velocity = displacement \( \div \) time

We have it as 8.25ms\(^{-1}\) at 14\(^\circ\) to the deck’s side, as shown here:

![Diagram](image3)

7.(a) component Y of the tension is \( Y = 6.3 \times 10^{-5} \times \sin 11.5 \)

\[
Y = 1.25 \times 10^{-5} \text{ N}
\]
(b) The weight of the spider is supported by the tension in the strand of the web it is clinging to. Since the spider is in a balanced position, then its weight must equal the total vertical component of the tensions. The spider weighs $2.5 \times 10^{-5} \text{N}$ as there are two $Y$ components to support the weight.

Since the tension is equal in size to the resultant, $R$, of the 60N and the 25N as shown in the diagram, we have:

\[ T = 65 \text{N at 22.6}^\circ \text{ to the vertical}. \]

9. By scale drawings.

[Diagram with vectors and calculations]

R measures 11.6m, $\theta$ measures 56\(^\circ\)

Coordinates
\[ R = (6.487 + 0, 0 + 9.617) = (6.487, 9.617) \]

Polar coordinates = (11.6m, 56\(^\circ\))

As the vectors are at right angles Pythagoras can be used

\[ R^2 = X^2 + Y^2 \]
\[ R^2 = 6.487^2 + 9.617^2 \]
\[ R^2 = 134.58 \quad R = 11.6 \text{ m} \quad \tan \theta = Y/X, \quad \theta = 56^\circ \]

10.

R measures 7.2m, $\theta$ measures 30\(^\circ\)

[Diagram with vectors and calculations]

Coordinates
\[ R = (6.235 + 0, 0 + 3.60) = (6.235, 3.60) \]

Polar coordinates = (7.2m, 30\(^\circ\))

Pythagoras:
\[ R^2 = X^2 + Y^2 \]
\[ R^2 = 6.235^2 + 3.60^2 \]
\[ R^2 = 51.9 \]
\[ R = 7.2 \text{ m} \quad \tan \theta = Y/X, \quad \theta = 30^\circ \]

11. Scale drawing:
R measures 5.6 ms$^{-1}$ 0 measures 26°

Resultant = $(5 + 0 + 0.5 \cos 85, \ 0 + 2 + 0.5 \sin 85) = (5.04, 2.5)$
Polar coordinates = (5.6, 26°)
As there are three vectors and they are not all at right angles this problem cannot be tackled using Pythagoras.

12. Visualise the problem clearly:

Start as if we know everything we need:

velocity = displacement $\implies$ R = displacement / time

but we don’t know R:
We can find R by scale diagram or by using coordinates.

R = $(0 + (-16.9 \cos 12), \ 90 + 16 \sin 12) = (-16.53, 93.51)$
Polar coordinates = (94.96, 80°)
R = displacement / time

95 = displacement
25 x 60
displacement = 95 x 25 x 60 = 142500m = 142.5km

13.

If the boat is to be held stationary, then the resultant of the three forces must be zero. The third force must therefore run from the head of the 6N force back to the tail of the 12N force. This is just the opposite vector to the one we get by adding the 12N and the 6N forces.

Take note of the minus signs:
third force = - (12 + 6sin30, 0 + 6cos30)
= (-15, 5.12)
Polar coordinates (15.87, -161°)

For this type of question you can also use the sine and cosine rules.

ANSWERS FOR TUTORIAL 3

1.

Pythagoras' theorem gives us
x² + (9.2)² = (10.5)²
Thus x = 5.1 The bricklayer lays a second line 5.1m long.

2. The resultant velocity of the fly is 3.5cms⁻¹ downwards and it has 17.5cm to move for which it takes 5s. During this time the rope is raised a height of (14.5 x 5)cm = 72cm
Thus the fly runs out of rope and falls off the end.

3. Displacement is distance travelled in a stright line from the refernce point. Displacement is a vector quantity and must therefore have a direction. You ought to know that this makes up a 3,4, 5 triangle, (work this out by Pythagoras) so the displacement is 5km east. The distance is a scalar quantity and indicates how far the object has travelled. The plane has travelled 3km and then a further 4km = 7km.
Answer E is correct.

4. Using rectangular and polar coordinates
the horizontal is R cos 50, the vertical is R sin 50
or using the other angle given:
the horizontal is $R \sin 40$, the vertical is $R \cos 40$,
$\sin 40 = \cos 50$ or $\cos (90-40)$ etc.
Check this with your teacher if you are unsure of this.

Therefore $E$ is the correct answer.

5. This is a typical higher question. There is far more information than is necessary to work out the answer. Use your gut feelings.
Remember from N5 that $E_p = mgh = Wh$, where $h$ is the vertical height.
$W = 800N$, $h = 5 \sin 20$, therefore $A$ is the correct answer.

6. The resultant force acting on the barge is supplied by whatever fraction of the cable force is acting down the North South line. The component of a cable’s force acting along the north south line is seen from the diagram to be $100000 \cos 26 = 2 \times 10^4 N$
As there are two cables each providing this force the total force is twice this value $= 1.8 \times 10^5 N$

VECTORS/ TUTORIAL ANSWERS:

ANSWERS FOR TUTORIAL 1

1. $X = R \cos \theta = R \sin \theta$ gives $X = 20.616 \cos 14.036^\circ = 20$
   $Y = 20.616 \sin 14.036^\circ = 5.09$ The rectangular coordinates are (20, 5.09)

2. $R^2 = (X^2 + Y^2)$
   $\tan \theta = Y/X$ gives $R^2 = 49.737^2 + 40.277^2 = 4096$
   $R = 64$
   $\tan \theta = 40.277$
   $\frac{49.737}{40.277} = 0.818$
   Thus $\theta = 39^\circ$
   The distance from the origin is 64 units and it is at an angle of $39^\circ$ to the X-axis.

3. The term polar coordinates refers to the position of a point given as the distance from the origin and the direction from the X-axis. It is customary to measure the angular component in the anti-clockwise direction. e.g.
(a) \((100, 36.87^\circ)\) gives

\[
\begin{align*}
X &= 100 \cos 36.87^\circ \\
   &= 80 \\
Y &= 100 \sin 36.87^\circ \\
   &= 60
\end{align*}
\]

The rectangular coordinates are \((80, 60)\)

(b) \((26.93, 68.2^\circ) = (10, 25)\)

(c) \((45.34, 41.42^\circ) = (34, 30)\)

N.B. Some calculators perform this transformation using buttons labelled R-P and P-R. Ask your teacher to show you if you are unsure.

4. \(R^2 = X^2 + Y^2\), which gives us: -

\[
\begin{align*}
R^2 &= 5.64^2 + 2.05^2 \\
    &= 31.81 + 4.20 \\
    &= 36.01 \\
\Rightarrow & \quad R = 6 \\
\tan \theta &= \frac{Y}{X} \\
\Rightarrow & \quad \tan \theta = \frac{2.05}{5.64} \\
    &= 0.36 \\
\Rightarrow & \quad \theta = 19.97
\end{align*}
\]

The pipe is 6m long at an angle of \(20^\circ\) to the longer wall.

5. Parallelogram.
ANSWERS FOR TUTORIAL 2

1. The only measurements on the list that are vector quantities are acceleration, force and momentum, (don’t worry about momentum you haven’t been taught about it yet!)

2.

Adding coordinates:
We usually take the horizontal line across the page as our x-axis and, if it is convenient, as it is in this case, we line up one of the vectors with it.

i) We start with the vectors arranged in no particular way.

ii) Our new diagram rotates the vectors to a more useful position.

iii) We can now put the vectors into their rectangular coordinates.

R measures 17.3 N θ measures 30°
\[ X = 10 \cos 60 = 5 \]
\[ Y = 10 \sin 60 = 8.66 \]

The sum of the coordinates is \( X = 10 + 5 = 15 \)N.
The sum of the Y coordinates is \( Y = 0 + 8.66 = 8.66 \)N.
The resultant is \((15 \text{N}, 8.66 \text{N})\).

Converting this to polar coordinates gives us \((17.32 \text{N}, 30^\circ)\) where the angular part is measured from the direction of whichever vector we chose as the X direction.

2. Scale diagrams

Follow the steps for question one.

\[ R \text{ measures } 22.9 \text{N } \theta \text{ measures } 35^\circ \]

Adding coordinates

Taking the bottom force direction as our X-axis, and finding the rectangular components of the top force gives us:
\[ X = 14 \cos 70 = 4.8 \text{N} \]
\[ Y = 14 \sin 70 = 13.2 \text{N} \]
The sum of the coordinates is \[ X = 14 + 4.8 = 18.8 \text{N} \]
The sum of the Y coordinates is \[ Y = 0 + 13.2 = 13.2 \text{N} \]
The resultant is \((18.8 \text{N}, 13.2 \text{N})\)
Converting this to polar coordinates gives us \((22.9 \text{N}, 35^\circ)\) where the angular part is measured from the direction of whichever vector we chose as the X direction.

4.

\[ R \text{ measures } 9.4 \text{ms}^{-1} \quad \theta \text{ measures } 20^\circ \]
Adding coordinates:

\[ (0 + 5\cos50, 5 + 5\sin50) = (3.2, 8.8) \]

Polar coordinates = \( (9.4 \text{ ms}^{-1}, 70^\circ) \)

5. (a) A scale drawing which looks like the one below, gives the resultant \( R \) as 4.6m at an angle of 27.2°

(b) A similar drawing to the above gives us \( R \) as 18.5m at 27.2°
(c) If one pair of vectors is a simple multiple of another pair, then their resultants will be the same multiple of each other.

6. (a) 4m from speed = distance ÷ time
distance = speed x time
d = 2 x 2 = 4m
(b) 16m from speed = distance ÷ time
distance = speed x time
d = 8 x 2 = 16m
(c) A scale diagram gives R as 16.5m at 14° to the side of the deck.
(d) Since velocity = displacement ÷ time
We have it as 8.25ms⁻¹ at 14° to the deck’s side, as shown here:

7. (a) component Y of the tension is Y = 6.3 x 10⁻⁵ x sin 11.5
\[ Y = 1.25 \times 10^{-5} \text{ N} \]
(b) The weight of the spider is supported by the tension in the strand of the web it is clinging to. Since the spider is in a balanced position, then its weight must equal the total vertical component of the tensions. The spider weighs 2.5x10⁻⁵N as there are two Y components to support the weight.
Since the tension is equal in size to the resultant, R, of the 60N and the 25N as shown in the diagram, we have:

\[ T = 65N \text{ at } 22.6^\circ \text{ to the vertical.} \]

9. By scale drawings.

Coordinates
\[ R = (6.487 + 0, 0 + 9.617) = (6.487, 9.617) \]
Polar coordinates = (11.6m, 56°)

As the vectors are at right angles Pythagoras can be used

\[ R^2 = X^2 + Y^2 \]
\[ R^2 = 6.487^2 + 9.617^2 \]
\[ R^2 = 134.58 \quad R = 11.6 \text{ m} \quad \tan \theta = Y/X, \theta = 56^\circ \]
R measures 7.2 m, \( \theta \) measures 30\(^\circ\)

Coordinates
\[
R = (6.235 + 0, 0 + 3.60) = (6.235, 3.60)
\]
Polar coordinates = (7.2 m, 30\(^\circ\))

Pythagoras:
\[
R^2 = X^2 + Y^2
\]
\[
R^2 = 6.235^2 + 3.60^2
\]
\[
R^2 = 51.9
\]
\[
R = 7.2 \text{ m}
\]
\[
\tan \theta = \frac{Y}{X}, \quad \theta = 30^\circ
\]

11. Scale drawing:

R measures 5.6 ms\(^{-1}\), \( \theta \) measures 26\(^\circ\)

Coordinates
Resultant = \((5 + 0 + 0.5 \cos85, 0 + 2 + 0.5 \sin85) = (5.04, 2.5)\)

Polar coordinates = \((5.6, 26^\circ)\)

As there are three vectors and they are not all at right angles this problem cannot be tackled using Pythagoras.

