2016

OUR DYNAMIC UNIVERSE part 2

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$$F = \frac{Gm_1m_2}{r^2}$$

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b) Resolving the motion of a projectile with an initial velocity into horizontal and vertical components and their use in calculations.
c) Comparison of projectiles with objects in free-fall.
d) Gravitational field strength of planets, natural satellites and stars.
e) Calculating the force exerted on objects placed in a gravity field.
f) Newton's Universal Law of Gravitation.

SUGGESTED ACTIVITIES

- ✓ Using software to analyse videos of projectiles (Tracker).
- ✓ Low orbit and geostationary satellites.
- ✓ Satellite communication and surveying.
- ✓ Environmental monitoring of the conditions of the atmosphere.
- ✓ Newton's thought experiment and an explanation of why satellites remain in orbit.
- ✓ Methods for measuring the gravitational field strength on Earth.
- ✓ Using the slingshot effect to travel in space.
- ✓ Lunar and planetary orbits.
- ✓ Formation of the solar system by the aggregation of matter.
- ✓ Stellar formation and collapse.
- ✓ The status of our knowledge of gravity as a force may be explored. The other fundamental forces have been linked but there is as yet no unifying theory to link them to gravity.

ASTRONOMICAL DATA

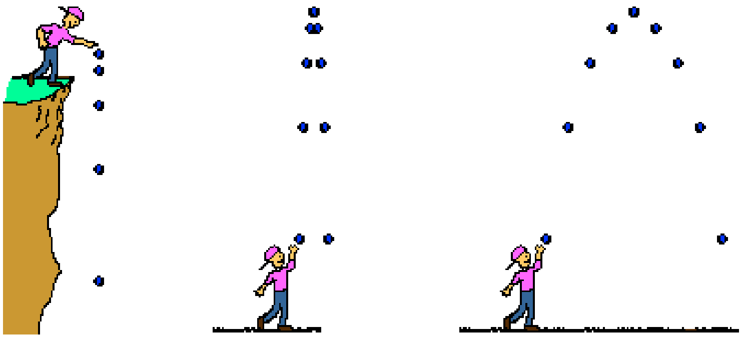
Mass of Earth	M_E	$6{\cdot}0 \times 10^{24}$ kg
Radius of Earth	R_E	$6{\cdot}4 \times 10^{6}$ m
Mass of Moon	M_M	$7{\cdot}3 \times 10^{6}$ kg
Radius of Moon	R_M	$1{\cdot}7 \times 10^{6}$ m
Mean radius of Moon orbit		$3{\cdot}84 \times 10^{8}$ m
Mass of Jupiter	M_J	$1{\cdot}0 \times 10^{27}$ kg
Radius of Jupiter	R_J	$7{\cdot}15 \times 10^{7}$ m
Mean radius of Jupiter orbit		$7{\cdot}8 \times 10^{11}$ m
Universal constant of gravitation	G	$6{\cdot}67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}

GRAVITATIONAL FIELD STRENGTH ON OTHER PLANETS

Planet	g (Nkg^{-1})
Mercury	3.7
Venus	8.8
Earth	9.8
(Moon)	1.6
Mars	3.8
Jupiter	26.4
Saturn	11.5
Uranus	11.7
Neptune	11.8

PROJECTILES

Generally projectiles have both horizontal and vertical components of motion. As there is only a single force, the force of gravity, acting in a single direction, only one of the components is being acted upon by the force. The two components are not undergoing the same kind of motion and must be treated separately. This force creates an acceleration equal to “g”, which on Earth has a value of $9.8ms^{-2}$



PROJECTILES FIRED VERTICALLY

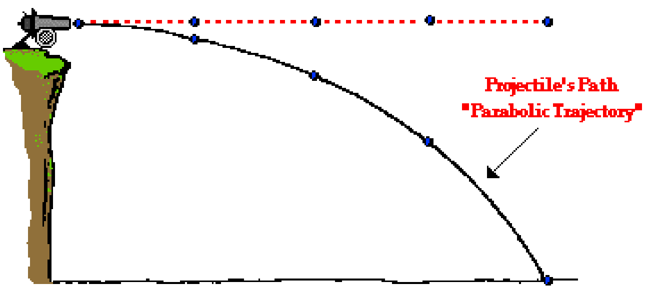
To summarise, for a vertical projectile:

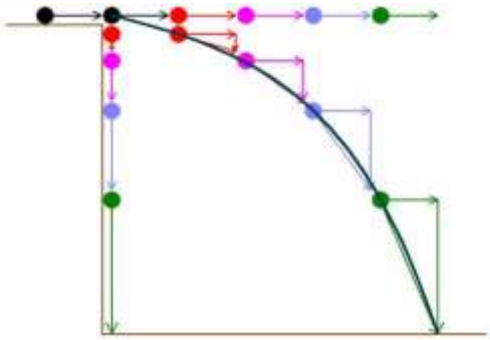
Direction of motion	Forces	Velocity	Acceleration
Horizontal	Air resistance negligible so no forces in the horizontal direction.	Constant (in this case $0ms^{-1}$)	None
Vertical	Air resistance negligible so only force of gravity acting in the vertical direction.	Changing with time	Constant or uniform acceleration of $-9.8\ m\ s^{-2}$

The second projectile situation to consider is the horizontal projectile.

PROJECTILES FIRED HORIZONTALLY

Here is a classic horizontal projectile scenario, from the time of Newton. In projectile motion we ignore all air resistance, or any force other than the force of gravity,weight, in our calculations, but you could be asked the effect of air resistance in a supplementary question.





Analysis of this projectile shows the two different components of motion.

Horizontally: there are no forces acting on the cannonball and therefore the **horizontal velocity is constant.**

Vertically: The force due to gravity is constant in the vertical plane and so the cannonball **undergoes constant vertical acceleration.**

The combination of these two motions causes the curved path of a projectile.

Example: The cannonball is projected horizontally from the cliff with a velocity of 100 ms^{-1}. The cliff is 20 m high.

Determine: (a) the vertical speed of the cannonball, just before it hits the water;

(b) if the cannonball will hit a ship that is 200 m from the base of the cliff.

Solution:

Horizontal	Vertical
s = ?	s = 20 m
v= 100 m s^{-1}	u = 0
t = ?	v= ?
	a = 9·8 m s^{-2}
	t= ?

$$v^2 = u^2 + 2as$$
$$v^2 = 0 + (2 \times 9{\cdot}8 \times 20)$$
$$v^2 = 392$$
$$\underline{v = 19.8 ms^{-1}}$$

(b)

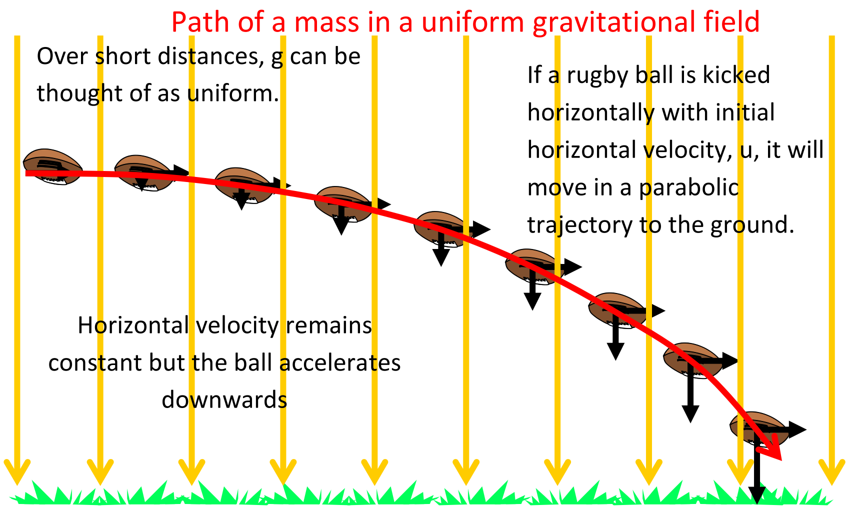
$$v = u + at$$
$$19.8 = 0 + 9.8t$$
$$t = \frac{19.8}{9.8} = \underline{2.02s}$$

$$s_h = \bar{v}t$$
$$s_h = 100 \times 2.02 = \underline{202m}$$

The cannonball will hit the ship.

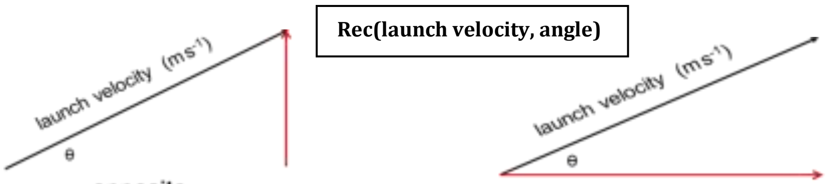
To summarise, for a horizontal projectile:

Direction of motion	Forces	Velocity	Acceleration
Horizontal	Air resistance negligible so no forces in the horizontal direction.	Constant	Zero
Vertical	Air resistance negligible so only force of gravity acting in the vertical direction.	Changing with time	Constant or uniform acceleration of -9.8 m s^{-2}



PROJECTILES AT AN ANGLE

The third and final projectile situation to consider is the projectile at an angle to the horizontal projectiles at an angle are an application for our knowledge of splitting vectors into their horizontal and vertical components.



$\sin\theta = \dfrac{\text{opposite}}{\text{hyp}}$

opposite = hyp $\sin\theta$

vertical component = launch velocity $\sin\theta$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hyp}}$

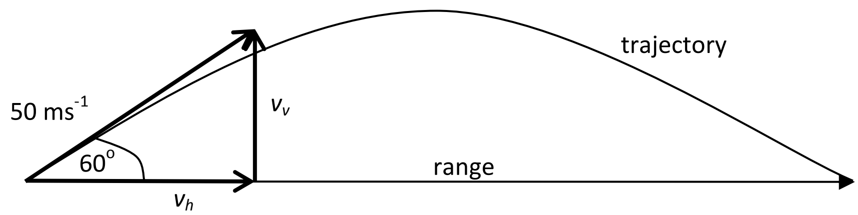
adjacent = hyp $\cos\theta$

horizontal component = launch velocity $\cos\theta$

The distance travelled horizontally (the range) is determined by the cosine component of the launch velocity. The time of flight is determined by the sine component of the launch velocity, **providing the angle to the horizontal is given.**

To summarise, for a projectile at an angle to the horizontal:

Direction of motion	Forces	Velocity	Acceleration
Horizontal	Air resistance negligible so no forces in the horizontal direction	Constant	Zero
Vertical	Air resistance negligible so only force of gravity acting in the vertical direction.	Changing with time	Constant or uniform acceleration of 9.8 m s^{-2}



The velocity at an angle <u>must</u> be split into its vertical and horizontal components before any further consideration of the projectile as only the vertical component is acting upon by the force of gravity.

Velocity at an angle is only used in calculations when finding horizontal and vertical components. You will never use the velocity at an angle (here 50 ms^{-1}) directly in any other calculation.

For projectiles fired at an angle above a horizontal surface:

1. The path of the projectile is symmetrical, in the horizontal plane, about the highest point. This means that:

initial vertical velocity = – final vertical velocity

$$u_v = -v_v$$

2. The time of flight = 2 × the time to highest point.

3. The vertical velocity at the highest point is zero.

To summarise, for any projectile:

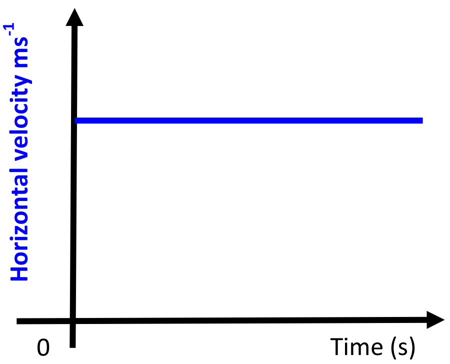
Direction of motion	Forces	Velocity	Acceleration
Horizontal	Air resistance negligible so no forces in the horizontal	Constant	Zero
Vertical	Air resistance negligible so only force of gravity acting in the vertical	Changing with time	Constant or uniform acceleration of $-9.8\ m\ s^{-2}$

TASK: LAUNCH VELOCITY

Determine experimentally and/or theoretically the angle, which gives the greatest range for a fixed, launch velocity.

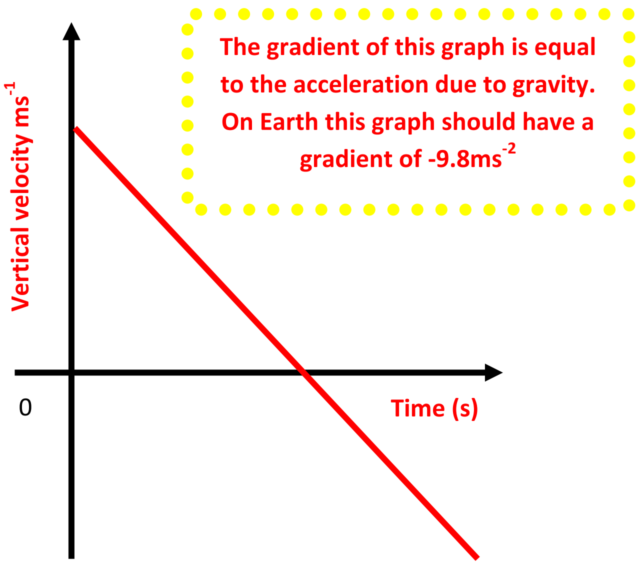
NOTES ON PROJECTILE MOTION

1. Objects travel sideways as well as falling.
2. The motion is symmetrical about the highest point.
3. The flight of a projectile is parabolic.
4. When dealing with projectiles you must deal with the **two motions separately**.
5. HORIZONTALLY, ignoring air resistance, the projectile moves with constant velocity.
6. VERTICALLY the motion is affected by a constant acceleration of $9.8ms^{-2}$ downwards on Earth. This is the acceleration due to gravity.

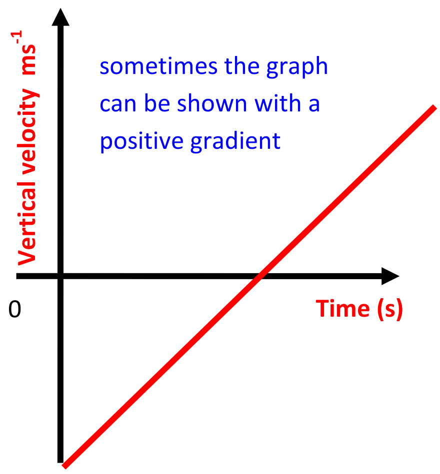


7. If we take air resistance into account then the speed tends to decrease in the horizontal direction, however this makes the calculations too difficult so for the higher course you must assume that you're your calculations are occurring in a vacuum!
8. When fired upward the object will decelerate at $9.8ms^{-2}$ reach a maximum height where its vertical velocity is zero. It will then accelerate downwards at $9.8ms^{-2}$.
9. You ought to realise that the forces on an object will be the same if an object accelerates downwards as they are if the object decelerates upwards!

10. Don't forget that even when the vertical velocity of the object is zero there is still a force on the object and it is still accelerating downwards at 9.8ms^{-2}.
11. The velocity of the object changes during the flight.
12. Remember that velocity is a vector quantity. We must split the resultant velocity (the velocity that the object is travelling in) into its components, in this case horizontal and vertical. We do this by taking components which has been dealt with elsewhere.



13. Notice there is only one gradient because there is only one acceleration, but this causes a deceleration in objects moving upwards, away from the Earth and it causes an acceleration in objects moving downwards, towards the centre of the Earth.

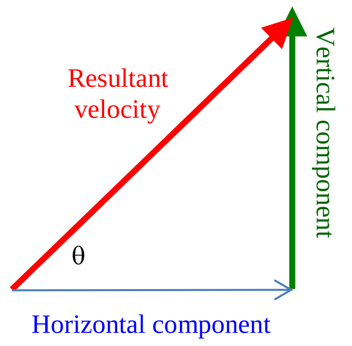


14. We then do <u>not</u> use the resultant again unless we are asked for the resultant at any further point.
15. The horizontal and vertical motions are independent of each other and the only link between the two motions is the TIME!
16. At any point during the flight the horizontal and vertical components are added together to give the resultant velocity of the object.

$$R^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x}$$

vertical component = $R \sin\theta$
where R is the resultant velocity
horizontal component = $R \cos\theta$
where R is the resultant velocity



When working out the projectile problems follow the method below.

a. Draw out a quick sketch of the problem.

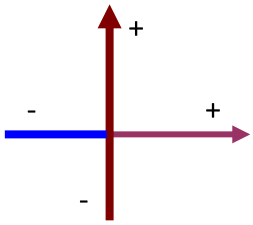
b. Find the horizontal and vertical components of the initial velocity. $u_h = v\cos\theta$ $u_v = v\sin\theta$

c. Draw up a table with the following information and fill in as much as you can.

Horizontal motion	Vertical motion
s_H	s_v
$\overline{v_H}$	u_v
T	v_v
	$a = 9.8ms^{-2} \downarrow$
	t
TIME IS THE COMMON FACTOR	
$\boxed{\overline{v_H} = \frac{s_H}{t}}$	$s_v = u_v t + \frac{1}{2}at^2$ $v_v^2 = u_v^2 + 2as_v$ $v_v = u_v + at$ $\overline{v_v} = \frac{v+u}{2} = \frac{s_v}{t}$

YOU MUST REMEMBER THAT DIRECTION IS IMPORTANT. You indicate direction by positive and negative signs.

You can work by convention i.e. axes, or take each question individually and decide which way you will take as positive. Show this in your summing up of the question. If an object is thrown **upwards** then *u* and *a* must have **opposite** signs.



17. Usually in questions they will ask you to find either the horizontal or vertical quantities but this will involve you working out something that is missing from the horizontal quantities. e.g. Find the horizontal range but you will only be given enough information to find the time from the vertical component. This time will then need to be substituted into the equation for horizontal components.
18. An important thing to remember is that at the top of the throw the vertical velocity will be zero. This can be equal to *v* for that part of the journey and *u* for the next part of the journey.This will occur half way through the journey if the **journey is symmetrical** (i.e. it lands at the same height that it was thrown.) These questions are becoming less common as they were considered too easy!
19. Other hints to remember, if the journey is symmetrical then the resultant velocity on landing is the same but in the opposite direction. i.e if an object is projected with an initial velocity of $75ms^{-1}$ at an angle of 60° to the horizontal, then it will land at a velocity of $-75ms^{-1}$ at an angle of 60° to the horizontal.
20. You can always break journey's into bits if they are too difficult to complete in one go, eg find the time it takes to go to the top of the throw and then find the time it takes to come back down.
21. The vertical displacement for a symmetrical journey is 0 m.
22. Remember you can use the version of the quadratic with it already set up for the equation ($s=ut+ ½ at^2$) if you feel confident and can remember it (it will not be on the relationship sheet)

we can rewrite the solution for the quadratic equation as:

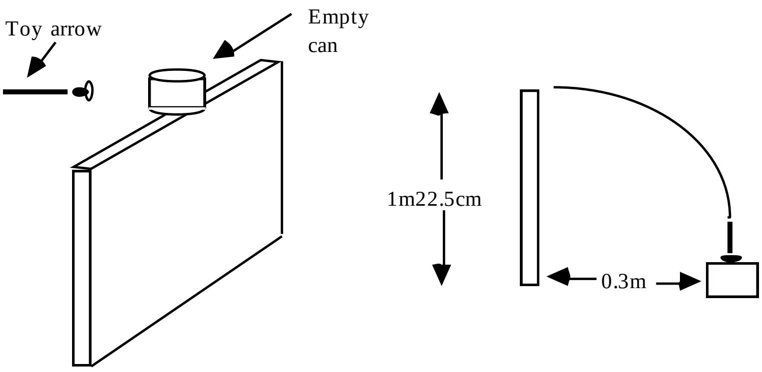
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-u_v \pm \sqrt{u_v^2 + 2as_v}}{a}$$

23. If you find yourself with a difficult quadratic to solve then you can use two equations instead, ie usually $v^2=u^2+2as$ and then $v=u+at$.
24. You don't get extra marks for being smart it is being correct that is important and carrying out a method that works!

25. Remember that an object that is thrown with **only a horizontal initial speed** only will land at the same time as an object dropped with zero initial vertical velocity.
26. For all our problems we ignore air resistance. Air resistance and wind would affect the journey but it is too hard to work out. You could be asked what effect this would have on the journey.
27. Sometimes you could be caught out with trying to square root a negative number. You can always try changing the sign convention and then seeing if that works out better. Remember that the square root of a number has two values, positive and negative. You can't ever end up with a negative time and expect the right answer!
28. If an object hits the ground its velocity on hitting is **not** zero!

PROJECTILES: WORKED EXAMPLES

1. Show that the horizontal velocity of the can is 0.6 m s^{-1} immediately after being struck by the toy arrow.



First, consider the vertical velocity in order to find the time.

$$s_v = u_v t + \tfrac{1}{2}at^2$$
$$1.225 = 0 \times t + \left(\tfrac{1}{2} \times 9.8 \times t^2\right)$$
$$t^2 = \frac{2 \times 1.225}{9.8} = 0.25$$
$$t = \sqrt{0.25} = 0.5\,s$$

Horizontally: $s_h = u_h t + \tfrac{1}{2}at^2$ or $s_h = u_h t$

$$0.3 = u_h t + \left(\tfrac{1}{2} \times 0 \times t^2\right)$$
$$u_h = \frac{0.3}{t} = \frac{0.3}{0.5} = 0.6\,m\,s^{-1}$$

2. How long does a stone take to reach the ground from the top of a 100m high building if, it is thrown vertically downwards at 5 ms^{-1}?

it is thrown upwards at an angle of 39.3° to the horizontal at 7.9 m s^{-1}?

$$s_v = u_v t + \tfrac{1}{2}at^2$$
$$100 = 5 \times t + \left(\tfrac{1}{2} \times 9.8 \times t^2\right)$$
$$100 = 5t + 4.9t^2 \quad \text{i.e. } 4.9t^2 + 5t - 100 = 0$$

We need to use the solution for a quadratic equation here i.e.

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve for t.

$$t = \frac{-5 \pm \sqrt{5^2 - (4 \times 4.9 \times -100)}}{9.8} = \frac{-5 \pm \sqrt{25 + 1960}}{9.8}$$
$$t = \frac{-5 \pm 44.55}{9.8} \quad \text{(a negative value for } t \text{ is not possible)}$$
$$\therefore t = \frac{-5 + 44.55}{9.8} = 4.04\,s.$$
$$\tfrac{1}{2}at^2 + u_v t - s_v = 0$$

(a) If this last method is too difficult then try breaking it up into smaller chunks!

If you can't do quadratics we need to use one of the other formulae.

$v^2 = u^2 + 2as$ doesn't contain time so that isn't the equation that we finally need but if we use $v = u + at$ we have 2 unknowns. We don't know either v or t so we have to work it out. So first find the final velocity on landing.

s = 100m
u = 5ms^{-1} downwards
v = ?
a = 9.8ms^{-2} downwards
t = ?

$$v^2 = u^2 + 2as$$
$$v^2 = 5^2 + 2 \times 9.8 \times 100$$
$$v^2 = 25 + 1960$$
$$v = \sqrt{1985} = 44.55\ ms^{-1}$$

Now put the new velocity into the other equation

$$v = u + at$$
$$44.5 = 5 + 9.8t$$
$$44.55 - 5 = 9.8t$$
$$\frac{39.55}{9.8} = t = 4.04s$$

Using

$\frac{1}{2}at^2 + u_v t - s_v = 0$

we can rewrite the solution for the quadratic equation as:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-u_v \pm \sqrt{u_v^2 + 2as_v}}{a}$$

Trying it on the above problem:

$$t = \frac{-5 \pm \sqrt{5^2 + 2 \times 9.8 \times 100}}{9.8} = \frac{-5 \pm \sqrt{25 + 1960}}{9.8}$$

$t = 4.04s$

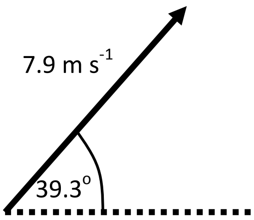
As you can see this is the same as above.

(b) Although the stone has both a horizontal **and** vertical component, we only need to consider the VERTICAL one.

The vertical component of the motion, v_v is given by:

$$v_v = 7.9 \sin 39.3$$
$$v_v = 5.00\, m s^{-1}$$

[Rec(7.9,39.3)]



Find how long it takes to reach the ground?
s=100m (downwards)
u =5ms^{-1}(upwards)
v=
a=9.8ms^{-2}(downwards)
t=

$$s_v = u_v t + \tfrac{1}{2}at^2$$
$$-100 = 5 \times t + \left(\tfrac{1}{2} \times -9.8 \times t^2\right)$$
$$-100 = 5t - 4.9t^2 \quad \text{i.e. } 4.9t^2 - 5t - 100 = 0$$

We need to use the solution for a quadratic equation here i.e.

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve for t.

$$t = \frac{5 \pm \sqrt{-5^2 - (4 \times 4.9 \times -100)}}{9.8} = \frac{5 \pm \sqrt{25 + 1960}}{9.8}$$

$t = \frac{5 \pm 44.55}{9.8}$ (a negative value for t is not possible)

$$\therefore t = \frac{5 + 44.55}{9.8} = 5.06s.$$

$\frac{1}{2}at^2 + u_v t - s_v = 0$

Find how long it takes for it to reach the top of the flight.

s = ?

u = 5 m s^{-1} upwards

v = 0 m s^{-1}

a = 9.8 m s^{-2} downwards

t =

$$v = u + at$$
$$0 = 5 + -9.8t$$
$$9.8t = 5$$
$$\frac{5}{9.8} = t = 0.510s$$

You now need to know what height this is above the 100m building.

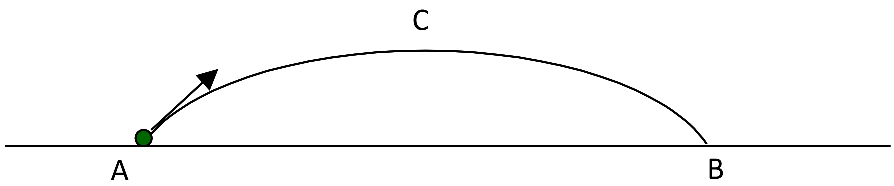
$$v^2 = u^2 + 2as$$
$$0^2 = 5^2 + 2 \times -9.8 \times s$$
$$19.6s = 25$$
$$s = \frac{25}{19.6} = 1.276m$$

So now you need to know the time it takes for the object to fall 1.276 + 100 m to the ground. remember that for this part of the journey the initial starting velocity of the object is zero, as it has reached the highest point.

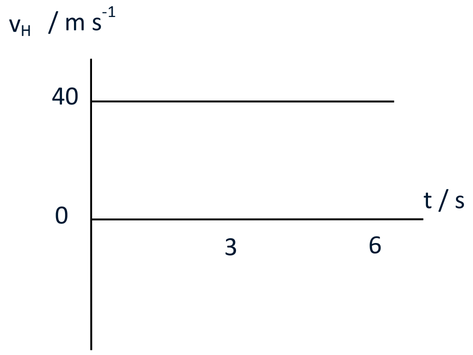
$$s = ut + \tfrac{1}{2}at^2$$
$$101.276 = 0 + \tfrac{1}{2} \times 9.8 + t^2$$
$$\frac{101.276}{4.9} = t^2 = 20.669$$
$$t = \sqrt{20.669} = 4.55s$$

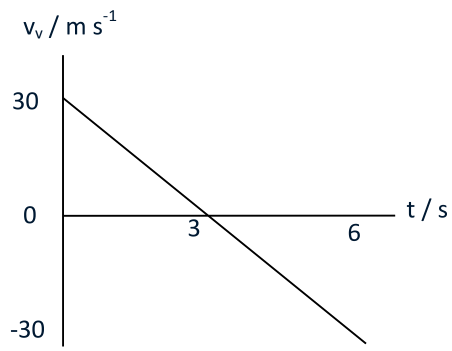
so total time = 4.55 + 0.51= <u>5.06 s</u>

3. A projectile is fired across level ground taking 6 s to travel from A to B. The highest point reached is C. Air resistance is negligible.



Velocity - time graphs for the flight are shown below





(a) Describe the horizontal and vertical motions of the projectile.

(b) Use a vector diagram to find the speed and angle at which the projectile was fired from point A.

(c) Find the speed at position C. Explain why this is the smallest speed of the projectile.

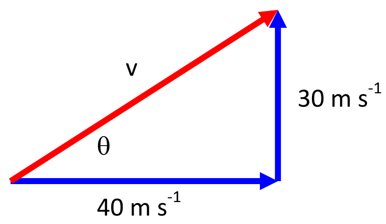
(d) Calculate the height above the ground of point C.

(e) Find the range AB.

Answer

(a) The horizontal motion is a constant speed (40 m s^{-1}). The vertical motion is a constant acceleration (9.8 m s^{-2}).