12. Visualise the problem clearly:
Start as if we know everything we need:

velocity = displacement \Rightarrow R = \frac{\text{displacement}}{\text{time}} = \frac{\text{displacement}}{25\text{min}}

but we don’t know R:
We can find R by scale diagram or by using coordinates.

\[ R = (0 + (-16.9\cos12), 90 + 16\sin12) = (-16.53, 93.51) \]
Polar coordinates = (94.96, 80°)

\[ R = \frac{\text{displacement}}{25\text{min}} = \frac{95}{25 \times 60} \]
displacement = 95 \times 25 \times 60 = 142500\text{m} = 142.5\text{km}

13.

If the boat is to be held stationary, then the resultant of the three forces must be zero. The third force must therefore run from the head of the 6N force back to the tail of the 12N force. This is just the opposite vector to the one we get by adding the 12N and the 6N forces.

Take note of the minus signs:
third force = - (12 + 6\sin30, 0 + 6\cos30) \\
= - (15, 5.12) \\
= (-15, -5.12) \\
Polar coordinates (15.87, -161^\circ) \\

For this type of question you can also use the sine and cosine rules.
ANSWERS FOR TUTORIAL 3

1. 

\[ \begin{align*} 
X^2 + (9.2)^2 &= (10.5)^2 \\
X &= 5.1 
\end{align*} \]

The bricklayer lays a second line 5.1m long.

2. The resultant velocity of the fly is \(3.5\text{cm}\text{s}^{-1}\) downwards and it has 17.5cm to move for which it takes 5s. During this time the rope is raised a height of \((14.5 \times 5)\text{cm} = 72\text{cm}\),

Thus the fly runs out of rope and falls off the end.

3. Displacement is distance travelled in a straight line from the reference point. Displacement is a vector quantity and must therefore have a direction. You ought to know that this makes up a 3,4, 5 triangle, (work this out by Pythagoras) so the displacement is 5km east. The distance is a scalar quantity and indicates how far the object has travelled. The plane has travelled 3km and then a further 4km = 7km.

Answer E is correct.

4. Using rectangular and polar coordinates
the horizontal is \(R \cos 50\), the vertical is \(R \sin 50\)

or using the other angle given:
the horizontal is \(R \sin 40\), the vertical is \(R \cos 40\),

\(\sin 40 = \cos 50 \text{ or } \cos (90-40) \text{ etc.}\).

Check this with your teacher if you are unsure of this.

Therefore E is the correct answer.
5. This is a typical higher question. There is far more information than is necessary to work out the answer. Use your gut feelings.

Remember from SG or Int I & II that $E_p = mgh = Wh$, where $h$ is the vertical height.

$W = 800\text{N}$, $h = 5 \sin 20$, therefore A is the correct answer.

6. The resultant force acting on the barge is supplied by whatever fraction of the cable force is acting down the North South line. The component of a cable’s force acting along the north south line is seen from the diagram to be $100000 \cos 26 = 2 \times 10^4 \text{N}$

As there are two cables each providing this force the total force is twice this value $= 1.8 \times 10^5 \text{N}$

Worked Answers
By scale diagram:
$R$ is $17.6 \text{N}$ and $\theta$ is $30^\circ$.

By calculating resultant vectors:
(assume $x$ to be positive from left to right and $y$ to be positive from bottom to top throughout these answers)

Resolve horizontally:
$F_H = 10 + 10 \cos 60$
$= 10 + 5$
$= 15 \text{N}$

Resolve vertically:
\[ F_v = 10 \sin 60 \]
\[ = 8.66 \text{ N} \]
\[ R^2 = 15^2 + 8.66^2 \]
\[ = 225 + 75 = 300 \]
\[ R = \sqrt{300} = 17.32 \text{ N} \]

\[ \tan \theta = \frac{8.66}{15} = 0.577 \]
\[ \theta = 30^\circ \]

By scale diagram:

R is 22.9 N and \( \theta \) is 35\(^\circ\).
By calculating resultant vectors:

Resolve horizontally:

\[ F_H = 14 + 14 \cos 70 \]
\[ = 14 + 4.8 \]
\[ = 18.8 \text{ N} \]

Resolve vertically:

\[ F_V = 14 \sin 70 \]
\[ = 13.16 \text{ N} \]

\[ R^2 = 18.8^2 + 13.16^2 \]
\[ = 353.4 + 173.2 = 526.6 \]
\[ R = \sqrt{526.6} = 22.9 \text{ N} \]

\[ \tan \theta = \frac{13.66}{18.8} = 0.7 \]
\[ \theta = 35^\circ \]
By scale diagram:

\[ R = \sqrt{6^2 + 4^2} = \sqrt{36 + 16} = \sqrt{52} = 7.2 \text{ N} \]

By calculating resultant vectors:
The vectors are already perpendicular so we don't need to calculate the components.

\[ R = \sqrt{52} = 7.2 \text{ N} \]

and just as a double-check:

\[ \tan 34^\circ = 0.666 = \frac{4}{6} \]
By scale diagram:

R is 9.4 m s\(^{-1}\) and \(\theta\) is 20°.

By calculating resultant vectors:

Resolve horizontally:

\[
F_H = 5 + 5 \cos 40 \\
= 5 + 3.8 \\
= 8.8 \text{ m s}^{-1}
\]

Resolve vertically:

\[
F_V = 5 \sin 40 \\
= 3.2 \text{ m s}^{-1}
\]

\[
R^2 = 8.8^2 + 3.2^2 \\
= 77.4 + 10.2 = 87.6 \\
R = \sqrt{87.6} = 9.4 \text{ m s}^{-1}
\]

\[
\tan \theta = \frac{3.2}{8.8} = 0.364 \\
\theta = 20°
\]

By scale diagram:
R is 7.5 N, \( \theta \) is 53\(^\circ\) north of east.

By calculating resultant vectors:

Resolve horizontally:

\[
\begin{align*}
F_x &= 4.5N \\
4.5N & \quad \text{and} \quad 3N \quad \text{are horizontal} \\
R & \quad \text{is the resultant vector} \\
\end{align*}
\]

Resolve vertically:

\[
\begin{align*}
F_y &= +9 + (-3) \\
&= 6N \\
\end{align*}
\]
\[ R^2 = 6^2 + 4.5^2 \]
\[ = 36 + 20.25 = 56.25 \]
\[ R = \sqrt{56.25} = 7.5 \text{ m s}^{-1} \]

\[ \tan \theta = \frac{4.5}{6} = 0.75 \]
\[ \theta = 36.9^\circ \text{ east of north} \]

NB. Note the different ways we have expressed the angles here.

By scale diagram:

\[
\begin{align*}
    \theta & \quad \text{45}\degree \\
    2 \text{ m} & \quad 3 \text{ m} \\
    R & \quad \text{4.6 m and } \theta \text{ is } 27^\circ.
\end{align*}
\]

By calculating resultant vectors:

\[
\begin{align*}
    F_H & = 2 + 3 \cos 45 \\
    & = 2 + 2.1 \\
    & = 4.1 \text{ m}
\end{align*}
\]

\[
\begin{align*}
    F_V & = 3 \sin 45 \\
    & = 2.1 \text{ m}
\end{align*}
\]
\[ R^2 = 4.1^2 + 2.1^2 \]
\[ = 16.8 + 4.4 = 21.2 \]
\[ R = \sqrt{21.2} = 4.6 \text{ m} \]

\[ \tan \theta = \frac{2.1}{4.1} = 0.512 \]
\[ \theta = 27.1^\circ \]

By scale diagram:

R is 18.4 m and \( \theta \) is 27\(^\circ\).

By calculating resultant vectors:

Resolve horizontally:
\[ F_H = 8 + 12 \cos 45 \]
\[ = 8 + 8.4 \]
\[ = 16.4 \text{ m} \]

Resolve vertically:
\[ F_V = 12 \sin 45 \]
\[ = 8.4 \text{ m} \]
\[ R^2 = 16.4^2 + 8.4^2 \]
\[ = 269 + 70.6 = 339.6 \]
\[ R = \sqrt{339.6} = 18.4 \text{ m} \]

\[ \tan \theta = \frac{8.4}{16.4} = 0.512 \]
\[ \theta = 27.1^\circ \]

a. If one pair of vectors is a simple multiple of another pair, then their resultants will be the same multiple of each other.
2. By scale diagram:

R (the least distance) is 11.6 m.

By calculating resultant vectors:

\[ R^2 = 9.617^2 + 6.487^2 \]
\[ = 92.5 + 42.1 = 134.6 \]
\[ R = \sqrt{134.6} = 11.6 \text{ m} \]

3.
By scale diagram:

\[ R = 7.2 \text{ m}. \]

By calculating resultant vectors:

\[ R^2 = 3.60^2 + 6.235^2 \]
\[ = 12.96 + 38.88 = 51.84 \]
\[ R = \sqrt{51.84} = 7.2 \text{ m} \]
4. **By scale diagram:**

R is 5.6 m s\(^{-1}\) and \(\theta\) is 23°.

**By calculating resultant vectors:**

Resolve horizontally:

\[
F_H = 5 + 0.5 \cos 85
\]
\[
= 5 + 0.04
\]
\[
= 5.04 \text{ m s}^{-1}
\]

**Resolve vertically:**

\[
F_V = 2 + 0.5 \cos 5
\]
\[
= 2 + 0.5
\]
\[
= 2.5 \text{ m s}^{-1}
\]
\[ R^2 = 5.04^2 + 2.5^2 = 25.4 + 6.25 = 31.65 \]
\[ R = \sqrt{31.65} = 5.6 \text{ m s}^{-1} \]
\[ \tan \theta = \frac{2.5}{5.04} = 0.496 \]
\[ \theta = 26.4^\circ \]

5. By scale diagram:

R is 95 m s\(^{-1}\). Since it takes 25 minutes the total distance is given by
\[ d = 95 \times (25 \times 60) = 142500 \text{ m or } 142.5 \text{ km} \]

By calculating resultant vectors:

Resolve horizontally:
\[ F_H = -16.9 \sin 78 \]
\[ = -16.5 \, m/s^1 \]

**Resolve vertically:**

\[ F_V = 90 + 16.9 \cos 78 \]
\[ = 90 + 3.5 \]
\[ = 93.5 \, m/s^1 \]

\[
\[ R^2 = 93.5^2 + 16.5^2 \]
\[ = 8742 + 272 = 9014 \]
\]
\[ R = \sqrt{9014} = 94.9 \, m/s^1 \]

\[ d = 94.9 \times 25 \times 60 = 142350 \, m \]
\[ = 142.35 \, km \]

6. By scale diagram:
R is 6.3 m s\(^{-1}\). As the distance covered is 56.36 m the time taken is given by \(\frac{56.36}{6.3} = 8.95 \text{s}\)

By calculating resultant vectors:

- **Resolve horizontally:**
  \[
  F_H = 3 \cos 20 + 5 \cos 60 \\
  = 2.8 + 2.5 \\
  = 5.3 \text{ m s}^{-1}
  \]

- **Resolve vertically:**
  \[
  F_V = 5 \sin 60 - 3 \sin 20 \\
  = 4.33 - 1.03 \\
  = 3.3 \text{ m s}^{-1}
  \]

\[ R^2 = 5.3^2 + 3.3^2 \]
\[ = 28.09 + 10.89 = 38.98 \]
\[ R = \sqrt{38.98} = 6.2 \text{ m s}^{-1} \]

\[ t = \frac{56.36}{6.2} = 9.1 \text{s} \]

7. By scale diagram:
R is $8.2 \text{ m s}^{-1}$ and $\theta$ is $14^\circ$.

**By calculating resultant vectors:**

\[
R^2 = 2^2 + 8^2 \\
= 4 + 64 = 68 \\
R = \sqrt{68} = 8.25 \text{ m s}^{-1} \\
\tan \theta = \frac{2}{8} = 0.25 \\
\theta = 14^\circ
\]

a. Velocity across the deck = $2 \text{ m s}^{-1}$.
\[
\therefore \quad d = vt = 2 \times 2 = 4 \text{ m}
\]

b. Velocity of the ferry = $8 \text{ m s}^{-1}$.
\[
\therefore \quad d = vt = 8 \times 2 = 16 \text{ m}
\]

c. Resultant velocity = $8.25 \text{ m s}^{-1}$.
\[
\therefore \quad d = vt = 8.25 \times 2 = 16.5 \text{ m} \text{ at } 14^\circ \text{ to the deckside.}
\]

d. $8.25 \text{ m s}^{-1}$ at $14^\circ$ to the deckside.
EQUATIONS OF MOTION / ANSWERS

ANSWERS TO TUTORIAL 1

1.  $a = \frac{v - u}{t}$

   Acceleration = \( \frac{(27\text{ms}^{-1} - 15\text{ms}^{-1})}{8\text{s}} \)
   
   = \( 12\text{ms}^{-1} / 8\text{s} \)
   
   = \( 1.5\text{ms}^{-2} \)

2.  Let its new velocity be \( v \text{ms}^{-1} \). Using this in the definition of acceleration gives us:

   \[
   a = \frac{v - u}{t} \\
   20 = \frac{v - 20}{0.5} \\
   
   \text{Thus } 20 \times 0.5 = v - 20 \\
   \text{and so } v = 10 + 2 \\
   \text{and so } v = 12
   \]

   Its new velocity is \( 12\text{ms}^{-1} \).

3.  $a = \frac{v - u}{t}$

   $15 = \frac{30 - 0}{t}$

   \[
   \text{Thus } t = 2
   \]

   The sportscar takes 2s to reach \( 30\text{ms}^{-1} \).
ANSWERS TO TUTORIAL 2

1. (a) This is half-way between the initial and final velocities and is 15ms\(^{-1}\).
   (b) At half time, which is 4s.
   (c) The area under the graph from zero to 4s is
      \[10 \times 4 + 0.5 \times 4 \times (15-10) \text{ m}\]
      \[= 40 + 10 \text{ m}\]
      \[= 50 \text{ m}\]
   (d) Area under the graph = \[10 \times 8 + 0.5 \times 8 \times (20-10)\]
      \[= 80 + 40 \text{ m}\]
      \[= 120 \text{ m}\]
   (e)

2. (a) The velocity increases steadily and then changes to a greater rate of increase, i.e. constant acceleration followed by a greater constant acceleration.
   (b)
3. It is a steadily increasing acceleration since the velocity increases by 25\(\text{ms}^{-1}\), then 30\(\text{ms}^{-1}\), etc.

4. (a) At the origin.
   (b) At 20s and again near 30s. You are looking for a gradient of zero, i.e. a horizontal line.

5. 
   
   \[ \text{VELOCITY MS}^{-1} \]
   
   \[ \text{TIME} \]
6.

<table>
<thead>
<tr>
<th>displacement (m)</th>
<th>0</th>
<th>0.2</th>
<th>0.8</th>
<th>1.8</th>
<th>3.2</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>time (s)</td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The displacement measurements are added together to get the displacement from the start.

E.g. \( t = 0.1, \ s = 0.2 \)
\( t = 0.2, \ s = 0.2 + 0.6 = 0.8 \)
\( t = 0.3, \ s = 0.2 + 0.6 + 1.0 = 1.8 \)
\( t = 0.4, \ s = 0.2 + 0.6 + 1.0 +1.4 = 3.2 \)
\( t = 0.5, \ s = 0.2 + 0.6 + 1.0 +1.4 + 1.8 = 5.0 \)

6 b) These results show constant acceleration. You can tell this as the displacement results measured from the start are increasing by a different amount with each time period.
ANSWERS TO TUTORIAL 3

1. \[ a = \frac{v - u}{t} \]
   \[ u = 5\text{cm} / 0.1\text{s} = 50\text{cms}^{-1} \]
   \[ v = 8\text{cm} / 0.02\text{s} = 400\text{cms}^{-1} \]
   \[ t = 0.5\text{s} \]
   Thus \[ a = \frac{400 - 50}{0.5} = 700 \]
   The acceleration is 700 cms\(^{-2}\)

2. (a) The object’s average velocity is half way between 24ms\(^{-1}\) and zero. It is 12ms\(^{-1}\).
    (b) It takes 8s to cover 96m at an average of 12ms\(^{-1}\).
    (c) \[ a = \frac{(0 - 24)}{8} = -3 \]
    The deceleration is 3ms\(^{-2}\)

3. (a) A 1Hz flash will show the object's position at one second intervals.
    Displacement is the product of the object's average velocity during the interval and the duration of the interval.
    Thus:

    | Interval | Initial vel. /cms\(^{-1}\) | Final vel. /cms\(^{-1}\) | Average vel. /cms\(^{-1}\) | Length / cm |
    |----------|---------------------------|---------------------------|--------------------------|-------------|
    | 1        | 0                         | 3                         | 1.5                      | 1.5         |
    | 2        | 3                         | 6                         | 4.5                      | 4.5         |
    | 3        | 6                         | 9                         | 7.5                      | 7.5         |
    | 4        | 9                         | 12                        | 10.5                     | 10.5        |
    | 5        | 12                        | 15                        | 13.5                     | 13.5        |
    | 6        | 15                        | 18                        | 16.5                     | 16.5        |
    (b) The figures quoted are displacements from the start.
        After 1 second ------ 1.5cm
        After 2 seconds------1.5cm + 4.5cm = 6cm
        After 3 seconds ------1.5cm + 4.5cm + 7.5cm = 13.5cm

4. The Y-axis shows the velocity of the falling object which will increase steeply to begin with but level off as the air drag increases.
5. This is a very common examination type question. The important features to look out for are:
   i) Starting velocity, in this question the ball is likely to have zero initial velocity.
   ii) A constant gradient put your ruler up against the lines. This gradient represents falling under gravity. It should therefore be constant.
   iii) The bounce should cause a rapid change of direction and hence change of velocity.
   iv) In this question the initial velocity and the gradient should have the same sign as they are in the same direction.
   v) After the bounce if the collision is inelastic energy will be lost and the velocity after the bounce will be less than that before the bounce.

   The answer is D.

6. You need to do the calculation error + reading whichever is the biggest has the largest percentage error. In this question there is no point in working out C1, length of card, or the distance, as C1 is the same error as C2 but has a larger value. Remember the larger the reading the smaller the percentage error.

   The answer is B.

7.

   \[
   \begin{array}{ccccccc}
   \text{Vector} & a & P & v & \text{displacement} & F & \\
   \text{Scalar} & E_w & E_p & E_H & \text{distance} & \text{mass} & E_k & t & \text{speed} \\
   \end{array}
   \]

   The answer is C.

8. Another common examination type question. It is best to familiarise yourself with these graphs.

   The answer is D.

   It cannot be A, B, or C as the velocity time graph is NOT the gradient of the displacement time graph given. E is incorrect as the displacement is gradually getting less, which indicates a negative gradient.
9. Carefully look at the question and graph and go through in your head what the graph represents. Between 0 and P, the trolley is accelerating, this corresponds to the initial push up the slope. Between P and Q the trolley is decelerating up the slope, i.e. in the same direction. At Q the trolley’s velocity goes from 0 to \(-1.5 \text{ ms}^{-1}\) this represents the trolley rolling back down the slope.

The maximum displacement of the trolley is at 2 seconds.

b) For 0 to P \(a = (v - u) / t\) = \((3-0)/1\) = 3
   For P to Q \(a = (v - u) / t\) = \((0-3)/2\) = 1.5
   For Q to R \(a = (v - u) / t\) =\((-1.5-0)/5.25\) = -0.67

\[
\begin{array}{c|c|c|c}
\text{Acceleration / ms}^{-2} & 3 & 0 & -0.67 \\
\hline
0 & P & Q & R
\end{array}
\]
10a) We find the total distance by adding up the distances travelled during each period of the cat's motion. This is calculated by using its average speed during each period.

\[
\text{average speed} = \frac{\text{distance}}{\text{time}}
\]

\[
\Rightarrow \frac{0 + 60}{2} = \frac{S_1}{1.5}
\]

\[
\Rightarrow S_1 = 45
\]

also

\[
\Rightarrow 60 = \frac{S_2}{3.5}
\]

\[
\Rightarrow S_2 = 210
\]

also

\[
\Rightarrow \frac{60 + 50}{2} = \frac{S_2}{t}
\]

\[
\Rightarrow S_2 = 55 \times t
\]
**Extrcalculation:**

\[
\text{acceleration} = \frac{\text{final vel} - \text{initial vel}}{\text{time}}
\]

\[-20 = \frac{50 - 60}{\text{time}}\]

\[= \frac{-10}{\text{time}}\]

\[\text{time} = 0.5s\]

This lets us solve \(S_x = 55 \times t\)

which gives us \(S_x = 55 \times 0.5\)

\[= 27.5\]

the cat travels \(45cm + 210cm + 27.5cm = 282.5cm\)

b) The cockroach travels at constant speed and so:-

\[
\text{average speed} = \frac{\text{distance}}{\text{time}}
\]

\[\Rightarrow 50 = \frac{s}{0.2 + 1.5 + 3.5 + 0.5}\]

\[\Rightarrow S = 50 \times 5.7 = 285\]

The cockroach travels 285cm while the cat travels 282.5cm, so the cockroach is 2.5cm in front when the cat hits the door.
10c).