(b)



(c) [Pol(40,30)]

$v^2 = 30^2 + 40^2 = 900 + 1600 = 2500$

$v = \sqrt{2500} = 50\, m\, s^{-1}$

$\sin\theta = \frac{30}{50} = 0.6 \quad \theta = 36.9°$

(d) The speed at C is 40 m s^{-1} (the vertical speed is zero).

(e) Consider the vertical motion of the projectile:

s = ?
v = 0 m s^{-1}
u = 30 m s^{-1}
a = -9.8 m s^{-2}
t = 3 s

$s_v = u_v t + \frac{1}{2}at^2 = (30 \times 3) + \left(\frac{1}{2} \times -9.8 \times 3^2\right)$

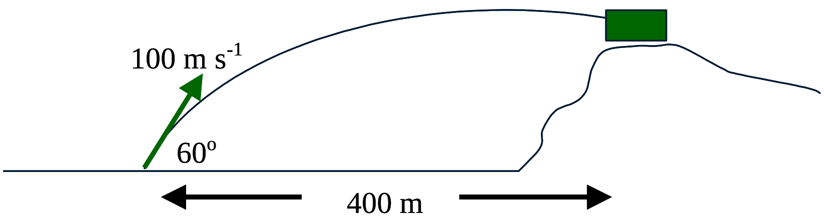
$s_v = 90 - 44.1 = 45.9\, m$

(f) Consider the horizontal motion of the projectile:

$s_H = \overline{v_H} \times t = 40 \times 6$

$s_H = 240\, m$

4. A missile is launched at 60° to the ground and strikes a target on a hill as shown below.



(a) If the initial speed of the missile was 100 ms-1 find:

(i) the time taken to reach the target

(ii) the height of the target above the launcher.

(a)

(i) Consider the horizontal motion of the projectile:

$u_H = u\cos\theta = 100\cos 60 = 50\, m\, s^{-1}$

$s_H = u_H \times t \qquad t = \frac{s_H}{u_H} = \frac{400}{50} = 8\, s$

(ii) Consider the vertical motion of the projectile:

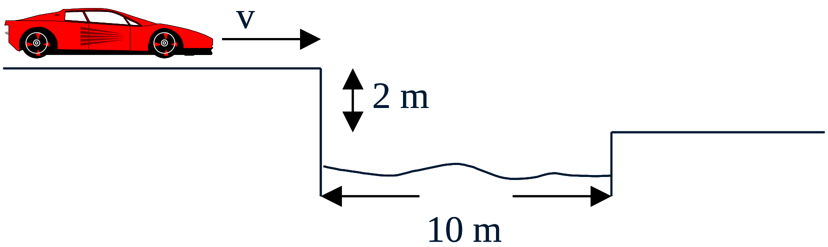
$u_V = u\sin\theta = 100\sin 60 = 86.6\, m\, s^{-1}$

$s_V = u_V t + \frac{1}{2}at^2$

$s_V = (86.6 \times 8) + \left(\frac{1}{2} \times -9.8 \times 8^2\right)$

$s_V = 692.8 - 313.6 = 379.2\, m$

5. A stunt driver hopes to jump across a canal of width 10 m. The drop to the other side is 2 m as shown.



(a) Calculate the horizontal speed required to make it to the other side.

(b) State any assumptions you have made.

(a)

$$s_V = u_V t + \tfrac{1}{2}at^2$$
$$2 = 0 + \left(\tfrac{1}{2} \times 9.8 \times t^2\right)$$
$$t^2 = \frac{2}{4.9} = 0.41$$
$$t = 0.64\,s$$

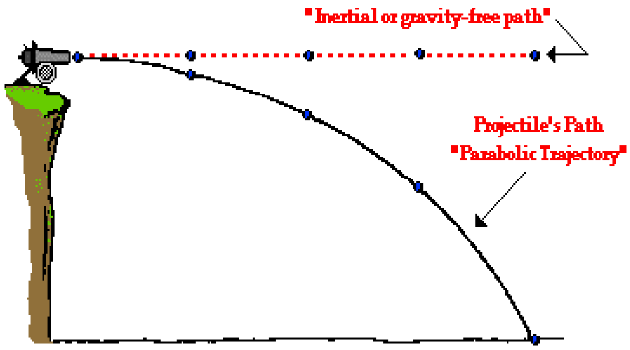
$$\overline{v_H} = \frac{s_H}{t} = \frac{10}{0.64} = 15.6\,m\,s^{-1}$$

(b) The calculations do not take into account the length of the car. In practice, the front of the car will accelerate downwards before the back.

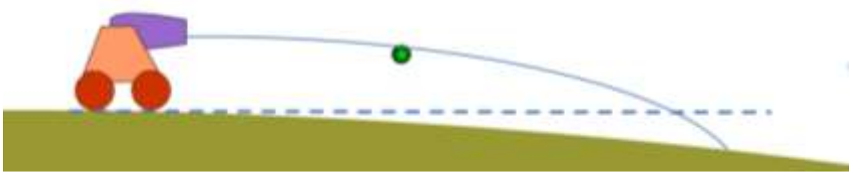
ORBITS AND NEWTON'S THOUGHT EXPERIMENT

Newton died in 1727, 230 years before the launch of Sputnik 1, the first man-made object to orbit Earth, in 1957. However, like all good Physicists, he did have a great imagination and conducted thought experiments.

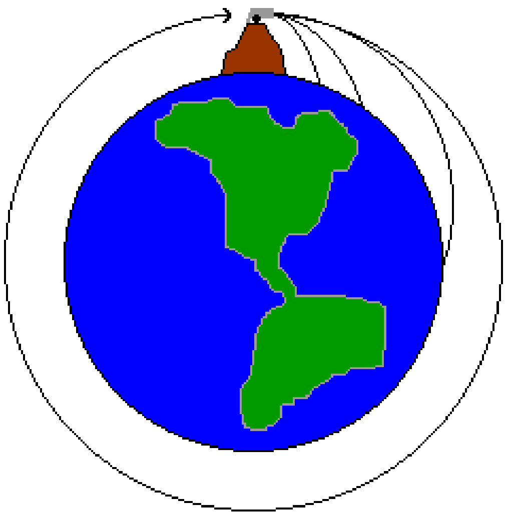
Newton considered the example of the cannon firing horizontally off a cliff. He knew that, as the Earth is approximately a sphere, the ground curves away from the projectile as it falls.



If we give the projectile a greater horizontal velocity, it will travel a greater distance before reaching the ground.



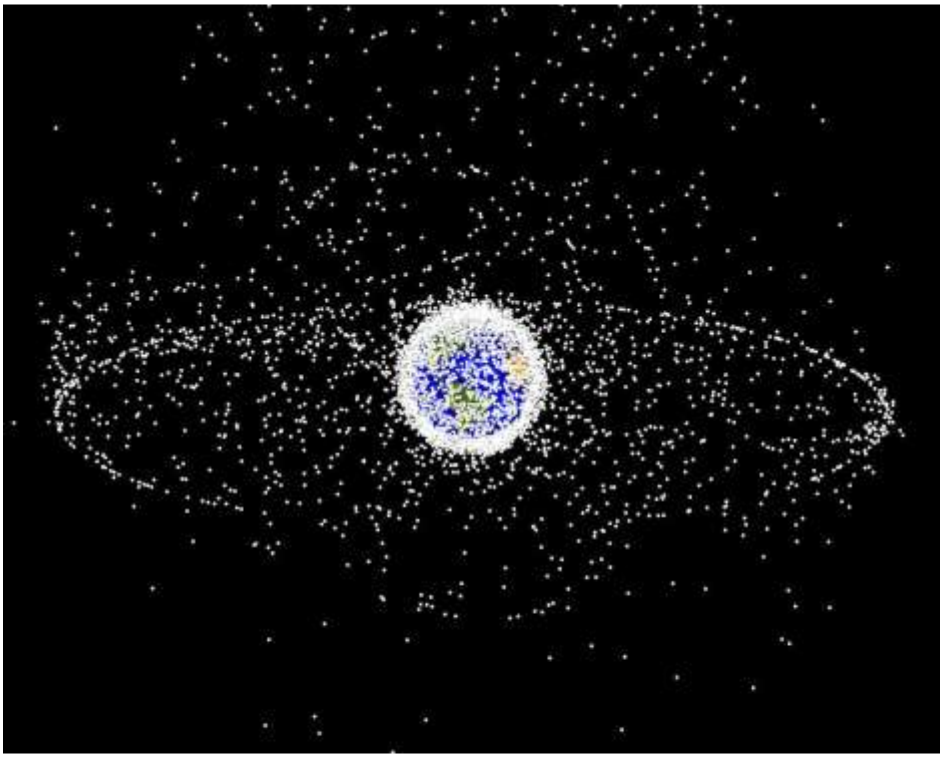
If that ground is also curving down and away from the projectile, it would take even longer for the projectile to land. (see the diagram to the side, note the additional distance that the cannonball would travel due to the curvature of the Earth).



Newton knew that there must be a horizontal launch velocity you could give a projectile which meant the Earth would curve away from the projectile at the same rate that the force of gravity accelerated it towards the ground. This would cause the projectile to go into orbit. You could also project the object at a velocity so that the object will never return to Earth without the presence of an external force, this is known as escape velocity and is dependent on the gravitational field.

On Earth escape velocity is roughly 11 kms^{-1}.

As of August 2014, we have 1200 active satellites in orbit around Earth and one space station. There are also approximately 21,000 objects larger than 10 cm orbiting the Earth and likely to be 500,000 bits between 1cm and 10cm in size. Their orbits are at different radii these radii mean they have certain orbital periods and this all depends on their velocity.



Note the geostationary and polar orbits that are rather busy and full!

Satellites in Low Earth Orbit (LEO), including the ISS, have a period of approximately 90 minutes.

Many communication satellites, for telecommunications and television, are in geostationary orbit. This orbit is at a greater radius, with a period of 24 hours and an altitude of almost 36,000 km. This allows the satellites to stay above the same point on the Earth's equator at all times and provide consistent communication across the globe.

Though high orbits, like geostationary, have been revolutionary for achieving successful global communications, they do have their drawbacks. They can lead to early failure of electronic components as they are not protected by the Earth's magnetic field and are exposed to very high levels of solar radiation and charge build-up. They also require a great deal of energy to achieve the altitudes required and very powerful amplifiers to ensure successful transmission back to Earth.

To avoid these orbits many organisations choose to use a 'constellation' of satellites in LEO, placed between the atmosphere and the inner Van Allen Belt. These have their own issues, as gases from the upper atmosphere cause drag which can degrade the orbit.

GRAVITY AND MASS

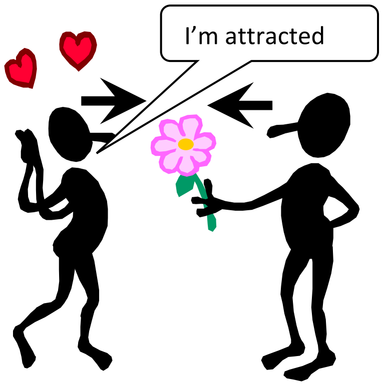
If there is one thing that causes confusion in physics then it could be the distinction between mass and weight.

Here we must be very careful about our use of terms.

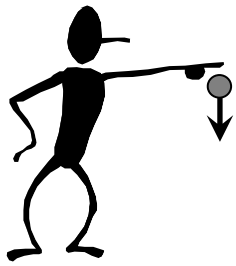
Do NOT use the term GRAVITY when you mean "The Force of Gravity". Try to think of gravity as a phenomenon rather than a force. If you wish to talk about the force of gravity on an object then you should use the term **the object's weight or the force of gravity.**

Mass is a measure of how much matter an object contains. This will only change if matter is added to or taken from the object.

The force of gravity is caused by mass, any object that has mass will have its own gravitational field. The magnitude of the field depends on the mass of the object: the larger the mass and the smaller the distance from that mass the greater the field strength.



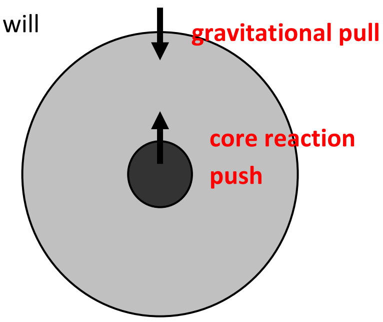
The field of an object will then exert a force on any mass in its vicinity. The forces associated with masses smaller than planetary masses are so small that very sensitive equipment needs to be used to measure them.



You will have done an experiment to measure the Earth's gravitational field using a set of masses and a Newton balance. If you measured the acceleration due to gravity earlier in the unit, then you have effectively measured the gravitational field strength since the magnitude of these two quantities is the same.

The Force of Gravity permeates the entire universe; scientists believe that stars were formed by the gravitational attraction between hydrogen molecules in space. As the mass accumulated the gravitational attraction on the gases increased so that the forces at the centre of the mass were big enough to cause the hydrogen molecules to fuse together. This process is nuclear fusion and generates the sun's energy. The energy radiating outwards from the centre of the sun counteracts the gravitational force trying to compress the sun inwards.

In time the hydrogen will be used up, the reaction will stop and the sun will collapse under its own force of gravity, but this shouldn't worry you unless you expect to live for approximately four billion years.



It is believed that our solar system formed from the debris generated from the formation of the sun. This debris joined together to form planets, due to the gravitational attraction between the particles.

Newton formulated the laws of gravity, inventing calculus along the way to help him get his laws in a simple form. Remember, Newton didn't invent gravity; he produced the calculations that explained his observations.

NEWTON'S UNIVERSAL LAW OF GRAVITATION

Newton's Law of Gravitation states that the gravitational attraction between two objects is directly proportional to the mass of each object and is inversely proportional to the square of their distance apart. Newton produced what is known as the Universal Law of Gravitation.

$$F = \frac{Gm_1m_2}{r^2}$$

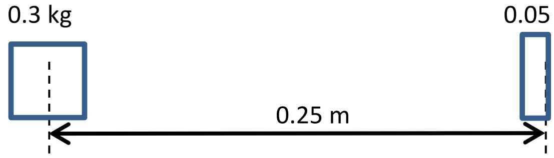
This relationship shows that the force of attraction between any two masses depends on the magnitude of the masses and the distance between their centres of mass. *G* is the universal constant of gravitation and has the value **6.67 x 10^{-11} m^{3}kg^{-1}s^{-2} or Nm^{2}kg^{-2}**

Gravitational force is always attractive, unlike electrostatic or magnetic forces.

Example: Consider a folder, of mass 0.3 kg and a pen, of mass 0.05 kg, sitting on a desk, 0.25 m apart. Calculate the magnitude of the gravitational force between the two masses.

Assume they can be approximated to point objects, where all their mass is concentrated in one place.