acceleration

\begin{center}
\begin{tikzpicture}
  \draw[->] (0,0) -- (6,0);
  \draw[->] (0,-2) -- (0,2);
  \draw (0,0) -- (0.2,0) -- (0.2,-2) -- (0,-2) -- cycle;
  \draw (1.7,0) -- (1.7,-2) -- (5.2,0) -- (5.2,-2) -- (5.7,0);
  \node at (0.2,-1) {0.2};
  \node at (1.7,-1) {1.7};
  \node at (5.2,-1) {5.2};
  \node at (5.7,-1) {5.7};
  \node at (0.2,1) {+40};
  \node at (0.2,-1.5) {-20};
\end{tikzpicture}
\end{center}
ANSWERS TO TUTORIAL 4

1.  (a) 
\[ \begin{align*} 
\text{u} & = 1.5 \text{ms}^{-1} \\
\text{v} & = 6.6 \text{ms}^{-1} \\
a & = ? \\
s & = ? \\
t & = 3 \text{s} 
\end{align*} \]

\[ a = \frac{(v-u)}{t} \Rightarrow a = \frac{(6.6 - 1.5)}{3} = 1.7 \]

The acceleration is 1.7ms\(^{-2}\)

(b) 
\[ \begin{align*} 
\text{u} & = 7 \text{ms}^{-1} \\
\text{v} & = 10 \text{ms}^{-1} \\
a & = 2 \text{ms}^{-2} \\
s & = ? \\
t & = ? 
\end{align*} \]

\[ a = \frac{(v-u)}{t} \Rightarrow 2 = \frac{(10 - 7)}{t} \Rightarrow t = 1.5 \]

It takes 1.5 seconds

2. 
\[ \begin{align*} 
\text{u} & = 3 \text{ms}^{-1} \\
\text{v} & = ? \\
a & = 4 \text{ms}^{-2} \\
s & = ? \\
t & = 5 \text{m} 
\end{align*} \]

\[ s = ut + \frac{1}{2}at^2 \Rightarrow s = 3 \times 5 + \frac{1}{2} \times 4 \times 5^2 = 65 \]

She slides 65m

3. 
\[ \begin{align*} 
\text{u} & = 0 \\
\text{v} & = 225 \text{ms}^{-1} \\
a & = ? \\
s & = 0.5 \text{m} \\
t & = ? 
\end{align*} \]

\[ v^2 = u^2 + 2as \Rightarrow 225^2 = 0 + 2 \times a \times 0.5 \Rightarrow 50625 = 0 + a \Rightarrow a = 50625 \]

The pellet accelerates at 50625ms\(^{-2}\)
4.  
\[ u = 3 \text{ms}^{-1} \]
\[ v = ? \]
\[ a = 200 \text{ms}^{-2} \]
\[ s = 13.75 \times 10^{-2} \text{m} \]
\[ t = ? \]

\[ v^2 = u^2 + 2as \Rightarrow v^2 = 3^2 + 2 \times 200 \times 13.75 \times 10^{-2} \]
\[ v^2 = 64 \]
\[ v = 8 \]

The ball is moving at 8\text{ms}^{-1} after the kick.

5.  
\[ u = -15 \text{ms}^{-1} \]
\[ v = 30 \text{ms}^{-1} \]
\[ a = 10 \text{ms}^{-2} \]
\[ s = ? \]
\[ t = ? \]

\[ v^2 = u^2 + 2as \Rightarrow 30^2 = (-15)^2 + 2 \times 10 \times s \]
\[ \Rightarrow s = 33.75 \]

The window is 33.75\text{m} above the ground.

6.  (a) The final velocity of the balloon is calculated from

\[ v = u + at \]
\[ \Rightarrow v = 1 + 2 \times 12 \]
\[ \Rightarrow v = 25 \]

When acceleration is constant, average velocity is halfway between the initial and final velocity:

\[ \bar{v} = \frac{u + v}{2} \]
\[ \Rightarrow \bar{v} = \frac{1 + 25}{2} \]
\[ \Rightarrow \bar{v} = 13 \]

The average velocity is 13\text{ms}^{-1}
(b) Displacement is the product of average velocity and time
\[ s = 13 \times 12 \]
\[ = 156 \]
The balloon rises 156m

7. (a)
\[ a = \frac{50 - 0}{3 \times 10^{-3}} \]
\[ = 16,666.7 \]
The ball’s acceleration is 16,667m\(^{-2}\)

(b) Since there is no air resistance, we assume the ball maintains its 50m\(^{-1}\) velocity and so has a displacement of 150 in the first 3s.

8. (a) Using the vertical component of the motion, we have
\[ u = 0 \]
\[ v = ? \]
\[ a = 10 \]
\[ s = ? \]
\[ t = 2 \text{s} \]
\[ s = ut + \frac{1}{2} at^2 \]
\[ \Rightarrow \]
\[ s = 0 + \frac{1}{2} \times 10 \times 2^2 \]
\[ = 20 \]
The cliff is 20m high

(b) Working with the horizontal components, we see that the stone has a constant horizontal velocity, which carries it 45m in 2s. Thus its initial velocity is 22.5m\(^{-1}\) horizontally.

9.
Using horizontal components, we find the time of flight since
\[
\frac{21.6 \times 10^3}{t} = 200 \times \cos(60) \]
\[\Rightarrow t = 216\]

Working with vertical components, we have a choice:
\[
\begin{align*}
\text{For motion up to the rock's highest point} \\
u &= 200 \cos(30) \\
v &= 0 \\
a &= ? \\
s &= ? \\
t &= 0.5 \times 216
\end{align*}
\]

\[
\begin{align*}
\text{For the complete up and down motion} \\
u &= 200 \cos(30) \\
v &= -200 \cos(30) \\
a &= ? \\
s &= 0 \\
t &= 216
\end{align*}
\]

Using the first option:
\[
v = u + at
\]
\[\Rightarrow a = \frac{0 - 200 \cos(30)}{108} = -1.604\]

The Moon's gravitational acceleration is 1.6ms\(^{-2}\).

Using the second option: -
\[ s = ut + \frac{1}{2}at^2 \]
\[ \Rightarrow \quad 0 = 200 \cos(30) \times 216 + \frac{1}{2} \times a \times 216^2 \]
\[ \Rightarrow \quad 0 = 37412297 + 23328a \]
\[ \Rightarrow \quad a = -1.604 \]

The Moon's gravitational acceleration is \(1.6\text{ms}^{-2}\).

10.
\[ s = ut + \frac{1}{2}at^2 \]

It takes 0.64s to reach the ground.

\[ 2 = 0 + \frac{1}{2} \times 9.8 \times t^2 \]
\[ 0.4 = t^2 \]
\[ t = 0.64s \]

11. The time needed to cross the river is \(3\text{m} \div 5.5 \text{ms}^{-1} = 0.54\text{s}\). The time to fall 1.5m is found by solving \(s=ut + 1/2at^2\) for \(t\).

\[ 1.5 = 0 + \frac{1}{2} \times 9.8 \times t^2 \]
\[ t = 0.55s \]

Since the rider takes longer to fall to the lower level than he does to cross, he misses the water, shame!

12.
\[ s = ut + \frac{1}{2}at^2 \]

\[ s = -1.5 \times 1.2 + \frac{1}{2} \times 9.8 \times 1.2^2 \]
\[ s = 5.26\text{m} \]
The surface of the water is 5.26m below the fish.

13.a) The average velocity of the trolley is given by:

\[ \bar{v} = \frac{(u + v)}{2} \]

\[ \bar{v} = \frac{0 + 2.5}{2} = 1.25 \]

Since the average velocity for the 5s of travel is 1.25 ms\(^{-1}\), the trolley covers 6.25m.

b) \[ v^2 = u^2 + 2as \]

\[ 0 = 4.81^2 + 2 \times 9.8 \times s \]

\[ s = 1.18 \]

The maximum height reached is 1.18m.

c) The ball moves forward at a constant velocity of 2.5 ms\(^{-1}\) as the trolley accelerates so we need to calculate the distance covered by both.

The flight is calculated from \( v = u + at \).

\[ -4.81 = 4.81 - 9.8t \]

\[ t = 0.98s \]

The ball moves forward \( 2.5 \times 0.98m = 2.45m \)

The distance moved forward by the trolley is given by:

\[ s = ut + \frac{1}{2}at^2 \]

\[ s = 2.5 \times 0.98 + \frac{1}{2} \times a \times 9.8^2 \]
Finding the acceleration of the trolley using $v=u+at$ gives

$$2.5 = 0 + a \times 0.5$$
$$a = 0.5$$

**Going back and finishing the first equation gives us**

$$s = ut + \frac{1}{2}at^2$$

$$s = 2.5 \times 0.98 + \frac{1}{2} \times 0.5 \times 9.8^2$$

$$s = 2.69$$

Thus the ball-bearing lands 24cm behind the trolley.

14. Time taken for the can and arrow to drop is found using

$$s = ut + \frac{1}{2}at^2$$

$$1.225 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$t = 0.5$$

The can’s horizontal velocity component is constant and it covers 0.3m in 0.5s. It’s horizontal velocity, according to $v=s/t$ is

$$0.3m \div 0.5s = 0.6 \text{ ms}^{-1}$$

15. $v = u + at$

$$v = 11.11 + 1.5 \times 6$$

$$v = 20.11$$

The train is moving at approximately 20.1 ms$^{-1}$ (74kmh$^{-1}$).

16. The 20p takes 4s to reach the bottom of the well.

$$s = ut + \frac{1}{2}at^2$$

$$0.8 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$t = 4$$
17. a) Vertically downwards at 5ms\(^{-1}\)

\[ s = ut + \frac{1}{2}at^2 \]

\[ 100 = 5 \times t + \frac{1}{2} \times 10 \times t^2 \]
\[ 100 = (5t + 5t^2) \]
\[ 100 = 5(t + t^2) \]
\[ \frac{100}{5} = t + t^2 = 20 \]
\[ t = 4 \]

or \( t = -5 \) (which is not possible)

b) Vertically upwards at 5ms\(^{-1}\)

\[ s = ut + \frac{1}{2}at^2 \]

\[ 100 = -5 \times t + \frac{1}{2} \times 10 \times t^2 \]
\[ 100 = (-5t + 5t^2) \]
\[ 100 = 5(-t + t^2) \]
\[ \frac{100}{5} = -t + t^2 = 20 \]
\[ t = 5 \]

or \( t = -4 \) (which is not possible)
Or if you are not confident with the use of quadratic equations you can get by this by using 2
equations, eg.

a) Vertically downwards at 5ms \(^{-1}\)

\[ v^2 = u^2 + 2as \]
\[ v^2 = 5^2 + 2 \times 10 \times 100 \]
\[ v = 2000 \]
\[ v = 44.7 \]

Then find \( t \) from \( v = u + at \)

\[ v = u + at \]
\[ 44.7 = 5 + 10t \]
\[ 39.7 = 10t \]
\[ t = 4 \]

The stone takes four seconds to reach the ground.

b) Vertically upwards at 5ms \(^{-1}\)

As this involves squaring and square rooting negative numbers the easiest way of doing this is to
work out the details of the flight up to its highest point and then find its time to fall

\[ v = u + at \]
\[ 0 = 5 + -10 \times t \]
\[ 10t = 5 \]
\[ t = 0.5 \]

This is the time the stone takes to reach the top of the flight.
\[v^2 = u^2 + 2as\]
\[0^2 = 5^2 + 2 \times 10 \times s\]
\[20s = 25\]
\[s = 1.25\]

This is the extra displacement of the stone.

Then find \(t\) from \(s = ut + \frac{1}{2}at^2\)

\[s = ut + \frac{1}{2}at^2\]
\[101.25 = 0 + \frac{1}{2} \times 10 \times t^2\]
\[101.25 = 5t^2\]
\[t^2 = 20.25\]
\[t = 4.5\]

The time for the stone to fall the total distance from the top of the flight to the bottom is 4.5s, however, don’t forget to add in the time the stone takes to get to the top.

Total time = 0.5 + 4.5 = 5

The stone takes five seconds to reach the ground.
NEWTON’S SECOND LAW/ TUTORIAL ANSWERS:

ANSWERS FOR TUTORIAL 1

1. \( F=ma \)  gives  \( F = 10^5 \times 5 \)
   The force needed is \( 5 \times 10^5 \) N

2. \( F=ma \)  gives  \( 100=2 \times 10^4 \times m \)
   \( m=0.005 \)
   The pellet has a mass of 5g

3. \( 140 = 80a \)
   \( a = 1.75 \)
   The acceleration is \( 1.75 \text{ ms}^{-2} \)

ANSWERS FOR TUTORIAL 2

1. Friction=0.97N
   Weight=1.47 N

\[ W = mg = 150/1000 \times 9.8 = 1.47 \text{ N} \]
The resultant force is \( (1.47 - 0.97) = 0.5 \text{ N} \)
\[ F = m \times a = 0.15 \times 3.3 \text{ ms}^{-2} \]

2. \( F = 100 - 20 = 80N \)
   \( a = 1000 \text{ ms}^{-2} \)
   \( F = m \times a = 80 = m \times 1000 \)
   \( m = 0.08 \text{ kg} = 80 \text{ g} \)
3. \( F = ? \)
   \( m = \frac{W}{g} = \frac{9.8}{7.84} = 0.8 \text{ kg} \)
   \[
   a = \frac{(v-u)}{t} \\
   a = \frac{(7.5-0)}{5} \\
   a = 1.5
   \]
   The accelerating force is 1.2 N

4. Friction  
   \( = 95.5 \text{ N} \)

   Weight

   \[
   F = mg \\
   W = (0.01 \times 10^{-3}) \times 9.8 \\
   W = 9.8 \times 10^{-5} \text{ N} \\
   \]
   Unbalanced (or accelerating) Force = \( W - F \)
   \[
   = 98 \text{ N} - 95.5 \text{ N} \\
   = 2.5 \text{ N} \\
   \]
   \[
   F = ma \\
   = 2.5 \text{ N} \times 0.01 \times 10^{-3} \times a \\
   a = 0.25
   \]
   The feather accelerates at 0.25 ms\(^{-2}\)

5. One way to tackle this is to add the two forces using a vector diagram, than apply that force to the mass to find out what acceleration it causes. Alternatively, we can find the accelerations caused by the individual forces and then find the resultant acceleration using a vector diagram since acceleration is a vector quantity.
The acceleration caused is \(4 \text{ms}^{-2}\) at an angle of 53° to the 6N force as shown on the diagram.

6. (a) The 12N unbalanced force causes an acceleration of \(4 \text{ms}^{-2}\) to the right in the 3kg vehicle.

(b) There is no unbalanced force so \(a=0\)

7. This problem can be answered using a scale diagram or using components. Using a scale diagram

The resultant of these two forces is 115.5N at 90° to the 200N force
or using components
(Right positive, up positive)

Horizontal Force \( = 200 + (-231 \cos 30) \)
\( = 200 - 200 \)
\( = 0 \)

Vertical Force \( = 0 + 231 \sin 30 \)
\( = 115.5 \text{N} \)

\( F = ma \)
\( 115.5 = 1.5 \times a \)
\( a = 77 \text{ ms}^{-2} \)

8. This problem can be answered using a scale diagram or using components.

Using a scale diagram

The resultant of the three forces is 14.9N downwards.

or using components

(Right positive, down positive)

Horizontal Force \( = 35 + 0 + (-39 \cos 26) \)
\( = 35 + 0 + -35 \)
\( = 0 \)

Vertical Force \( = 0 + 32 + (-39 \sin 26) \)
\( = 32 + -17.1 \)
\( = 14.9 \text{N} \)

\( F = ma \)
\( 14.9 = 100 \times a \)
\( a = 0.149 \text{ ms}^{-2} \) downwards
9. Have this question checked by your teacher.

10. Have this question checked by your teacher.

11. a) The mass of the block can be found using $W=mg$. The weight shown in the diagram as the force downwards.

\[ W=mg \]
\[ 200 = m \times 9.8 \]
\[ m = 20.4 \text{kg}. \]

b) The block can move up or down the slope. The resultant is given by

\[ R = W\sin \theta - F \]
\[ R = (200 \sin 54) - 121 \]
\[ R = 40.8 \text{ N down the slope}. \]

c) \[ F = ma \]
\[ 40.8 = 20.4 \times a \]
\[ a = 2 \text{ m/s}^2 \text{ down the slope}. \]

12. This problem is quite easy but it needs tackling in stages. There are many ways to start this but all the piece must be found.

Find $m$

\[ W=mg \]
\[ 686 = m \times 9.8 \]
\[ m = 70 \text{kg}. \]

Find the unbalanced force

\[ F = ma \]
\[ F = 70 \times 2.5 \]
\[ F = 175 \text{ N} \]

Find the component of weight acting down the slope

\[ \text{Component} = W\sin \theta \]
\[ = 686 \sin 30 \]
\[ = 343 \text{N} \]

The frictional force must be the difference between the component of weight trying to accelerate the block down the slope and the unbalanced accelerating force.
Friction = 343 – 175
= 168N

13. This problem is again quite easy but it needs tackling in stages. There are many ways to start this but all the piece must be found.

Find m
W=mg
735 = m x 9.8
m = 75kg.

Find the acceleration of the block down the slope.

\[ a = \frac{(v - u)}{t} \]
\[ a = \frac{(6 - 0)}{2} \]
\[ a = 3 \]

Find the unbalanced force
F= ma
F = 75 x 3
F=225 N

Find the component of weight acting down the slope
Component = Wsin2
= 735 sin 30
= 367.5N

The frictional force must be the difference between the component of weight trying to accelerate the block down the slope and the unbalanced accelerating force.

\[ \text{Friction} = 367.5 - 225 \]
\[ = 142.5N \]
ANSWERS FOR TUTORIAL 3

1. (a) \( E_k = \frac{1}{2} m v^2 \)
   
   \[ m = 75000 + 25000 = 100000 \text{ kg}. \]
   \[ v = 8 \text{ kms}^{-1} = 8000 \text{ ms}^{-1} \]
   
   \[ E_k = \frac{1}{2} m v^2 \]
   
   \[ = \frac{1}{2} \cdot 100000 \cdot (8000)^2 = 320 \times 10^{10} \text{ J} \]
   
   (b) work done = loss in \( E_k = 320 \times 10^{10} \text{ J} \)
   
   (c) 53 minutes and 20 seconds = 3180 + 20 = 3200 seconds

\[ a = \frac{v - u}{t} = \frac{0 - 8000}{3200} = -2.5 \]

\[ F = ma \]

\[ F = 100000 \times -2.5 \]

\[ F = -250000 \]

The average frictional force is 250000 N.

(d) \( E_w = F \times s \)

\[ 320 \times 10^{10} = 250000 \times s \]

\[ = 12800 \text{ km} \]

2. \( \epsilon E_p = \epsilon mgh \)

\[ = 0.1 \times 10 \times 40 = 40 \text{ J} \]

3. \( t = 10 \text{ minutes} = 600 \text{ s} \)

\[ P = \frac{E_w}{t} = \frac{Fs}{t} \]

\[ P = \frac{100 \times 100}{600} \]

\[ P = 16.666 \]

The average power of the man pushing the lawn mower is 17 W
4. The kinetic energy is converted into heat as the objects do work against the force of friction. Since their original energies are the same, they each do the same amount of work as they stop so their stopping distances must be equal.

5.(a) The minimum upward force exerted by the fork-lift truck would be equal to the weight of the case.

\[ W = mg \]
\[ W = 500 \times 10 = 5000N \]

(b) \[ \varepsilon_{Ep} = \varepsilon_{mgh} = 500 \times 10 \times 1.5 = 7500 \text{ J} \]

(c) \[ E_k = \frac{1}{2}mv^2 \]
\[ = \frac{1}{2} \times 500 \times 2.0^2 = 1000 \text{ J} \]

(d) When the fork-lift truck stops this kinetic energy is transferred to the surroundings as heat.

(e)

\[ P = \frac{E}{t} \]
\[ 25000 \times 3 = E \]
\[ E = 75000 \text{ J} \]

The energy into the system is 75000 J, the energy gained by the package is 7500 J.

\[ \text{efficiency} = \frac{E_{\text{out}}}{E_{\text{in}}} \times 100\% \]
\[ \text{efficiency} = \frac{7500}{75000} \times 100\% \]
\[ \text{efficiency} = 10\% \]

The system is 10% efficient.

6. (a) Work done = force \times \text{distance}
\[ = 10N \times 3m \]
\[ = 30J \]
(b) Kinetic energy since we can see from the second graph that the speed is increasing.

(c) The velocity is steady during this part of the motion and so the forces on the object are balanced. This means that the work done is against the force of friction which only produces heat.
ANSWERS TO TUTORIAL 1

1. \[ a = \frac{(v - u)}{t} \]
  Acceleration = \( \frac{(27\text{ms}^{-1} - 15\text{ms}^{-1})}{8}\text{s} \)
   = \( \frac{12\text{ms}^{-1}}{8}\text{s} \)
   = \( 1.5\text{ms}^{-2} \)

2. Let its new velocity be \( v \text{ms}^{-1} \). Using this in the definition of acceleration gives us:
   \[ a = \frac{(v - u)}{t} \]
   \[ 20 = v - 2 \]
   \[ 0.5 \]
   Thus \( 20 \times 0.5 = v - 2 \)
   and so \( v = 10 + 2 \)
   = 12
   Its new velocity is 12ms\(^{-1}\).

3. \[ a = \frac{(v - u)}{t} \]
   \[ 15 = \frac{30 - 0}{t} \]
   Thus \( t = 2 \)
   The sportscar takes 2s to reach 30ms\(^{-1}\).
ANSWERS TO TUTORIAL 2

1. (a) This is half-way between the initial and final velocities and is 15 ms\(^{-1}\).
   (b) At half time, which is 4s
   (c) The area under the graph from zero to 4s is
       \[ 10 \times 4 \text{m} + 0.5 \times 4 \times (15-10) \text{m} \]
       \[ = 40 \text{m} + 10 \text{m} \]
       \[ = 50 \text{m} \]
   (d) Area under the graph = 10x8m + 0.5x8x(20-10)
       \[ = 80 \text{m} + 40 \text{m} \]
       \[ = 120 \text{m} \]
   (e)

   ![Graph of acceleration vs. time]

2. (a) The velocity increases steadily and then changes to a greater rate of increase, i.e. constant acceleration followed by a greater constant acceleration.
   (b)

   ![Graph of acceleration vs. time]
3. It is a steadily increasing acceleration since the velocity increases by 25ms\(^{-1}\), then 30ms\(^{-1}\), etc.