Solution:



m_1 = 0.3 kg
m_2 = 0.05 kg
r = 0.25 m
G = 6.67 x 10^{-11}m^3 kg^{-1} s^{-2}
F = ?

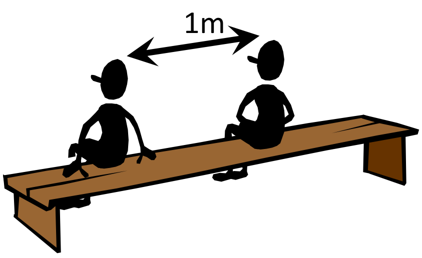
$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{6.67\times10^{-11}\times0.3\times0.05}{0.25^2}$$

$$F = 1.6\times10^{-11}\,N$$

We do not notice the gravitational force between everyday objects because it is so small, in fact it is the weakest of the four fundamental forces of our universe. This is just as well, or you would have to fight against the force of gravity every time you walk past a large building! The force of gravity only becomes really apparent when *very* large masses are involved, e.g. planetary masses.

For example the force of attraction between two pupils of average mass [60kg] sitting 1 metre apart as 2.4 x 10^{-7}N. If there were no resistive forces and one pupil were able to move under the influence of this force towards the other, it would take 22 360s to cover the 1m distance.



$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{6.67\times10^{-11}\times60\times60}{1^2}$$

$$F = 2.4\times10^{-7}\,N$$

$$F = ma$$

$$2.4\times10^{-7} = 60\times a$$

$$a = 4.0\times10^{-9}\,ms^{-1}$$

$$s = ut + \tfrac{1}{2}at^2$$

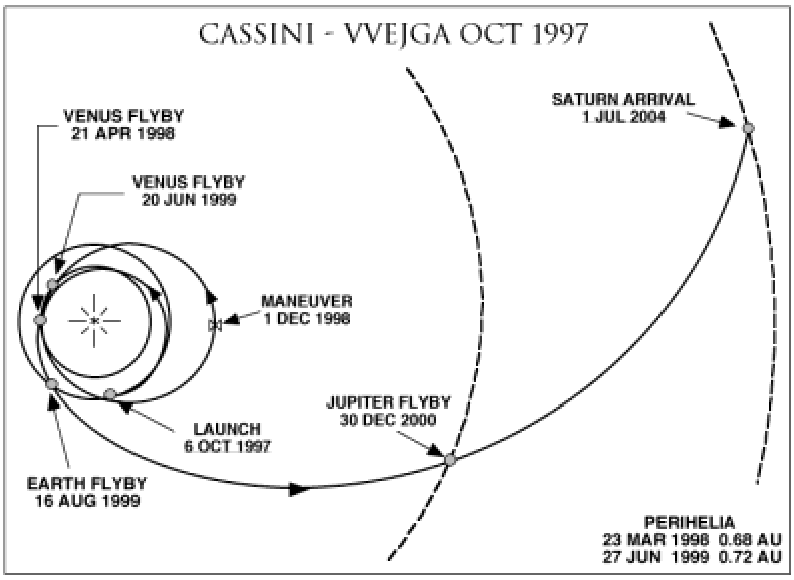
$$1 = 0 + \tfrac{1}{2}\times4.0\times10^{-9}\times t^2$$

$$t^2 = \frac{2}{4.0\times10^{-9}} = 5.0\times10^8$$

$$t = 2.3\times10^4\,s \quad \text{or 6 hours 12 min and 40s}$$

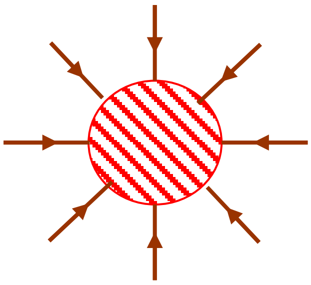
When this is applied to gas molecules or dust particles then the aggregation of larger and larger masses becomes easier to understand.

Another application of the gravitational force is the use made of the 'slingshot effect' by space agencies to get some 'free' energy to accelerate their spacecraft. Simply put they send the craft close to a planet, where it accelerates in the gravitational field of the planet. Here's the clever part, if the trajectory is correct the craft then speeds past the planet with the increased speed. Don't get it right and you still get a spectacular crash into the planet, which could be fun but a bit on the expensive side!



GRAVITATIONAL FIELDS

A gravitational field is the region around a mass where another mass experiences a force. A gravitational field is a region where gravitational forces exist.



The strength or intensity of the gravitational field, g, is defined as the force acting per unit mass placed at a point in the field.

The size of the field is given by:

$$g = \frac{F_w}{m}$$

where g= gravitational field strength in $\mathrm{Nkg^{-1}}$

F_w = Force on the mass in Newtons

m= mass being attracted in kilograms

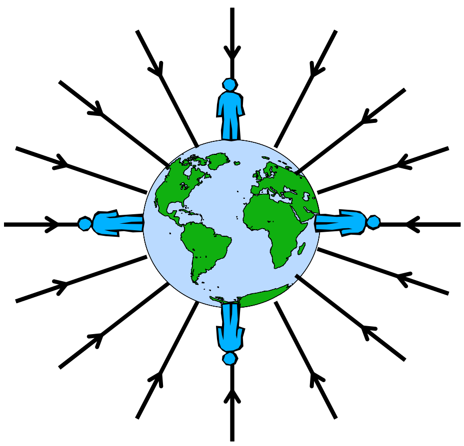
If a mass, m, experiences a gravitational force, F, then gravitational field strength, $g = \frac{F}{m}$ with units of N $\mathrm{kg^{-1}}$ ($\mathrm{m\ s^{-2}}$).

We can visualise the direction of a field by drawing FIELD LINES. For an isolated point mass the lines are always directed towards the mass.

The field lines are RADIAL.

(Gravitational lines around a "point" mass).

For a radial field:

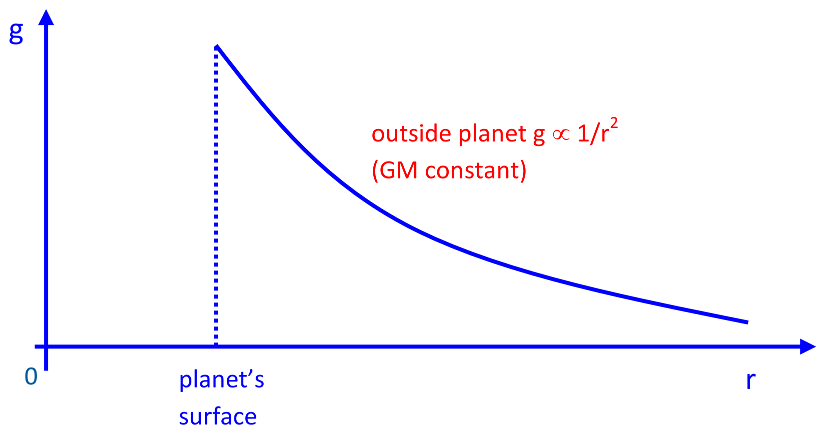


$$F = \frac{GMm}{r^2}$$

$$\therefore \frac{F}{m} = \frac{GM}{r^2}$$

$$\therefore g = \frac{GM}{r^2}$$

As you get further away from the mass the strength of the field gets weaker. The strength of the field can be shown using field lines and the closer the field lines the stronger the field.. These field lines do not really exist (look out of the window!) but they do show a concept that is real. (This is also similar to magnetic field lines when you draw the direction that a compass will point around a magnet) These field lines are a convenient, fictional concept that represents a real field.



Gravitational field strength is the force on a 1kg mass placed in the field.

On Earth the gravitational field strength is the force of attraction between 1 kg of mass and the Earth. Remember the law of gravitation is a force of attraction between *two* masses, this means the 1 kg mass is attracting the Earth towards it, as well as the other way round.

Example 1: Show, using the universal law of gravitation, that the gravitational field strength on Earth is 9·8 N kg^{-1}.

$$F = \frac{Gm_1m_2}{r^2}$$

but

$$mg = \frac{Gm_1m_2}{r^2}$$

$$\cancel{m}g = \frac{Gm_1\cancel{m_2}}{r^2}$$

$$g = \frac{6.67\times10^{-11}\times6{\cdot}0\times10^{24}}{(6{\cdot}4\times10^{6})^2}$$

$$g = 9.77\,Nkg^{-1}$$

Example 2:

(a) Calculate the gravitational field strength on the surface of the Moon.

(b) Calculate the gravitational force between the Moon and the Earth.

(a)

$$F = \frac{Gm_1m_2}{r^2}$$

but

$$mg = \frac{Gm_1m_2}{r^2}$$

$$\cancel{m}g = \frac{Gm_1\cancel{m_2}}{r^2}$$

$$g = \frac{6.67\times10^{-11}\times7{\cdot}3\times10^{6}}{(1{\cdot}7\times10^{6})^2}$$

$$g = 1.68Nkg^{-1}$$

(b)

$$F = \frac{Gm_1m_2}{r^2}$$

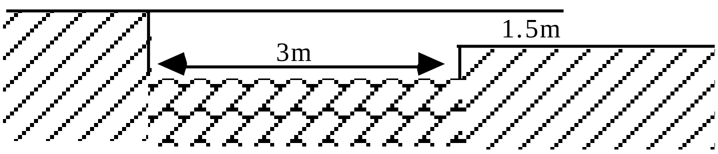
$$F = \frac{6.67\times10^{-11}\times6{\cdot}0\times10^{24}\ \times7{\cdot}3\times10^{6}}{(3.84\times10^{8})^2}$$

$$F = 1.98\times10^{4}\,N$$

EQUATIONS OF MOTION / TUTORIAL 4

1)
   a) A child's ball rolls down a pavement and passes a mark at 1.5 ms^{-1}. If its velocity is 6.6 ms^{-1} three seconds later, what is its acceleration?
   b) The same ball rolls round a corner at 7ms^{-1} onto a new slope which gives it an acceleration of 2 ms^{-2}. How long does it take for its velocity to rise to 10ms^{-1}?
2) A water chute has a constant slope which gives the sliders an acceleration of 4 ms^{-2}. If a girl jumps on the chute at a velocity of 3 ms^{-1}, how far will she slide in 5s?
3) An air-gun pellet has an initial velocity of zero and a muzzle velocity of 225 ms^{-1}. If the barrel of the rifle is 0.5m long, what is the pellet's acceleration?
4) A football player catches up on a ball rolling at 3 ms^{-1} and kicks it ahead along the same line with an acceleration of 200 ms^{-2} while his foot is in contact with it. If his boot moves with the ball over a distance of 13.75cm, how fast is the ball moving after the kick?
5) A ball is thrown upwards out of a window at 15 ms^{-1} and strikes the ground with a velocity of 30 ms^{-1}. How high is the window?
6) A balloon is released with an initial upwards velocity of 1 ms^{-1} and accelerates at 2 ms^{-2} for the first 12s of its climb.
   (a) What is its average velocity during these 12s?
   (b) What is its displacement during this time?
7) A golf ball is driven from a tee and reaches a velocity of 50 ms^{-1} in a time of 3ms.
   a) What is the ball's acceleration?
   b) How far does the ball travel during the first 3s after it is struck assuming there is no air resistance during its flight?
8) A stone is thrown horizontally off a cliff and lands 2s later 45m out from the spot vertically below the thrower.
   a) How high is the cliff?
   b) What is the stone's initial velocity?
9) 9. A meteroid strikes the Moon's surface and the impact ejects a 5kg rock with a velocity of 200 ms^{-1} at 60^{o} to the horizontal. If the rock lands 21.6km away, what is the gravitational acceleration at the Moon's surface?
10) A person holds a golf ball at arm's length so that the ball is 2m above the ground. How long does it take the reach the ground if he drops it?

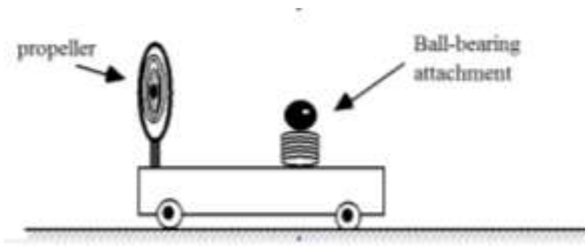
11) A mountain biker comes down a hill and then shoots horizontally off a riverbank at 5.5 ms^{-1}. The far bank is 1.5m vertically below his starting height and 3m away horizontally:



Does he make it across?

12) A fisher is standing on a bridge reeling in a hooked fish, which is rising vertically at 1.5 ms^{-1} when it manages to wriggle off. If the fish hits the water 1.2s later, how far above the surface was it when it escaped?

13) A trolley is fitted with a mechanism which fires a ball bearing vertically upwards 5 seconds after the trolley is released. The trolley itself is fitted with a propeller that accelerates it along a horizontal track:

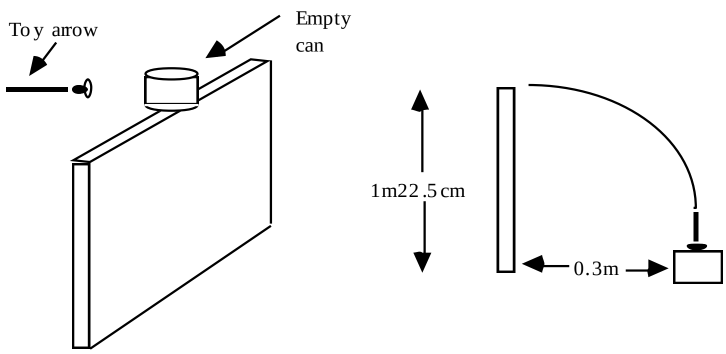


(a) If the ball-bearing is released when the vehicle reaches a velocity of 2.5ms^{-1}, calculate how far the trolley has moved along the track to this point.

(b) If the ball-bearing starts with a vertical velocity of 4.8ms^{-1}, find the maximum height it reaches.

(c) How far from the trolley does the ball-bearing land assuming that the trolley keeps at the same acceleration?

14)



Show that the horizontal velocity of the can is 0.6ms^{-1} immediately after being struck by the toy arrow.

15) A train moving at 40kmh^{-1} has an acceleration of 1.5ms^{-2}.
   a) What is its speed after 6s?
   b) What is its speed after travelling 100m?

16) A lucky 20p is dropped down a wishing well 80m deep. How long does it take to reach the bottom? (Ignore any effects of air resistance on the coin.)

17) How long does a stone take to reach the ground from the top of a 100m high building if,
   a) it is thrown vertically downwards at 5ms^{-1}?
   b) it is thrown vertically upwards at 5ms^{-1}?