4. (a) At the origin.
(b) At 20s and again near 30s. You are looking for a gradient of zero, i.e. a horizontal line.

5. 

\[
\begin{array}{|c|c|}
\hline
\text{TIME} & \text{VELOCITY } ms^{-1} \\
\hline
\end{array}
\]
6.

<table>
<thead>
<tr>
<th>displacement (m)</th>
<th>0</th>
<th>0.2</th>
<th>0.8</th>
<th>1.8</th>
<th>3.2</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>time (s)</td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The displacement measurements are added together to get the displacement from the start.

e.g. \( t = 0.1, \ s = 0.2 \)
\( t = 0.2, \ s = 0.2 + 0.6 = 0.8 \)
\( t = 0.3, \ s = 0.2 + 0.6 + 1.0 = 1.8 \)
\( t = 0.4, \ s = 0.2 + 0.6 + 1.0 + 1.4 = 3.2 \)
\( t = 0.5, \ s = 0.2 + 0.6 + 1.0 + 1.4 + 1.8 = 5.0 \)

6 b) These results show constant acceleration. You can tell this as the displacement results measured from the start are increasing by a different amount with each time period.
ANSWERS TO TUTORIAL 3

1. \[ a = \frac{v - u}{t} \]
   \[ u = \frac{5 \text{cm}}{0.1 \text{s}} = 50 \text{cms}^{-1} \]
   \[ v = \frac{8 \text{cm}}{0.02 \text{s}} = 400 \text{cms}^{-1} \]
   \[ t = 0.5 \text{s} \]
   Thus \[ a = \frac{400 - 50}{0.5} = 700 \]
   The acceleration is \( 700 \text{ cms}^{-2} \)

2. (a) The object's average velocity is half way between \( 24 \text{ms}^{-1} \) and zero. It is \( 12 \text{ms}^{-1} \).
   (b) It takes 8s to cover 96m at an average of \( 12 \text{ms}^{-1} \).
   (c) \[ a = \frac{0 - 24}{8} = -3 \]
   The deceleration is \( 3 \text{ms}^{-2} \)

3. (a) A 1Hz flash will show the object's position at one second intervals.
   Displacement is the product of the object's average velocity during the interval and the duration of the interval.
   Thus:

<table>
<thead>
<tr>
<th>Interval</th>
<th>Initial vel. /cms(^{-1})</th>
<th>Final vel. /cms(^{-1})</th>
<th>Average vel. /cms(^{-1})</th>
<th>Length / cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>6</td>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>9</td>
<td>7.5</td>
<td>7.5</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>12</td>
<td>10.5</td>
<td>10.5</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>15</td>
<td>13.5</td>
<td>13.5</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>18</td>
<td>16.5</td>
<td>16.5</td>
</tr>
</tbody>
</table>

   (b) The figures quoted are displacements from the start.
   After 1 second ---- 1.5cm
   After 2 seconds----1.5cm + 4.5cm = 6cm
   After 3 seconds----1.5cm + 4.5cm + 7.5cm = 13.5cm

4. The Y-axis shows the velocity of the falling object which will increase steeply to begin with but level off as the air drag increases.

5. This is a very common examination type question. The important features to look out for are:
   i) Starting velocity, in this question the ball is likely to have zero initial velocity.
   ii) A constant gradient put your ruler up against the lines. This gradient represents falling under gravity. It should therefore be constant.
   iii) The bounce should cause a rapid change of direction and hence change of velocity.
   iv) In this question the initial velocity and the gradient should have the same sign as they are in the same direction.
v) After the bounce if the collision is inelastic energy will be lost and the velocity after the bounce will be less than that before the bounce.

The answer is D.

6. You need to do the calculation error + reading whichever is the biggest has the largest percentage error. In this question there is no point in working out C1, length of card, or the distance, as C1 is the same error as C2 but has a larger value. Remember the larger the reading the smaller the percentage error.

The answer is B.

7.

<table>
<thead>
<tr>
<th>Vector</th>
<th>a</th>
<th>P</th>
<th>v</th>
<th>displacement</th>
<th>F</th>
<th>Scalar</th>
<th>Ew</th>
<th>Ep</th>
<th>Ev</th>
<th>distance</th>
<th>mass</th>
<th>Ek</th>
<th>t</th>
<th>speed</th>
</tr>
</thead>
</table>

The answer is C.

8. Another common examination type question. It is best to familiarise yourself with these graphs.

The answer is D.

It cannot be A, B, or C as the velocity time graph is NOT the gradient of the displacement time graph given. E is incorrect as the displacement is gradually getting less, which indicates a negative gradient.
9. Carefully look at the question and graph and go through in your head what the graph represents. Between 0 and P, the trolley is accelerating, this corresponds to the initial push up the slope. Between P and Q the trolley is decelerating up the slope, i.e. in the same direction. At Q the trolley’s velocity goes from 0 to –1.5 ms\(^{-1}\) this represents the trolley rolling back down the slope.

The maximum displacement of the trolley is at 2 seconds.

\[ a = \frac{(v - u)}{t} \]

\[ a_{0 \text{ to } P} = \frac{3 - 0}{1} = 3 \]

\[ a_{P \text{ to } Q} = \frac{0 - 3}{2} = -1.5 \]

\[ a_{Q \text{ to } R} = \frac{-1.5 - 0}{5.25} = -0.67 \]

Acceleration / ms\(^{-2}\)
10a) We find the total distance by adding up the distances travelled during each period of the cat’s motion. This is calculated by using its average speed during each period.

\[
\text{average speed} = \frac{\text{distance}}{\text{time}}
\]

\[
\Rightarrow \frac{0 + 60}{2} = \frac{S_1}{1.5}
\]
\[
\Rightarrow S_1 = 45
\]
also

\[
\Rightarrow 60 = \frac{S_2}{3.5}
\]
\[
\Rightarrow S_2 = 210
\]
also

\[
\Rightarrow \frac{60 + 50}{2} = \frac{S_2}{t}
\]

\[
\Rightarrow S_2 = 55 \times t
\]
Extra calculation:

\[ \text{acceleration} = \frac{\text{final vel} - \text{initial vel}}{\text{time}} \]

\[ -20 = \frac{50 - 60}{\text{time}} \]

\[ = \frac{-10}{\text{time}} \]

\[ \text{time} = 0.5s \]

This lets us solve \( S_3 = 55 \times t \)

which gives us \( S_3 = 55 \times 0.5 \)

\[ = 27.5 \]

the cat travels \( 45cm + 210cm + 27.5cm = 282.5cm \)
b) The cockroach travels at constant speed and so:

The cockroach travels 285cm while the cat travels 282.5cm, so the cockroach is 2.5cm in front when the cat hits the door.

**acceleration**

\[
\begin{array}{c|c|c|c|c|c}
0.2 & 1.7 & 5.2 & 5.7 \\
\hline
+40 & & & & \\
\hline
-20 & & & & \\
\end{array}
\]

\[
average \ speed = \frac{distance}{time}
\]

\[
\Rightarrow 50 = \frac{s}{0.2 + 1.5 + 3.5 + 0.5}
\]

\[
\Rightarrow s = 50 \times 5.7 = 285
\]
ANSWERS TO TUTORIAL 4

1. (a) \[ u = 1.5 \text{ms}^{-1}, \quad v = 6.6 \text{ms}^{-1} \]
\[
\begin{align*}
\text{a} &= \frac{(v-u)}{t} \\
\text{s} &= ? \\
\text{t} &= 3 \text{s}
\end{align*}
\]
\[ a = \frac{(6.6 - 1.5)}{3} = 1.7 \]

The acceleration is 1.7ms\(^{-2}\)

(b) \[ u = 7 \text{ms}^{-1}, \quad v = 10 \text{ms}^{-1} \]
\[
\begin{align*}
\text{a} &= \frac{(v-u)}{t} \\
\text{s} &= ? \\
\text{t} &= ?
\end{align*}
\]
\[ a = \frac{(10 - 7)}{t} \Rightarrow t = 1.5 \]

It takes 1.5 seconds

2. \[ u = 3 \text{ms}^{-1}, \quad v = ? \]
\[
\begin{align*}
\text{a} &= 4 \text{ms}^{-2} \\
\text{s} &= ut + \frac{1}{2}at^2 \\
\text{s} &= ? \\
\text{t} &= 5 \text{m}
\end{align*}
\]
\[ s = 3 \times 5 + \frac{1}{2} \times 4 \times 5^2 = 65 \]

She slides 65m

3. \[ u = 0 \]
\[
\begin{align*}
\text{v} &= 225 \text{ ms}^{-1} \\
\text{a} &= ? \\
\text{s} &= 0.5 \text{m} \\
\text{t} &= ?
\end{align*}
\]
\[ v^2 = u^2 + 2as \Rightarrow 225^2 = 0 + 2 \times a \times 0.5 \]
\[ \Rightarrow 50625 = 0 + a \Rightarrow a = 50625 \]
The pellet accelerates at 50625ms$^{-2}$

4.

\[ u = 3 \text{ms}^{-1} \]

\[ \begin{align*}
  v &= ? \\
  a &= 200 \text{ms}^{-2} \\
  s &= 13.75 \times 10^{-2} \text{m} \\
  t &= ? 
\end{align*} \]

\[
\begin{align*}
  v^2 &= u^2 + 2as \\
  &= 3^2 + 2 \times 200 \times 13.75 \times 10^{-2} \\
  &= 64 \\
  \Rightarrow v &= 8
\end{align*}
\]

The ball is moving at 8ms$^{-1}$ after the kick.

5.

\[ u = -15 \text{ms}^{-1} \]

\[ \begin{align*}
  v &= 30 \text{ms}^{-1} \\
  a &= 10 \text{ms}^{-2} \\
  s &= ? \\
  t &= ? 
\end{align*} \]

\[
\begin{align*}
  v^2 &= u^2 + 2as \\
  30^2 &= (-15)^2 + 2 \times 10 \times s \\
  \Rightarrow s &= 33.75
\end{align*}
\]

The window is 33.75m above the ground.

6. (a) The final velocity of the balloon is calculated from

\[
\begin{align*}
  v &= u + at \\
  \Rightarrow v &= 1 + 2 \times 12 \\
  \Rightarrow v &= 25
\end{align*}
\]

When acceleration is constant, average velocity is halfway between the initial and final velocity:
\[
\bar{v} = \frac{u + v}{2} \\
\Rightarrow \quad \bar{v} = \frac{1 + 25}{2} \\
\Rightarrow \quad \bar{v} = 13
\]

The average velocity is 13m\(s^{-1}\)

(b) Displacement is the product of average velocity and time

\[
\Rightarrow \quad s = 13 \times 12 \\
\quad = 156
\]

The balloon rises 156m

7. (a) 

\[
a = \frac{50 - 0}{3 \times 10^{-3}} \\
\quad = 16666.7
\]

The ball's acceleration is 16,667m\(s^{-2}\)

(b) Since there is no air resistance, we assume the ball maintains its 50m\(s^{-1}\) velocity and so has a displacement of 150 in the first 3s.

8. (a) Using the vertical component of the motion, we have

\[
u = 0 \\
v = ? \\
a = 10 \\
s = ? \\
t = 2s \\
\Rightarrow \quad s = ut + \frac{1}{2}at^2 \\
\quad = 0 + \frac{1}{2} \times 10 \times 2^2 \\
\quad = 20
\]

The cliff is 20m high
(b) Working with the horizontal components, we see that the stone has a constant horizontal velocity, which carries it 45m in 2s. Thus its initial velocity is 22.5m\(^{-1}\) horizontally.

9.

![Diagram of a projectile motion with 200ms\(^{-1}\) at 60° and a horizontal range of 21.6km.]

Using horizontal components, we find the time of flight since

\[
\frac{21.6 \times 10^3}{t} = 200 \times \cos(60) \\
\Rightarrow t = 216
\]

Working with vertical components, we have a choice:

\[\begin{align*}
  u &= 200 \cos(30) \\
  v &= 0 \\
  a &= ? \\
  s &= ? \\
  t &= 0.5 \times 216
\end{align*}\]

For motion up to the rock's highest point

\[\begin{align*}
  u &= 200 \cos(30) \\
  v &= -200 \cos(30) \\
  a &= ? \\
  s &= 0 \\
  t &= 216
\end{align*}\]

For the complete up and down motion

Using the first option: -
\[ v = u + at \]
\[ \Rightarrow a = \frac{0 - 200 \cos(30)}{108} = -1.604 \]

The Moon's gravitational acceleration is \(1.6 \text{ms}^{-2}\).

Using the second option:

\[ s = ut + \frac{1}{2} at^2 \]
\[ \Rightarrow 0 = 200 \cos(30) \times 216 + \frac{1}{2} \times a \times 216^2 \]
\[ \Rightarrow 0 = 37412297 + 23328a \]
\[ \Rightarrow a = -1.604 \]

The Moon's gravitational acceleration is \(1.6 \text{ms}^{-2}\).

10.

\[ s = ut + \frac{1}{2} at^2 \]

\[ 2 = 0 + \frac{1}{2} 9.8t^2 \]  It takes 0.64s to reach the ground.

\[ 0.4 = t^2 \]
\[ t = 0.64 \text{s} \]
11. The time needed to cross the river is 3 m \div 5.5 \text{ ms}^{-1} = 0.54 \text{s}. The time to fall 1.5 m is found by solving \( s = ut + \frac{1}{2}at^2 \) for \( t \).

\[
1.5 = 0 + \frac{1}{2} \times 9.8 \times t^2
\]

\[t = 0.55 \text{s}\]

Since the rider takes longer to fall to the lower level than he does to cross, he misses the water, shame!

\[
s = ut + \frac{1}{2}at^2
\]

\[s = -1.5 \times 1.2 + \frac{1}{2} \times 9.8 \times 1.2^2
\]

\[s = 5.26 \text{ m}\]

12.

The surface of the water is 5.26 m below the fish.

13.a) The average velocity of the trolley is given by:

\[
v = \frac{(u + v)}{2}
\]

\[v = \frac{0 + 2.5}{2} = 1.25 \text{ ms}^{-1}\]

Since the average velocity for the 5s of travel is 1.25 ms\(^{-1}\), the trolley covers 6.25 m.

\[
v^2 = u^2 + 2as
\]

\[0 = 4.81^2 + 2 \times 9.8 \times s
\]

\[s = 1.18 \text{ m}\]

b)

The maximum height reached is 1.18 m
c) The ball moves forward at a constant velocity of 2.5 m\(\text{s}^{-1}\) as the trolley accelerates so we need to calculate the distance covered by both.

The flight is calculated from \(v=u+at\).

\[
\Rightarrow -4.81 = 4.81 - 9.8t \\
\Rightarrow t = 0.98s 
\]

The ball moves forward \(2.5 \times 0.98m = 2.45m\)

The distance moved forward by the trolley is given by:

\[
s = ut + \frac{1}{2}at^2 \\
s = 2.5 \times 0.98 + \frac{1}{2} \times a \times 9.8^2 
\]

Finding the acceleration of the trolley using \(v=u+at\) gives

\[
2.5 = 0 + a \times 0.98 \\
a = 0.5 
\]

Going back and finishing the first equation gives us

\[
s = ut + \frac{1}{2}at^2 \\
s = 2.5 \times 0.98 + \frac{1}{2} \times 0.5 \times 9.8^2 \\
s = 2.69 
\]

Thus the ball-bearing lands 24cm behind the trolley.
14. Time taken for the can and arrow to drop is found using

\[ s = ut + \frac{1}{2}at^2 \]

1.225 = 0 + \frac{1}{2} \times 9.8 \times t^2

t = 0.5

The can’s horizontal velocity component is constant and it covers 0.3m in 0.5s. It’s horizontal velocity, according to \( v = \frac{s}{t} \) is

0.3m \( \div \) 0.5s = 0.6 ms \(^{-1}\)

15. \[ v = u + at \]

40 km \( \text{h}^{-1} \) = 4000m per 3600s

\[ v = 11.11 + 1.5 \times 6 \]

\[ v = 20.11 \]

The train is moving at approximately 20.1 ms \(^{-1}\) (74 km \( \text{h}^{-1} \)).

16. \[ s = ut + \frac{1}{2}at^2 \]

0.8 = 0 + \frac{1}{2} \times 9.8 \times t^2

t = 4

The 20p takes 4s to reach the bottom of the well.
17. a) Vertically downwards at 5ms\(^{-1}\)

\[ s = ut + \frac{1}{2}at^2 \]

\[ 100 = 5 \times t + \frac{1}{2} \times 10 \times t^2 \]

\[ 100 = (5t + 5t^2) \]

\[ 100 = 5(t + t^2) \]

\[ \frac{100}{5} = t + t^2 = 20 \]

\[ t = 4 \]

or \( t = -5 \) (which is not possible)

b) Vertically upwards at 5ms\(^{-1}\)

\[ s = ut + \frac{1}{2}at^2 \]

\[ 100 = -5 \times t + \frac{1}{2} \times 10 \times t^2 \]

\[ 100 = (-5t + 5t^2) \]

\[ 100 = 5(-t + t^2) \]

\[ \frac{100}{5} = -t + t^2 = 20 \]

\[ t = 5 \]

or \( t = -4 \) (which is not possible)
Or if you are not confident with the use of quadratic equations you can get by this by using 2 equations, eg.

\[ v^2 = u^2 + 2as \]
\[ v^2 = 5^2 + 2 \times 10 \times 100 \]
\[ v = 2000 \]
\[ v = 44.7 \]

Then find \( t \) from \( v = u + at \)

\[ v = u + at \]
\[ 44.7 = 5 + 10t \]
\[ 39.7 = 10t \]
\[ t = 4 \]

The stone takes four seconds to reach the ground.

b) Vertically upwards at 5ms\(^{-1}\)

As this involves squaring and square rooting negative numbers the easiest way of doing this is to work out the details of the flight up to its highest point and then find its time to fall

\[ v = u + at \]
\[ 0 = 5 - 10 \times t \]
\[ 10t = 5 \]
\[ t = 0.5 \]

This is the time the stone takes to reach the top of the flight.
\[v^2 = u^2 + 2as\]
\[0^2 = 5^2 + 2 \times 10 \times s\]
\[20s = 25\]
\[s = 1.25\]

This is the extra displacement of the stone.

\[s = ut + \frac{1}{2}at^2\]
\[101.25 = 0 + \frac{1}{2} \times 10 \times t^2\]
\[101.25 = 5t^2\]
\[t^2 = 20.25\]
\[t = 4.5\]

Then find \(t\) from \(s = ut + \frac{1}{2}at^2\)

The time for the stone to fall the total distance from the top of the flight to the bottom is 4.5s, however, don’t forget to add in the time the stone takes to get to the top.

Total time = 0.5 + 4.5 = 5

The stone takes five seconds to reach the ground.
MOMENTUM & IMPULSE / TUTORIAL ANSWERS

ANSWERS TO TUTORIAL 1

1. Impulse = change in momentum

   \[ \Delta \text{mv} = 0.2 \times 15 - 0 \]
   \[ = 3 \]

   The impulse = 3kgms\(^{-1}\)

2. a) Change in momentum = impulse and so the momentum change in this situation is 520kgms\(^{-1}\)

   b) Since running into the player means that the runner slows down, his new momentum is 520kgms\(^{-1}\) less than his original. That is 80kgx8ms\(^{-1}\).520kgms\(^{-1}\) = 120kgms\(^{-1}\).

3. a)

   \[ \Delta (\text{mv}) = 2 \times 3.5 - 2 \times 0.5 \]
   \[ = 7 - 1 \]
   \[ = 6 \]

   The momentum change is 6kgms\(^{-1}\)

   b)

   \[ \bar{F} = \Delta (\text{mv}) \Rightarrow \bar{F} \times 1.5 = 6 \]
   \[ \Rightarrow \bar{F} = 4 \]

   The average force acting is 4N.

4. Let the time taken to stop be \( t \) seconds.

   \[ \bar{F}t = \Delta (\text{mv}) \Rightarrow 180t = 90 \times 5 - 0 \]
   \[ \Rightarrow t = 2.5 \]

   The football player takes 2.5s to stop.

5. \[ \bar{F}t = \Delta (\text{mv}) \Rightarrow 2000 \times 8 - 5 \]
   \[ \Rightarrow 6000 \]
An impulse of 6000 Ns is needed to change the velocity of the car.

6. \[ Ft = \Delta(mv) \quad \Rightarrow \quad \Delta p = m(v - u) \]

Impulse equals \( \Delta p \) change in momentum. Therefore the space probe’s momentum change is \( 5 \times 10^9 \) kgms\(^{-1}\).

\[ \Delta p = 5(11 - 8) \]
\[ \Delta p = 15 \]

7. a) The momentum change of the object is 15kgms\(^{-1}\).

b) \[ Ft = \Delta p \]
\[ Ft = 15 \]
\[ F \times 1.5 = 15 \]
\[ F = 10 \]

The average force acting on the object is 10N.

c) Impulse equals change in momentum, so the impulse given to the object is 15Ns.

ANSWERS TO TUTORIAL 2.

1. Before:
Total momentum before = (3 \times 0.5 + 2v) \text{kgms}^{-1} \\
= 1.5 + 2v \\

Total momentum after = (3 \times 2 + 2 \times 0.75) \text{kgms}^{-1} \\
= 7 + 1.5 \\

Since momentum is conserved, 1.5 + 2v = 7 + 1.5 \\
2v = 5 \\
\therefore v = 2.5 \\

The original velocity of the 2kg vehicle is 2.5 \text{ms}^{-1}.

3. Total momentum before explosion = zero \\
Total momentum after = 0.02 \times 10 + 0.05v \\
Since momentum is conserved, 0.2 + 0.05v = zero \\
\therefore v = -4 \\

The body of the pen moves off at 4 \text{ms}^{-1} in the opposite direction to the lid.
The resultant force \( R = (820 - 492) \text{N} \)
\[ R = 328 \text{N} \]

\[ F = ma \Rightarrow 328 = 82 \times a \]
\[ \Rightarrow a = 4 \]

\[ v^2 = u^2 + 2as \Rightarrow v^2 = 0 + 2 \times 4 \times 2 \]
\[ = 16 \]
\[ \Rightarrow v = 4 \]

The climber is travelling at \( 4 \text{ms}^{-1} \).