GRAVITY AND MASS/ TUTORIAL 5

In the following questions, when required, use the following data:

Gravitational constant = $6{\cdot}67 \times 10^{-11}$ N m^2 kg^{-2}

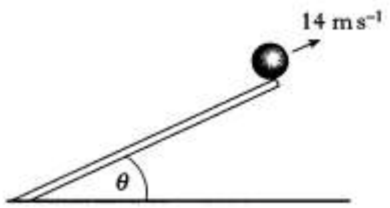
1) State the inverse square law of gravitation.
2) Show that the force of attraction between two large ships, each of mass $5{\cdot}00 \times 10^7$ kg and separated by a distance of 20 m, is 417 N.

3) Calculate the gravitational force between two cars parked 0·50 m apart. The mass of each car is 1000 kg.
4) In a hydrogen atom an electron orbits a proton in a circle with a radius of $5{\cdot}30 \times 10^{-11}$ m. The mass of an electron is $9{\cdot}11 \times 10^{-31}$ kg and the mass of a proton is $1{\cdot}67 \times 10^{-27}$ kg.
   Calculate the gravitational force of attraction between the proton and the electron in a hydrogen atom.
5) The distance between the Earth and the Sun is $1{\cdot}50 \times 10^{11}$ m. The mass of the Earth is $5{\cdot}98 \times 10^{24}$ kg and the mass of the Sun is $1{\cdot}99 \times 10^{30}$ kg. Calculate the gravitational force between the Earth and the Sun.
6) Two protons exert a gravitational force of $1{\cdot}16 \times 10^{-35}$ N on each other.
   The mass of a proton is $1{\cdot}67 \times 10^{-27}$ kg. Calculate the distance separating the protons.

GRAVITATION EXAM QUESTIONS

For past paper questions please refer to the LA Homework Booklet, available at www.mrsphysics.co.uk/higher

1. A ball is rolled up a slope so that it is travelling at 14 m s^{-1} as it leaves the end of the slope.

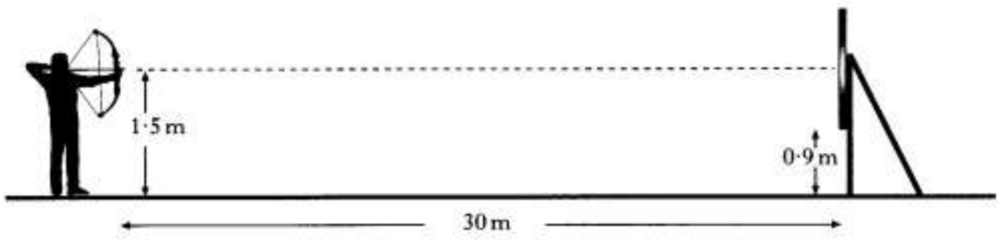


(a) The slope is set so that the angle to the horizontal, θ is 30°.

Calculate the vertical component of the velocity of the ball as it leaves the slope.

(b) The slope is now tilted so that the angle to the horizontal, θ, is increased. The ball is rolled so that it still leaves the end of the slope at 14 m s^{-1}. Describe and explain what happens to the maximum height reached by the ball. 2 **(3)**

5. An archer fires an arrow at a target which is 30 m away.

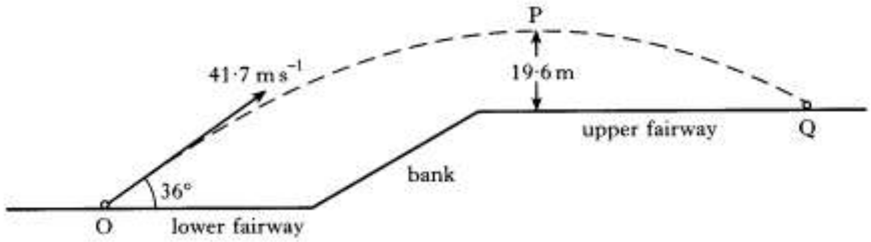


The arrow is fired horizontally from a height of 1·5 m and leaves the bow with a velocity of 100 m s^{-1}.

The bottom of the target is 0·90 m above the ground.

Show by calculation that the arrow hits the target.

2. The fairway on a golf course is in two horizontal parts separated by a steep bank as shown below.



A golf ball at point O is given an initial velocity of 41·7 m s^{-1} at 36° to the horizontal.

The ball reaches a maximum vertical height at point P above the upper fairway. Point P is 19·6 m above the upper fairway as shown. The ball hits the ground at point Q.

The effect of air resistance on the ball may be neglected.

(a) Calculate:

(i) the horizontal component of the initial velocity of the ball;

(ii) the vertical component of the initial velocity of the ball.

(b) Show that the time taken for the ball to travel from point O to point Q is 4·5 s. 3

(c) Calculate the horizontal distance travelled by the ball.

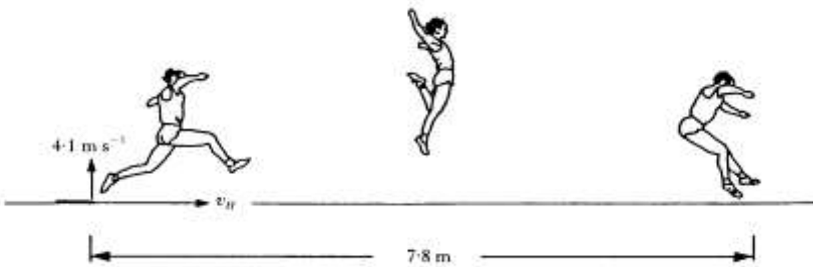
3. (a) A long jumper devises a method for estimating the horizontal component of his velocity during a jump.

His method involves first finding out how high he can jump vertically.

He finds that the maximum height he can jump is 0·86 m.

(i) Show that his initial vertical velocity is 4·1 m s^{-1}. 2

(ii) He now assumes that when he is long jumping, the initial vertical component of his velocity at take-off is 4·1 m s^{-1}.



The length of his long jump is 7·8 m.

Calculate the value that he should obtain for the horizontal component of his velocity, v_H.

(b) His coach tells him that, during his 7·8 m jump, his maximum height above the ground was less than 0·86 m. Ignoring air resistance, state whether his actual horizontal component of velocity was greater or less than the value calculated in part (a) (ii). You must justify your answer.

SECTION 4: GRAVITATION ANSWERS

PROJECTILES

1. (a) 7·8 s
   (b) 2730 m
2. (a) 5·0 s
   (b) 123 m
3. (b) 24·7 m s^{-1} at an angle of 52.6º below the horizontal
4. (a) v_{horiz} = 5·1 m s^{-1}, v_{vert} = 14·1 m s^{-1}
5. (b) 50 m s^{-1} at 36.9º above the horizontal
   (c) 40 m s^{-1}
   (d) 45 m
   (e) 240 m
6. (a) 20 m s^{-1}
   (b) 20.4 m
   (c) 4·1 s
   (d) 142 m
7. (a) 8 s
   (b) 379 m
8. (a) 15·6 m s^{-1}
12. 2 s

GRAVITY AND MASS

1. $F = \dfrac{Gm_1m_2}{r^2}$
3. 2·67 × 10^{-4} N
4. 3·61 × 10^{-47} N
5. 3·53 × 10^{22} N
6. 4·00 × 10^{-15} m

CHAPTER 7: SPECIAL RELATIVITY

These notes are produced with material from EducationScotland, Dick Orr, and Arfur Dogfrey and lots of reading from other sources. Note most books transcribe the terms t and t′ which appears to give a different equation. It is the same equation but the terms are reversed. Always use the formula given in the SQA Relationship sheet.

CHAPTER 7: SPECIAL RELATIVITY

5 SPECIAL RELATIVITY

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}} \qquad l' = l\sqrt{1-\frac{v^2}{c^2}}$$

a) The speed of light in a vacuum is the same for all observers,
b) The constancy of the speed of light led Einstein to postulate that measurements of space and time for a moving observer are changed relative to those for a stationary observer.
c) Length contraction and time dilation.

SUGGESTED ACTIVITIES

- ✓ Galilean invariance, Newtonian relativity and the concept of absolute space. Newtonian relativity can be experienced in an intuitive way. Examples include walking in a moving train and moving sound sources.
- ✓ At high speeds, non-intuitive relativistic effects are observed. Length contraction and time dilation can be studied using suitable animations. Experimental verification includes muon detection at the surface of the Earth and accurate time measurements on airborne clocks. The time dilation equation can be derived from the geometrical consideration of a light beam moving relative to a stationary observer.

SPECIAL RELATIVITY

The Dummies Guide to Special Relativity : **http://conduit9sr.tripod.com/**

Before we start this part of the course, please accept that the Newtonian Mechanics you have been taught so far is true – up to a point. The following material takes the subject a bit further…….

Many people know that Einstein was renowned for his theories on Relativity but for some people that is far as their knowledge goes.

Useful Definitions and ideas

inertial reference frames	Simply two places that are moving at constant speeds relative to one another.
absolute reference frame	A unique, universal frame of reference from which everything could be defined or measured against. Einstein's theories prove no such reference frame exists.
the ether (aether)	Early theories suggested that electromagnetic waves (light) required a medium: (a space-filling substance or field) to travel through. This ether was believed to be an absolute reference frame. Modern theories have no requirement for this idea, and indeed the Michelson–Morley experiment performed in 1887 provided no evidence for such a field.
Time dilation	A difference in a time interval as measured by a stationary observer and a moving observer.
Length contraction	A difference in a length along the moving axis as measured by a stationary observer and a moving observer.

https://bambinidisatana.com/wp-content/uploads/2013/10/Galileo-Galilei-z3.jpg?6746a2

Galilean Invariance

Galileo was one of the first scientists to consider the idea of relativity.

He stated that the laws of Physics should be the same in all **inertial frames of reference**.

He first described this principle in 1632 using the example of a ship, travelling at **constant velocity**, without rocking, on a smooth sea; any **observer** doing experiments below the deck would not be able to tell whether the ship was **moving** or **stationary**.

In other words, the **laws of Physics** are the same whether **moving at constant speed** or at **rest**.

http://www.biografiasyvidas.com/biografia/n/fotos/newton.jpg

Newtonian Relativity

Newton followed this up by expanding on Galileo's ideas.

He introduced the idea of **absolute** or **universal space time**.

He believed that it was the same time at all points in the **universe** as it was on **Earth**, not an unreasonable assumption.

According to Newton, **absolute time** exists independently of any perceiver and progresses at a consistent pace throughout the universe. **Absolute space**, in its own nature, without regard to anything external, remains always **similar** and **immovable**.

Einstein wrote two papers on relativity. His theory on Special Relativity was published in 1905 in a paper titled, "On the Electrodynamics of Moving Bodies" by Albert Einstein. **Special Relativity** deals with the concepts of **space and time as viewed by observers moving relative to one another with uniform velocities**. It is called "Special" as it deals with only one situation and that is one where objects are not accelerating, so are continuing at constant velocity. In 1916 Einstein published another paper, on General Relativity. **The Theory of General Relativity is the one which redefined the laws of gravity. The Theory of General Relativity also says that large objects cause outer space to bend in the same way a marble laid onto a large thin sheet of rubber would cause the rubber to bend**. The larger the object, the further space bends. Just like a bowling ball would make the rubber sheet bend much more than the marble would.

If you want more information refer to the other material in the Our Dynamic Universe Folder. Don't give up too early, try to get to grips with one of the major works of the twentieth century. However, do be aware that the literature is not consistent on the choice of definition of terms so that you can find various forms of the equations (I know I found them) use intuition as your best form of defence!

NOTATION

t time interval measured in the **same** frame of reference as the event (e.g. the pulse of light, throwing up a ball, running a race)

t' time interval measured in a frame of reference moving relative to the one containing the event(e.g. on a train)

l length measured in the **same** frame of reference as the object (e.g. rod)

l' length measured in a frame of reference moving relative to the one containing the object (e.g. rod)

v relative velocity of the two frames of reference

(*Note*: Recall that no one frame of reference is any more 'stationary' or 'moving' than any other. There is no 'absolute rest'.)

It is Einstein's theory of Special Relativity that we need to introduce in this course.

SQA CfE Higher Physics	SQA CfE Advanced Higher Physics
SPECIAL RELATIVITY	**GENERAL RELATIVITY**
Uniform Motion Only	**Any motion –e.g. accelerate,curve etc**

Before we can go deeply into the idea of Special Relativity we must introduce the concept of Frames of Reference.

FRAMES OF REFERENCE

How we see things in the world is determined by our position or viewing point. We call this our frame of reference. Relativity is all about observing events and measuring physical quantities, such as distance and time, from different reference frames. Frames of reference refer to any laboratory, vehicle, platform, spaceship, planet etc. and often have a relative velocity to another 'frame of reference'. These are called *inertial frames of reference* when the movement is in a *straight line with a constant velocity*.

If you ask most people to explain what the term "motion" means, they will probably say something about movement, transport, travel, and other similar words that convey the idea of movement from one point to another. However, most people would omit to mention that all motion is **relative**. That is, motion can only be judged by comparing the position of the moving object with some other reference point or object.

The following figure illustrates this.

1. Car moves relative to the road
2. Person in car moves relative to the car
3. Car moves relative to the house
4. Person in the house moves relative to the house

One can go further - the person in the car moves relative to the road, the car moves relative to the house, and so on.

In the language of physics, the reference against which the motion of an object is measured is called a "frame of reference". Thus, in the figure above, for example, the road is a frame of reference against which the motion of the car can be measured. Alternatively, the house could be a frame of reference against which the motion of the car is measured.

We have seen that there is a special frame of reference, referred to as an "inertial" frame of reference. **An inertial frame of reference is one in which Newton's first law of motion holds, i.e you are travelling at constant speed or are at rest.**

An example will illustrate this. Imagine sitting in your stationary car on a flat road, with the gears in neutral. The car remains stationary, as there are no unbalanced forces acting on the car. Now put the car in gear and floor the accelerator. Forces from the engine act on the car and it picks up speed. Now if you were holding a cup of coffee at this point you are likely to spill this down your top. You will experience

Newton's Laws of Motion. Remember this as a memory aid. If you'd spill your coffee you are not in an inertial reference frame. As the car continues to pick up speed, wind resistance increases on the car, and eventually the car will reach its maximum speed. At this point, the force from the engine is exactly balanced by the frictional and wind resistance forces, and hence the net force on the car is zero. The speed remains constant, at maximum speed, in accordance with Newton's first law.

In this example, the road (or surface of the earth), against which the motion of the car is measured, is an inertial frame of reference. Frames of reference need not be stationary. It can be shown that if a stationary frame of reference is inertial, then it is also inertial if it moves at constant speed. Thus, in the figure above, the car is an inertial frame of reference whether it is stationary or moving relative to the road.

Here is another example, you probably feel that you are currently stationary, but remember that the Earth rotates on its axis once a day so at the equator a person would be moving at 1670 kmh^{-1}. The Earth is also moving with respect to the Sun, orbiting the Sun once a year requires the Earth to be moving at approximately 100 000 kmh^{-1}, with respect to the Sun. We are in the outer spiral of the Milky Way galaxy, so from an inertial reference frame in another galaxy we would observe someone on the Earth as travelling 800 000 kmh^{-1}. Which speed is correct? They are all correct- it depends on your frame of reference

However, any accelerating frame of reference is not inertial. Thus, the car in the figure above, if it is accelerating, is not an inertial frame of reference. Remember that if you spill your coffee you are not in an inertial frame of reference.

Note that without a frame of reference, it would not be possible to judge the state of motion of an object moving at constant speed. Newton believed that there was one overall, preferred, reference frame, but Einstein disagreed.

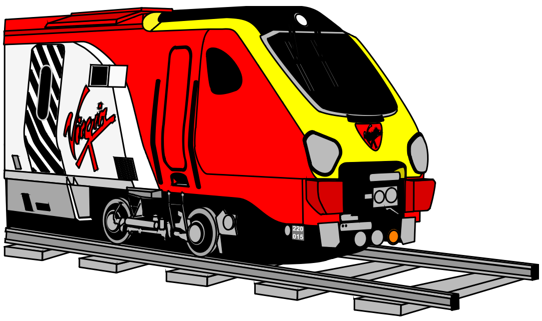
In the famous "Principia Mathematica" Isaac Newton stated.

> "Absolute space, in its own nature, without relation to anything external, remains always similar and immovable."

Newton said all other frames that were moving at constant velocity relative to this frame (and hence to each other) acted just as this "absolute" frame did. He never explained how to determine the absolute inertial reference frame. Thus his definition was useful in deriving his theory but was not an operational definition.

The point of all this is that Einstein asserted that there is no "preferred" inertial frame of reference against which motion should be measured. In other words, all inertial frames of reference are **equivalent**. It does not matter which inertial frame of reference is used to judge the motion of an object, in our example either the road or the house are acceptable frames of reference against which to judge the motion of the car.

Here is another example of the same event seen by three different observers, each in their own frame of reference:



Event 1: You are reading your Kindle on the train. The train is travelling at 60 mph.

Observer	Location	Observation
1	Passenger sitting next to you	You are stationary
2	Person standing on the platform	You are travelling towards them at 60 mph
3	Passenger on train travelling at 60 mph in opposite direction	You are travelling towards them at 120 mph.

This example works well as it only involves objects travelling at relatively low speeds. The comparison between reference frames does not work in the same way, however, if objects are moving close to the speed of light.

Event 2: You are reading your Kindle on an interstellar train. The train is travelling at 2×10^8 m s^{-1}.

Observer	Location	Observation
1	Passenger sitting next to you	You are stationary
2	Person standing on the platform	You are travelling towards them at 2×10^8 m s^{-1}
3	Passenger on train travelling at 2×10^8 m s^{-1} in opposite direction	*You are travelling towards them at 4×10^8 m s^{-1}* ✗

The observation made by observer three is impossible as an object cannot travel faster than the speed of light from any reference frame and it would certainly be impossible to *watch* something travel faster than light.

POSTULATES OF SPECIAL RELATIVITY

Einstein took this one stage further, and came up with his first postulate of special relativity, which generalises our discussion to ALL of the laws of physics:

Postulate 1:

All the laws of physics are the same in all inertial frames of reference.

We can give a simple example of this principle in action. Consider a person standing at the road side and a person sitting in a car moving at constant speed. This postulate states that both people must observe the same laws of physics for all phenomena. If the car is accelerating, then it does not constitute an inertial frame of reference, and the person on the ground and in the car need not observe the same laws of physics.

The second postulate is, in a sense, a corollary (consequence) of the first postulate:

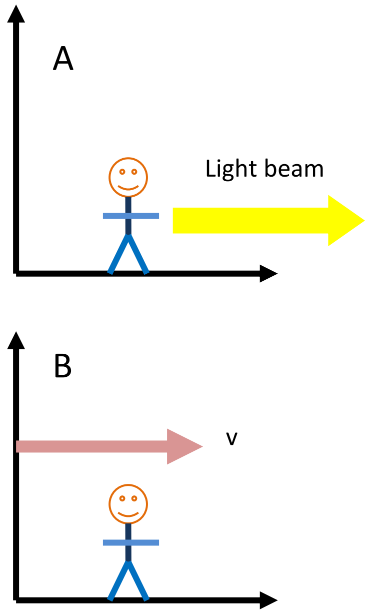
Postulate 2:

The speed of light (in a vacuum) is the same in all inertial frames of reference.

This postulate leads to a surprising difficulty when we consider what we expect to happen in everyday life. Suppose we are driving along at 50 mph, approaching a person on the pavement. At the instant we pass the person, he throws a ball at 50 mph, relative to him and in the direction we and the car are travelling. In the frame of reference of the car, therefore, the ball is stationary.

The second postulate says that light does not behave in this way. Suppose that, instead of throwing a ball, the person on the pavement switches on a torch. The second postulate says that both we and him have to measure the same speed of light from the torch - no matter how fast we are travelling in the car.

Our consideration of throwing the ball suggests that if the speed of light is c, and our speed is v then the person on the pavement sees light travelling at speed c, but we should see it travelling at speed $c - v$. The second postulate says that this is **not** the case. The second postulate says that we see the light travelling at speed c.



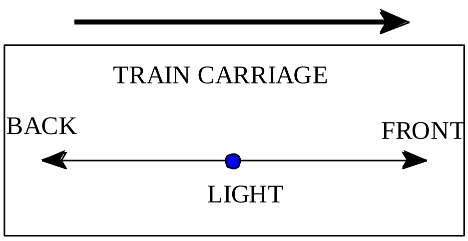
The figure (left) illustrates this, but in the conventional notation used to represent frames of reference, in the context of relativity. In this example, frame A is the pavement, and frame B is the car.

Person in frame A fires a light beam (e.g. switching on a torch) at the instant that frame B passes by. The second postulate implies that BOTH observers would measure the same speed for the light beam!

Einstein showed:

1. everyone sees the same speed for light (given the symbol *c*)
2. what happens when you try to go faster than c.

To understand this, our concepts of time, length, and mass must be changed. Consider two observers viewing the same event. One observer is stationary and one moving. If *c* is the same for both observers then they must experience different distances and times. Two events can be simultaneous in one frame of reference but when viewed from another frame of reference the events are not simultaneous. This is called the *Relativity of Simultaneity*.



Imagine a train travelling with both its front and back doors open. A light is uncovered from under a dark casing, causing a beam of light to be emitted. The beam is emitted from the centre of a train carriage. A person on the train would see the beam of light arrive at the end walls of the train at the same time. But a person viewing the same event on the railway banking would see the beam of light exit from the back door before the front door because the train is moving forward. If the person on the train walks from the middle of the carriage to the front door he has travelled half a carriage relative to the train. Relative to the bank the person has walked further.

These ideas seem very contrary to classical Newtonian Mechanics which you have dealt with so far.

Newtonian Mechanics states that:

1. the time interval between events is independent of the observer.
2. the space interval is independent of the observer.

Newton	**Einstein**
d and t constant	c constant
c relative	d and t relative

THE PRINCIPLES OF RELATIVITY

To summarise

Special relativity (which applies to observers/reference frames in relative motion with constant velocity) has two postulates:

1. The laws of physics are the same for all observers in all parts of the universe. ***All the laws of physics are the same in all inertial frames of reference.***

2. Light always travels at the same speed in a vacuum, 3.0×10^{8} m s^{-1} (299,792,458 m s^{-1} to be more precise). (Light does slow down inside transparent material such as glass.)

This means that no matter how fast you go, you can never catch up with a beam of light, since it always travels at $3{\cdot}0 \times 10^{8}$ m s^{-1} *relative to you*.

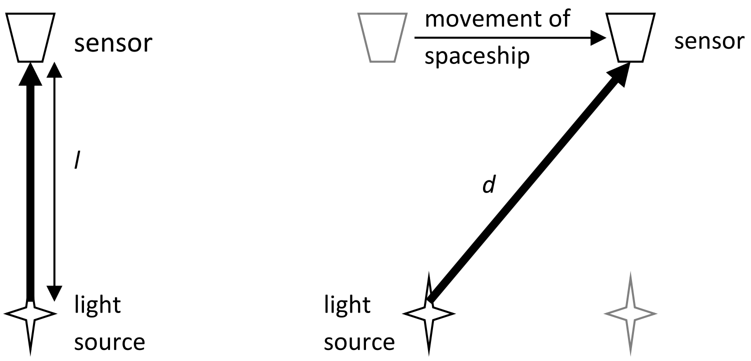
The best known experimental evidence that started the proof for the Theory of special relativity is the Michelson-Morley interferometer experiment. It actually failed to show the expected results, despite many people repeating the experiment. (*Read more about this experiment later in this chapter*).

Example: A "space car" is travelling through space at 90% of the speed of light (2.7×10^{8} m s^{-1}) with its headlights on. The occupants of the car will see the beams of the headlights travel away from them at 3×10^{8} m s^{-1}, so "common sense" or Newtonian Mechanics would say that the light from the headlights is travelling at 5.7×10^{8} m s^{-1}. But it is not, it is travelling at 3×10^{8} m s^{-1}.

An observer on Earth will also observe light from the headlamps travel at 3×10^{8} m s^{-1}.The speed of light, *c*, is constant in and between all reference frames and for all observers.

These principles have strange consequences for the measurement of distance and time between reference frames.

TIME DILATION



Your point of view, where the source and sensor are moving along together

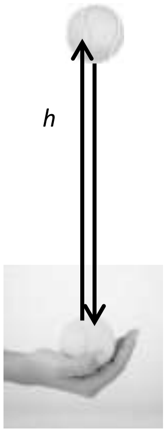
Your stationary colleague's point of view, where the sensor moves after the light is fired

A very simple thought experiment shows that one consequence of the speed of light being the same for all observers is that time experienced by all observers is not necessarily the same. There is no universal clock that we can all refer to – we can simply make measurements of time as we experience it.

Imagine you are on a spaceship travelling at constant speed (v) relative to a colleague on a space station. You are investigating a timing device based on the fact that the speed of light is constant. You fire a beam of light at a sensor and time how long it takes to arrive. From your point of view the light has less distance (l) to travel than from the point of view of your colleague on the space station (d). If you both observe the speed of light as the same then you cannot agree on when it arrives, i.e. both of you experience time in a different way.

Time is different for observers in different reference frames because the path they observe for a moving object is different.

Example 1



Event: Inside a moving train carriage, a tennis ball is thrown straight up and caught in the same hand.

Observer 1, standing in train carriage, throws tennis ball straight up and catches it in the same hand.

In Observer 1's reference frame they are stationary and the ball has gone straight up and down.

Observer 1 sees the ball travel a total distance of $2h$.

The ball is travelling at a speed *s*.

The period of time for the ball to return to the observers hand is:

$$t = \frac{2h}{v_1}$$

Observer 2, standing on the platform watches the train go past at a speed, v_2, and sees the passenger throw the ball. However, to them, the passenger is also travelling horizontally, at speed v. This means that, to Observer 2, the tennis ball has travelled a horizontal distance, as well as a vertical one.

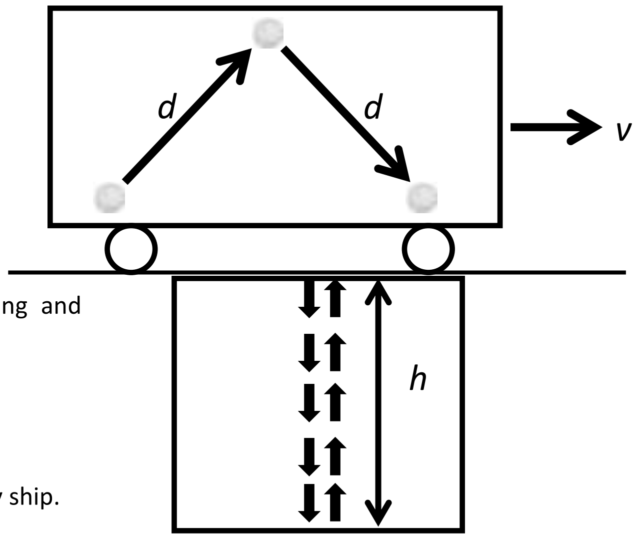
Observer 2 sees the ball travel a total distance of $2d$.

The period of time for the ball to return to the observers hand is: $t' = \frac{2d}{v_1}$

For observer 2, the ball has travelled a greater distance, in the same time.

Example 2:

Event: You are in a spaceship travelling to the left at speed. Inside the spaceship cabin, a pulsed laser beam is pointed vertically up at the ceiling and is reflected back down. The laser emits another pulse when the reflected pulse is detected by a photodiode.

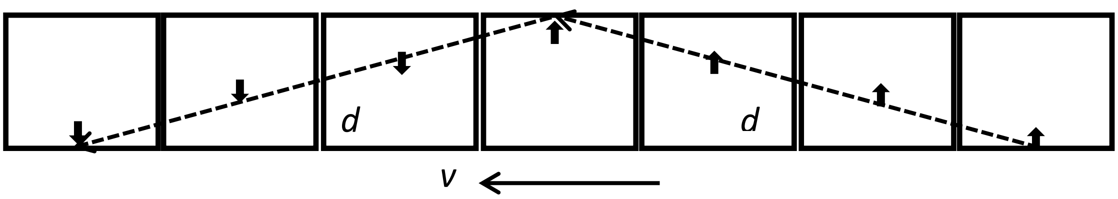


Reference frame 1: You, inside the cabin.

The beam goes straight up, reflects off the ceiling and travels straight down.

Period of pulse: $t = \frac{2h}{c}$

Reference frame 2: Observer on another, stationary ship.