2. a)

b) Because she is \textit{accelerating} and a change in velocity only occurs if there is a resultant force greater than zero.

c) The \( F \) in \( F = ma \) is the resultant force.

\[ F = 80 \times 5 \]
\[ = 400 \]

The resultant force is \( 400 \text{N} \).
d)

The 528N upward force balances the lady’s weight and provides the accelerating force. Thus

\[ 528 = W + 400 \]

\[ W = 128 \]

The gravitational field strength is therefore 128 Newton for 80kg, which is 1.6Nkg⁻¹.

3. a) Using a scale diagram

\[ F=250N \text{ at an angle of } 65^\circ \]

Using components.

*Left is positive, up is positive.*

Horizontal Force  = 50 + 200 \cos45 – 120 \cos45

= 106.6N

Vertical Force  = 0 + 200 \sin45 + 120 \sin45

= 226N
\[ R^2 = H^2 + V^2 \]
\[ R^2 = 106.6^2 + 226^2 \]
\[ R^2 = 11364 + 51200 \]
\[ R = 250N \]

\[ \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{226}{106.6} = 2.12 \]
\[ \theta = 65^\circ \]

b) \( F = ma \) gives us \( 250 = 2000a \) so that the value of \( a \) is 0.125

The acceleration should be 0.125\( \text{m/s}^2 \) but is measured at only 0.1\( \text{m/s}^2 \). The standby thruster must in fact be giving some thrust which we can calculate in the usual way.

\[ F = ma \] gives \( F = 2000 \times 0.1 = 200 \)

Thus the satellite is getting a resultant thrust of 200N when the calculated value is 250N so that the standby thruster is giving a small thrust of 50N.

4 Using \( v^2 = u^2 + 2as \) we get \( 10^2 = 0 + 2a \times 0.25 \)
\[ a = 200 \]

Applying \( F = ma \) to the 6kg dummy gives
\[ F = 6 \times 200 \]
\[ F = 1200 \]

The force on the dummy is 1200N

5. \( v^2 = u^2 + 2as \) gives \( (3 \times 10^7)^2 = 0 + 2a \times 0.15 \)
\[ a = 3 \times 10^{15} \]

\[ F = ma \] \( F = 9.1 \times 10^{-31} \times 3 \times 10^{15} \)
\[ = 2.7 \times 10^{-15} \]

The electric force on the electron is \( 2.7 \times 10^{-15} \)N (Notice that the gravitational force on it is only \( 9.1 \times 10^{-30} \)N since gravity is the weakest of the natural forces.)
6. \( F = ma \)
   \[ F = 50000 \times 0.01 \]
   \[ = 500 \]
   The resultant force on the barge is 500N

   \[
   \begin{align*}
   300N & \quad \text{F2} = 400N \\
   500N & 
   \end{align*}
   \]
   The second horse pulls the barge with a force of 400N

7. \[ v^2 = u^2 + 2as \]
   \[ 15^2 = 0 + 2a \times 22.5 \]
   \[ a = 5 \]

   \[ F = ma \]
   \[ F = 1600 \times 5 \]
   \[ = 8000 \]
   Since 8000N of thrust are required, 8 bags of propellant are needed.

8. a) Momentum before = \((0.1 \times 20 + \text{zero})\)
   momentum after = \((\text{zero} + 0.5v)\)
   
   Since momentum is conserved, \(0.5v = 2\)
   
   Thus \(v = 4\)
   
   The stick moves back at \(4\text{ms}^{-1}\)

   b) The puck loses \(2\text{kgms}^{-1}\) of momentum as it stops then gains \(2\text{kgms}^{-1}\) as it leaves. Its total momentum change is, therefore, \(4\text{kgms}^{-1}\).

   \[ \Delta p = m(v - u) \]
   \[ \Delta p = 0.1(20 - (-20)) \]
   \[ \Delta p = 4 \]

   Remember that momentum and impulse are vector quantities.
9. a) \[
\Delta (mv) = (4 \times 8 - 4 \times 3)
\]
\[
= 20
\]
\[
Ft = 10t
\]
\[
Ft = \Delta (mv)
\]
\[
\Rightarrow 10t = 20
\]
\[
\Rightarrow t = 2
\]

The force acted for 2s

b) Use impulse \( = Ft = 20\text{Ns} \)

or impulse = momentum change = \((32 - 12)\text{kgms}^{-1}\)

\[
= 20\text{kgms}^{-1}
\]

10. a) Let the final speed of the wagon be \( v \text{ms}^{-1} \)

Then since momentum is conserved,

\[
1000\text{kg} \times 5\text{ms}^{-1} = (1000\text{kg} + 4000\text{kg}) \times v \text{ms}^{-1}
\]

\[
5000 = 5000v
\]

\[
v = 1
\]

The velocity of the full wagon is 1ms\(^{-1}\).

b) As the coal slides over the bed of the wagon, the horizontal friction between them acts to slow the wagon and speed up the coal.

11. a) If the velocity of the astronaut is \( v \text{ms}^{-1} \), then since momentum is conserved,

\[
0.050\text{kg} \times 100\text{ms}^{-1} = (100 - 0.05)\text{kg} \times v \text{ms}^{-1}
\]

\[
\Rightarrow 5 = 99.95v
\]

\[
\Rightarrow v = 0.05
\]

The astronaut moves at 0.05ms\(^{-1}\) towards his ship.
b) 
\[ \frac{\dot{v}}{s} = \frac{t}{t} \Rightarrow 0.05 = \frac{100}{t} \]
\[ \Rightarrow t = 2000 \]

The astronaut takes 2000s to reach his ship.

c) Time remaining = \(1550g/0.8 \text{ gs}^{-1}\)
\[= 1937.5s\]

d) If he releases double the mass of gas, his velocity doubles so that his journey is completed in safety.

12. a) Since energy is conserved, the kinetic energy of the pieces is equal to their previous kinetic energy plus the potential energy stored in the explosive.

b) Since momentum is conserved, the total momentum of the pieces is the same as the total momentum before the explosion.

13. Momentum is conserved.

a) If the balls are moving in the same direction
momentum before = momentum after
\[(2 \times 5) + (8 \times 2) = 10v\]
\[10 + 16 = 10v\]
\[v = 2.6 \text{ ms}^{-1}\]

\[E_k \text{ before} = \frac{1}{2} mv^2 + \frac{1}{2} mv^2\]
\[= \left(\frac{1}{2} \times 2 \times 5^2\right) + \left(\frac{1}{2} \times 8 \times 2^2\right)\]
\[= 41 \text{ J}\]

\[E_k \text{ after} = \frac{1}{2} mv^2\]
\[= \left(\frac{1}{2} \times 10 \times 2.6^2\right)\]
\[= 33.8 \text{ J}\]
7.2 J of energy are lost.

Momentum is conserved.

b) If the balls are moving in opposite directions

momentum before = momentum after

\[(2 \times 5) + (8 \times -2) = 10v\]

\[10 - 16 = 10v\]

\[v = 0.6 \text{ ms}^{-1}\]

Energy is a scalar quantity so direction is not important when calculating size.

\[E_k \text{ before} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2\]

\[= (\frac{1}{2} \times 2 \times 5^2) + (\frac{1}{2} \times 8 \times 2^2)\]

\[= 41 \text{ J}\]

\[E_k \text{ after} = \frac{1}{2}mv^2\]

\[= (\frac{1}{2} \times 10 \times 0.6^2)\]

\[= 1.8 \text{ J}\]

39.2 J of energy are lost.

14. a) Find \(v\) as the putty lands in the bucket.

\[v^2 = u^2 + 2as\]

\[v^2 = 0^2 + 2 \times 9.8 \times 2\]

\[v^2 = 39.2\]

\[v = 6.26\]

The putty lands in the bucket with a velocity of 6.26 ms\(^{-1}\)

Now we can find the velocity of the bucket as the putty lands in it.

momentum before = momentum after
\[(4 \times 6.26) + (6 \times 0) = (6+4)v\]
\[25 = 10v\]
\[v = 2.5 \text{ ms}^{-1} \text{ downwards}\]

b) \(E_k\) before = \(\frac{1}{2}mv^2 + \frac{1}{2}mv^2\)
\[= (\frac{1}{2} \times 4 \times 6.26^2) + (\frac{1}{2} \times 8 \times 0^2)\]
\[= 78.4 \text{ J}\]

\(E_k\) after = \(\frac{1}{2}mv^2\)
\[= (\frac{1}{2} \times 4 \times 2.5^2)\]
\[= 12.5 \text{ J}\]

65.9J of kinetic energy are lost by the putty.

c) Impulse = change in momentum
\[\text{Impulse} = Ft\]
\[Ft = m(v-u)\]
\[F(6 \times 10^{-3}) = 4(6.3 - 2.5)\]
\[F = 2520 \text{ N}\]

\[F = ma\]
\[2520 = 10a\]
\[a = 252 \text{ ms}^{-2}\]

15. Momentum is conserved

momentum before = momentum after
\[0.02 \times 50) + (0.380 \times 0) = 0.4v\]
\[1 = 0.4v\]
\[v = 2.5 \text{ ms}^{-1}\]

Now find the acceleration of the block
\[ v^2 = u^2 + 2as \]
\[ 0^2 = 2.5^2 + 2 \times a \times 0.125 \]
\[ -6.25 = a \]
\[ a = -25 \]

Now find the frictional force decelerating the pellet

\[ F = ma \]
\[ F = 0.4 \times -25 \]
\[ F = -10N \]

The frictional force is 10N opposing the direction of motion.

16. \( E_k \) at the bottom = \( \frac{1}{2} m v^2 \)
\[ = \left( \frac{1}{2} \times 200 \times 20^2 \right) \]
\[ = 40000 \text{ J} \]
\( E_p \) at the top = mgh
\[ = (200 \times 9.8 \times 15^2) \]
\[ = 29400 \text{ J} \]

E loss due to friction is the difference between the two values

\[ E_k - E_p = 40000 - 29400 = 10600 \text{ J} \]

The energy loss due to friction is 10600J
\[ s = ut + \frac{1}{2}at^2 \]
\[ 0.8 = 0 + \frac{1}{2} \times 10 \times t^2 \]
\[ 0.8 = 5t^2 \]
\[ t = 0.4 \]

17.a) 

\[ : \quad v = \frac{s}{t} \]
\[ v = \frac{0.2}{0.4} \]
\[ v = 0.5 \]

The horizontal velocity is constant

Hence it has been shown that the velocity of the putty after the impact is 0.5ms\(^{-1}\).

b) The principle of the conservation of momentum is that During any collision the momentum of the system before is equal to the momentum of the system after the collision.

ii) Momentum before = momentum after

\[ (0.1 \times 0) + (5 \times 10^{-4} \times u) = (0.1+5 \times 10^{-4}) \times 0.5 \]
\[ 5 \times 10^{-4} u = 0.1005 \times 0.5 \]
\[ 5 \times 10^{-4} u = 0.05025 \]
\[ u = 100.5 \text{ ms}^{-1} \]

c) The size of the putty could be reduced.