Period of pulse: $t' = \frac{2d}{c}$

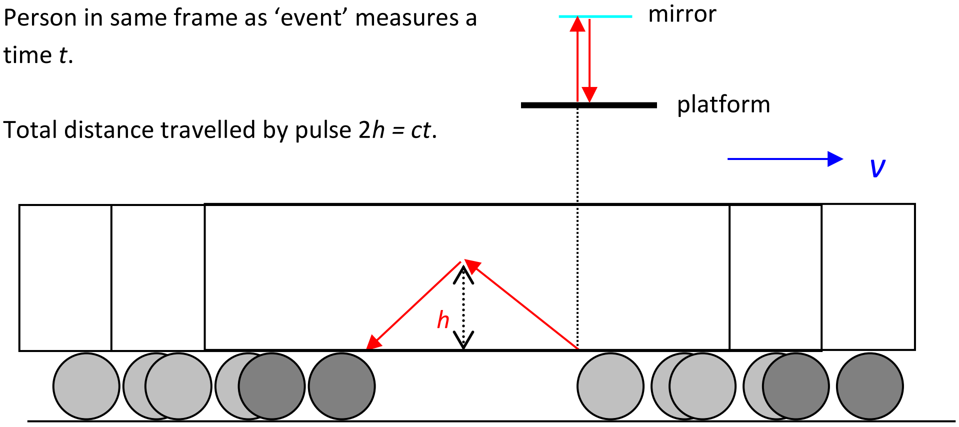
The time for the experiment as observed by the stationary ship (reference frame 2), *t'*, is greater than the time, *t*, observed by you when moving with the photodiode, i.e. what you might observe as taking 1 second could appear to take 2 seconds to your stationary colleague. Note that you would be unaware of any difference until you were able to meet up with your colleague again and compare your data.

ANOTHER THOUGHT EXPERIMENT

Consider a person on a platform who shines a laser pulse upwards, reflecting the light off a mirror. The time interval for the pulse to travel up and down is *t* (no superscript).

Person in same frame as 'event' measures a time *t*.

Total distance travelled by pulse $2h = ct$.



Travellers in this different frame of reference observe the 'event' (eg out of the window of the train), which takes place in the platform frame of reference and measure a time *t'*.

A different frame of reference, for example a train moving along the x-axis at high speed v, passes. From the point of view of travellers on board the train, the light travels as shown in the diagram above.

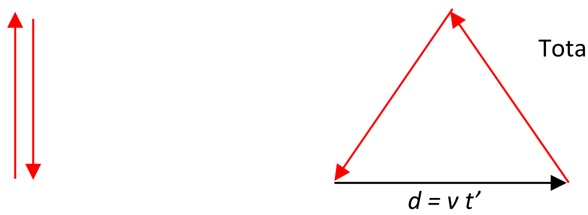
The time taken for the light to travel up and back, as measured by travellers in this frame, is t' (t prime).

In the time t' that it takes for the light to travel up and back down the train in this frame, the train has travelled a distance d.

Both observers measure the *same* speed for the speed of light.

Platform frame of reference

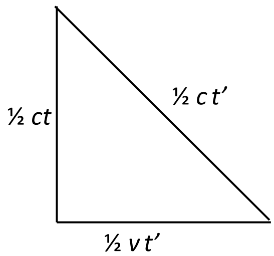
Train frame of reference



Total distance travelled by pulse= *ct'*

Horizontal distance travelled by train = *vt'*

A right- angled triangle can be formed where the vertical side is the height, *h*, of the pulse (½ *ct*), the horizontal side is half of the distance, *d*, gone by the train (½ *vt'*) and the hypotenuse is half the distance gone by the pulse as seen by the travellers on the train (½*ct'*).



Applying Pythagoras to the triangle gives:

$$(\tfrac{1}{2}ct')^2 = (\tfrac{1}{2}ct)^2 + (\tfrac{1}{2}vt')^2$$

$$(ct')^2 = (ct)^2 + (vt')^2$$

$$(c^2 - v^2)\,t'^2 = c^2 t^2$$

$$\left(1 - \frac{v^2}{c^2}\right) t'^2 = t^2$$

$$t'\sqrt{1-(v/c)^2} = t \qquad (1)$$

$$t' = \frac{t}{\sqrt{1-\dfrac{v^2}{c^2}}} \qquad (2)$$

Where

t time interval measured in the **same** frame of reference as the event (e.g. the pulse of light, throwing up a ball, running a race).

t' time interval measured in a frame of reference moving relative to the one containing the event(e.g. on a train).

v relative velocity of the two frames of reference.

WHAT ASSUMPTIONS HAVE WE MADE?

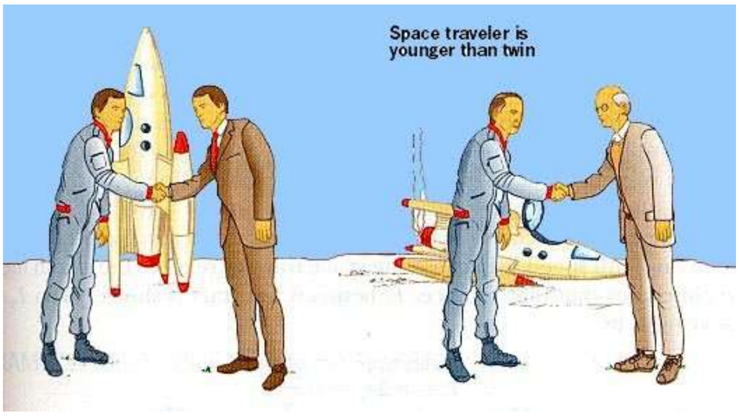
(i) The two frames are moving relative to each other along the *x*-axis, i.e. the train passes the platform. There is no bending or circular motion involved.

(ii) We require two travellers on the train since the start and finish places are separate. This is fine since two clocks can be synchronised in the same frame of reference.

This theory leads to some other interesting situations. Measured time will change for a moving system depending on who is observing the motion.

TIME DILATION AND THE LORENZ TRANSFORMATION

One famous situation leading from this theory is the thought experiment regarding the twin paradox. (which is not currently possible): At the age of 25 you leave your twin behind on Earth to go on a space mission. You are in a spaceship travelling at 90% of the speed of light and you go on a journey for 20 years as *you* measure it. When you get back you will find that 46 years have elapsed on Earth. Your clock will have run slower than those on Earth, but both your clock **and** the one on Earth are correct. You are 45, but your twin is 71, not just in terms of what the clock says but biologically as well. This is "real" time to both of you (even though it seems different).



Here is this paradox in more detail but with a slightly different scenario.. It is taken from Russell Stannard's excellent book "Relativity- a very short introduction" Oxford. (2008) ISBN 978–0–19–923622–0

Imagine an astronaut in a high-speed spacecraft and a mission controller on the ground. They both have identical clocks. The astronaut is to carry out a simple experiment. On the floor of the craft she is to fix a lamp which emits a pulse of light. The pulse travels directly upwards at right angles to the direction of motion of the craft (see Figure 1). There the pulse strikes a bullseye target fixed to the ceiling. Let us say that the height of the craft is 4 metres. With the light travelling at speed, c she finds that the time taken for this trip, t, as measured on her clock, is given by t= 4/c. (From t=d/v)

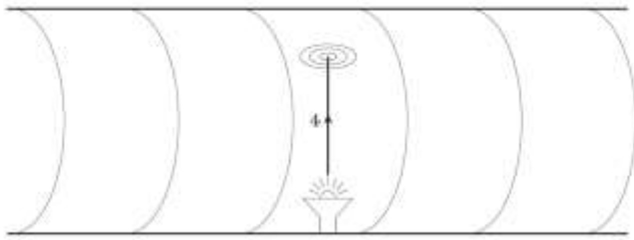


Figure 1

Now let's see what this looks like from the perspective of the mission controller. As the craft passes him overhead, he too observes the trip performed by the light pulse from the source to the target.

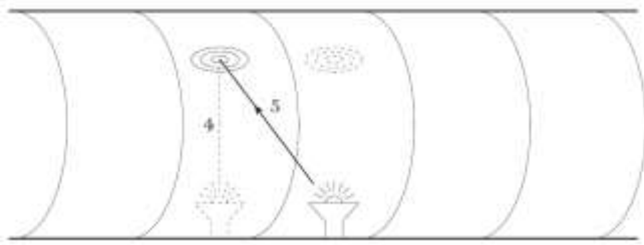


Figure 2

According to his perspective, during the time taken for the pulse to arrive at the target, the target will have moved forward from where it was when the pulse was emitted. For him, The astronaut arranges for a pulse of light to be directed towards a target such that the light travels at right angles to the direction of motion of the spacecraft According to the mission controller on earth, as the spacecraft passes overhead, the target moves forward in the time it takes for the light pulse to perform its journey. The pulse, therefore, has to traverse a diagonal path the path is not vertical; it slopes (see Figure 2). The length of this sloping path will clearly be longer than it was from the astronaut's point of view. Let us say that the craft moves forward 3 metres in the time that it takes for the light pulse to travel from the source to the target. Using Pythagoras' theorem, where $3^2 + 4^2 = 5^2$*, we see that the distance travelled by the pulse to get to the target is, according to the controller, 5 metres.*

So what does he find for the time taken for the pulse to perform the trip? Clearly it is the distance travelled, 5 metres, divided by the speed at which he sees the light travelling. This we have established is c (the same as it was for the astronaut). Thus, for the controller, the time taken, t, registered on his clock, is given by t = 5/c.

But this is not the time the astronaut found. She measured the time to be t= 4/c. So, they disagree as to how long it took the pulse to perform the trip. According to the controller, the reading on the astronaut's clock is too low; her clock is going slower than his. And it is not just the clock. Everything going on in the spacecraft is slowed down in the same ratio. If this were not so, the astronaut would be able to note that her clock was going slow (compared, say, to her heart beat rate, or the time taken to boil a kettle, etc.). And that in turn would allow her to deduce that she was moving – her speed somehow affecting the mechanism of the clock. But that is not allowed by the principle of relativity. All uniform motion is relative. Life for the astronaut *must proceed in exactly the same way as it does for the mission controller. Thus we conclude that everything happening in the spacecraft – the clock, the workings of the electronics, the astronaut's ageing processes, her thinking processes – all are slowed down in the same ratio. When she observes her slow clock with her slow brain, nothing will seem amiss. Indeed, as far as she is concerned, everything inside the craft keeps in step and appears normal. It is only according to the controller that everything in the craft is slowed down. This is time dilation. The astronaut has her time; the controller has his. They are not the same. In that example we took a specific case, one in which the astronaut and spacecraft travel 3 metres in the time it takes light to travel the 5 metres from the source to the target. In other words, the craft is travelling at a speed of 3/5c, i.e. 0.67c. And for that particular speed we found that the astronaut's time was slowed down by a factor 4/5, i.e. 0.8. It is easy enough to obtain a formula for any chosen speed, v..We apply Pythagoras' theorem to triangle ABC. The distances are as shown in Figure 4. Thus:*

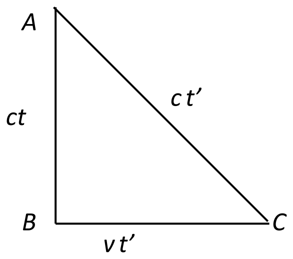
$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 - BC^2$

$c^2t^2 = (c^2 - v^2)t'^2 \qquad (1)$

$t^2 = (1 - v^2/c^2)t'^2$

$t = t' \sqrt{(1 - v^2/c^2)}$



From this formula we see that if v is small compared to c, the expression under the square root sign approximates to one, and t≈ t. Yet no matter how small v becomes, the dilation effect is still there. This means that, strictly speaking, whenever we undertake a journey – say, a bus trip – on alighting we ought to re-adjust our watch to get it back into synchronization with all the stationary clocks and watches. The reason we do not is that the effect is so small. For instance, someone opting to drive express trains all their working life will get out of step with those following sedentary jobs by no more than about one-millionth of a second by the time they retire, hardly worth bothering about. At the other extreme, we see from the formula that, as v approaches c, the expression under the square root sign approaches zero, and tends to zero. In other words, time for the astronaut would effectively come to a standstill. This implies that if astronauts were capable of flying very close to the speed of light, they would hardly age at all and would, in effect, live for ever. The downside, of course, is that their brains would have almost come to a standstill, which in turn means they would be unaware of having discovered the secret of eternal youth.

So much for the theory! But is it true in practice? Emphatically, yes. In 1977, for instance, an experiment was carried out at the CERN laboratory in Geneva on subatomic particles called muons. These tiny particles are unstable, and after an average time of 2.2×10^{-6} seconds (i.e. 2.2 millionths of a second) they break up into smaller particles. They were made to travel repeatedly around a circular trajectory of about 14 metres diameter, at a speed of v = 0.9994c. The average lifetime of these moving muons was measured to be 29.3 times longer than that of stationary muons – exactly the result expected from the formula we have derived, to an experimental accuracy of 1 part in 2000. In a separate experiment carried out in 1971, the formula was checked out at aircraft speeds using identical atomic clocks, one carried in an aircraft, and the other on the ground. Again, good agreement with theory was found. These and innumerable other experiments all confirm the correctness of the time dilation formula.

http://alternativephysics.org/book/TimeDilationExperiments.htm

At first one might think that if her time is going slow, then when she observes what is happening on the ground, she will perceive time down there to be going fast. But wait. That cannot be right. If it were, then we would immediately be able to conclude who was actually moving and who was stationary. We would have established that the astronaut was the moving observer because her time was affected by the motion whereas the controller's wasn't. But that violates the principle of relativity i.e. that for inertial frames, all motion is relative. Thus, the principle leads us to the, admittedly somewhat uncomfortable, conclusion that if the controller concludes that the astronaut's clock is going slower than his, then she will conclude that his *clock is going slower than hers. But how, you might ask, is that possible? How can we have two clocks, both of which are lagging behind the other?!*

To address this problem we must first recognize that in the set-up we have described we are not comparing clocks directly side-by-side. Though the astronaut and controller might indeed have synchronized their two clocks as they were momentarily alongside each other at the start of the space trip, they cannot do the same for the subsequent reading; the spacecraft and its clock have flown off into the distance. The controller can only find out how the astronaut's clock is doing by waiting for some kind of signal (perhaps a light signal) to be emitted by her clock and received by himself. He then has to allow for the fact that it has taken time for that signal to travel from the craft's new location to himself at mission control. By adding that transmission time to the reading of the clock when it emitted the signal, he can then calculate what the time is on the other clock now, and compare it with the reading on his own. It is only then that he concludes that the astronaut's clock is running slow. But note this is the result of a calculation, not a direct visual comparison. And the same will be true for the astronaut. She arrives at her conclusion that it is the controller's clock that is running slow only on the basis of a calculation using a signal emitted by his clock .Which doubtless still leaves a nagging question in your mind, namely 'But whose clock is really going slow?' With the set-up we have described, that is a meaningless question. It has no answer.

As far as the mission controller is concerned, it is true that the astronaut's clock is the one going slow; as far as the astronaut is concerned, it is true that it is the mission controller's clock that is going slow. And we have to leave it at that.

Not that people have left it at that. Enter the famous twin paradox. This proposal recognizes that the seemingly contradictory conclusions arise because the times are being calculated. But what if the calculations could be replaced by direct side-by-side comparisons of the two clocks – at the end of the journey as well as at the beginning? That way there would be no ambiguity. What this would require is that the spacecraft, having travelled to, say, a distant planet, turns round and comes back home so that the two clocks can be compared directly. In the original formulation of the paradox it was envisaged that there were twins, one who underwent this return journey and the other who didn't. On the traveller's return one can't have both twins younger than each other, so which one really has now aged more than the other, or are they still both the same age?

The experimental answer is provided by the experiment we mentioned earlier involving the muons travelling repeatedly round the circular path. These muons are playing the part of the astronaut. They start out from a particular point in the laboratory, perform a circuit, and return to the starting point. And it is these travelling muons that age less than an equivalent bunch of muons that remain at a single location in the laboratory. So this demonstrated that it is the astronaut's clock which will be lagging behind the mission controller's when they are directly compared for the second time.

So does this mean that we have violated the principle of relativity and revealed which observer is really moving, and consequently which clock is really slowed down by that motion? No. And the reason for that is that the principle applies only to inertial observers. The astronaut was in an inertial frame of reference while cruising at steady speed to the distant planet, and again on the return journey while cruising at steady speed. But – and it is a big 'but' – in order to reverse the direction of the spacecraft at the turn-round point, the rockets had to be fired, loose objects lying on a table would have rolled off, the astronaut would be pressed into the seat, and so on. In other words, for the duration of the firing of the rockets, the craft was no longer an inertial reference frame; Newton's law of inertia did not apply. Only one observer remained in an inertial frame the whole time and that was the mission controller. Only the mission controller is justified in applying the time dilation formula throughout. So, if he concludes that the astronaut's clock runs slow, then that will be what one finds when the clocks are directly compared. Because of that period of acceleration undergone by the astronaut, the symmetry between the two observers is broken – and the paradox resolved. At least it is partially resolved. The astronaut knows that she has violated the condition of remaining in an inertial frame throughout, and so has to accept that she cannot automatically and blindly use the time dilation formula (in the way that the mission controller is justified in doing). But it still leaves her with a puzzle. During the steady cruise out, she is able, from her calculations, to conclude that the controller's clock *was going slower than her own. During the steady cruise home, she can conclude that the controller's clock will be losing even more time compared to her own (the time dilation effect not being dependent on the direction of motion – only on the moving clock's speed relative to the observer). That being so, how on earth (literally) did the mission controller's clock get ahead of her own? What was responsible for that? Is there any way the astronaut could calculate in advance that the controller's clock would be ahead of hers by the end of the return journey? The answer is yes; there is. But we shall have to reserve the complete resolution of the twin paradox for later – when we have had a chance to see what effect acceleration has on time.*

At velocities which we are more familiar with, a retired airline pilot who may have spent 35,000 hours travelling at, say, 180 m s^{-1} whilst in the air will be 23 μs younger than if they had stayed on the ground. Scott Kelly, a twin, who used to be 6 minutes younger than his older twin is now 6 minutes and 5 milliseconds younger, due to his 520 days on the ISS orbiting at 28,160 kmh^{-1} (17,500 mph). Even your calculators are unable to show this and you'd need to prove it using something like Microsoft Excel.

TIME DILATION FORMULA AND THE LORENTZ TRANSFORMATION

The formula linking these can be shown to be:

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

Note this is often written as:

$$t' = t\gamma$$

where γ is known as the Lorentz Factor. It is often used in the study of special relativity and is given by:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Example:

A rocket is travelling at a constant 2.7 × 10^{8} m s^{-1} compared to an observer on Earth. The pilot measures the journey as taking 240 minutes.

How long did the journey take when measured from Earth?

Solution:

t = 240 minutes

v = 2.7 × 10^{8} m s^{-1}

c = 3 × 10^{8} m s^{-1}

t' = ?

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{240}{\sqrt{1-\frac{(2\cdot 7\times 10^8)^2}{(3\times 10^8)^2}}} = 551\text{minutes}$$

An observer on Earth would measure the journey as taking 551 minutes, i.e. 551 minutes would have passed from their point of reference.

ANSWERING LORENZ FACTOR QUESTIONS

1) Make sure you know what the terms mean.

t time interval measured in the SAME frame of reference as the event (e.g. the pulse of light, throwing up a ball, running a race).

t' time interval measured in a frame of reference moving relative to the one containing the event(e.g. on a train)

l length measured in the SAME frame of reference as the object (e.g. rod)

l' length measured in a frame of reference moving relative to the one containing the object (e.g. rod).

v relative velocity of the two frames of reference.

(*Note*: Recall that no one frame of reference is any more 'stationary' or 'moving' than any other. There is no 'absolute rest'. We have chosen the platform to contain the 'event'.).

2) Think through the question and try to decide what you would expect to happen.

3) **Remember that** $\frac{v^2}{c^2}$ is the same as $\left(\frac{v}{c}\right)^2$

4) Note that the question can give the value of *v* in terms as a portion of *c.* e.g. the speed in the above example could have been given as 90% *c* or 0.9 times *c* or 9/10 of the speed of light. So using the question previously given we would calculate it as shown on the right.

$$t' = \frac{240}{\sqrt{1-0\cdot9^2}}$$

$$t' = \frac{240}{\sqrt{1-0\cdot81}}$$

$$t' = 551\text{minutes}$$

5) You might find it easier to square both sides which would give you the following equation

$$(t')^2 = \frac{(t)^2}{1-\frac{v^2}{c^2}}$$

6) Sometimes the ratio of t to t' can be given and again this would be calculated in the following way

$$\sqrt{1-\frac{v^2}{c^2}} = Ratio(t:t')$$

$$1-\frac{v^2}{c^2} = (Ratio(t:t'))^2$$

7) Sometimes the ratio of t' to t can be given and again this would be calculated in the following way

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = Ratio(t':t)$$

$$\frac{1}{1-\frac{v^2}{c^2}} = (Ratio(t':t))^2$$

8) Don't separate v^2 from c^2 until after you have dealt with the 1.

9) Remember that it is $1-\frac{v^2}{c^2}$ so if $\frac{v^2}{c^2}$ is taken to the other side it becomes +.

10) $1-\frac{v^2}{c^2}$ must give a value less than 1.

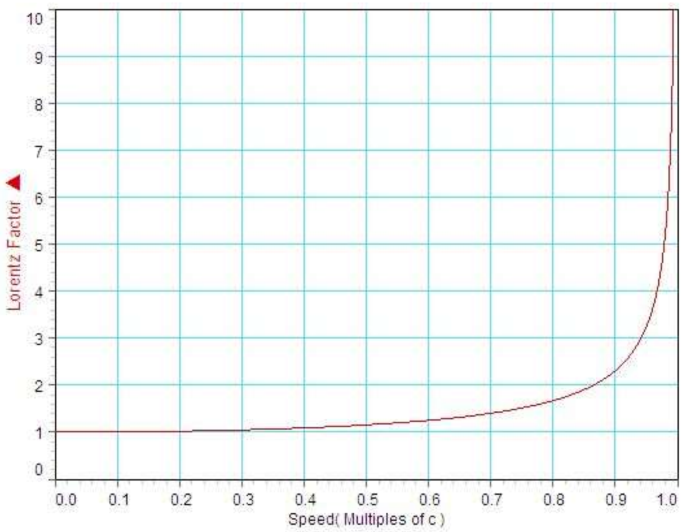
11) It is not necessary to put times into seconds, whatever the given units of time your calculated time quantity will have the same units as the equation works out ratios.

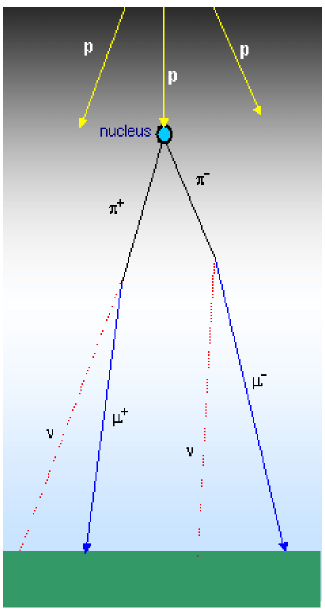
12) As we will see later the length transformation equation looks similar but is written in terms of t'=tγ compared with the time transformation written as l'=l/γ

13) Some people recommend working out the $\sqrt{1-\frac{v^2}{c^2}}$ bit first and then using t'=tγ or l'=l/γ, but remember that $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

WHY WE DO NOT NOTICE RELATIVISTIC TIME DIFFERENCES IN EVERYDAY LIFE?

A graph of the Lorentz factor versus speed (measured as a multiple of the speed of light) is shown below.





We can see that for small speeds (i.e. less than 0.1 times the speed of light) the Lorentz factor is approximately 1 and relativistic effects are negligibly small. Even 0.1 times the speed of light is 30,000,000 m s^{-1} or 108,000,000 km h^{-1} or about 67,500,000 mph – a tremendously fast speed compared to everyday life.

However, the speed of satellites is fast enough that even these small changes will add up over time and seriously affect the synchronisation of global positioning systems (GPS) and television satellites with users on the Earth. They have to be specially programmed to allow for the effects of special relativity (and also general relativity, which is not covered here). Very precise measurements of these small changes in time have been performed on fast-flying aircraft and agree with predicted results within experimental error.

Further evidence in support of special relativity comes from the field of particle physics and the detection of particles called muons at the surface of the Earth. Muons are created in the upper atmosphere by cosmic rays (high-energy protons from space).

The half-life of a muon is 2.2 μs and so moving at 0.994 c they would only expect to travel about 660m before half of them decayed. Muons formed at, say 12000 m would take 40 μs or about 20 half-lives to reach the ground. This would mean that only $1/2^{20}$ of the original number would be detected. The fact that the proportion reaching the ground is much higher than this means that the muons are living longer.

At 0.994c the formula for time dilation gives the half-life for the muons to be 20 μs. This means that at 0.994c the proportion reaching the ground should be 0.25, indicating the number of muons per second detected at the ground is much greater than expected. This is because the 'muon clock' runs slowly compared to the observer on Earth and the muon reaches the ground.

http://www.scivee.tv/node/2415

Example: Using the figures above, show, by calculation, why time dilation is necessary to explain the observation of muons at the surface of the Earth.

Solution:

t = 2.2 μs = 2.2×10^{-6} s

v = $0.994 \times 3.00 \times 10^{8}$ = 2.982×10^{8} m s^{-1}

d = ?

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{2.2\times10^{-6}}{\sqrt{1-(0.994)^2}}$$

$$t' = 20\mu\, s$$

$$d = vt$$
$$d = 2.982 \times 10^{8} \times 2.2 \times 10^{-6}$$
$$d = 656\,m$$

$$d' = vt'$$
$$d' = 2.982 \times 10^{8} \times 20 \times 10^{-6}$$
$$d' = 6.0 \times 10^{3}\,m = 6000\,m$$

In the reference frame of an observer on Earth the half-life of the muon is recorded as 20 μs during which it travels 6000 m. From this perspective, a period of only two half-lives is needed for the muons to reach the earth from 12000 m altitude.

It is useful to mention the significance of the term $\sqrt{1-(v/c)^2}$.

(The reciprocal $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is known as the gamma factor, or Lorenz Transformation.)

This term occurs in relativity equations and its size determines when relativity effects will be observed. At everyday speeds it is almost unity (one).

You should be able to *interpret* the final equation, stating what each of the symbols represent, and what the equation means in terms of the time interval for each observer.

Note: In our thought experiment above with the pulse emitted on the platform the laser pulse starts and finishes at the *same* place on the platform. Thus equation (1) is used to calculate the time interval *t'* registered in a frame of reference, eg the train, for an event which take place in a *different* frame of reference from the 'event'.

For example if $v = 0.4c$ then $(v/c)^2 = 0.16$ and $\sqrt{1-(v/c)^2} = \sqrt{1-0.16} = 0.917$.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{0.917} = 1.091$$

Let us use this value of $v = 0.4c$ in our thought experiment with a laser pulse time of 8.0 ns.

$$t' = t\gamma$$
$$t' = 8.0 \times 1.091$$
$$t' = 8.7ns$$

Thus, if $t = 8.0$ ns, we can calculate t', giving $t' = 8.7$ ns. A longer time interval is 'measured' by travellers on the train. This effect, known as *time dilation* (dilation = expanding), is a direct result of the postulate that the speed of light is measured to be the same by all observers.

Time dilation leads to observers being unable to agree about simultaneous events. Two events may appear to be simultaneous to one observer, but may *not* be simultaneous for others.

ANOTHER EXAMPLE AND SOME EFFECTS

Consider a space ship passing Earth at a velocity of 0.5*c*. It emits a pulse (or on Earth we observe 'ticks' of their clock) of duration $\Delta T = 2.0$ ns. We on Earth can 'measure' the duration t' we observe. Note that we are *not* in the same frame of reference as the 'event' so our time interval is t' not t. The duration of the event, in the frame of the event on the space ship, is t.

Using equation

$$t' = \frac{2.0\times10^{-9}}{\sqrt{1-0.5^2}} \qquad t' = \frac{2.0\times10^{-9}}{\sqrt{1-0.5^2}}$$
$$t' = \frac{2.0\times10^{-9}}{\sqrt{1-0.25}} \qquad t' = \frac{2.0\times10^{-9}}{0.87}$$
$$t' = 2.3n\ s \qquad t' = 2.3n\ s$$

gives us an observed time interval of t × 1 /0.87 = 2.3 ns.

We consider their clock is running slow, time is passing more quickly for us so they could end up 'younger'! Although time dilation can give rise to interesting discussions on time travel (into the future), the explanation of the twin paradox requires more consideration since any acceleration may involve general relativity. Also a returning twin would have to 'change' frames of reference.

When $\sqrt{1-(v/c)^2}$ is almost unity no effect is noticeable. It is useful to calculate this term (or the reciprocal) for various speeds, for example:

- a supersonic plane 422 m s^{-1} (900 mph);
- × 10^{8} m s^{-1};
- 0.3 × 10^{8} m s^{-1} (10%*c*);
- 1.0 × 10^{8} m s^{-1};
- 2.0 × 10^{8} m s^{-1},
- 2.8 × 10^{8} m s^{-1}
- and 99% *c*.

If you do these calculations you will clearly see that effects in everyday life are not noticeable. We need $v >$ 10%*c* for any noticeable effects.

LENGTH CONTRACTION

Another implication of Einstein's theory is the shortening of length when an object is moving. Consider the muons discussed above. Their large speed means they experience a longer half-life due to time dilation. An equivalent way of thinking about this is that the muons experience the height of the atmosphere as smaller (or contracted) by the same amount as the time has increased (or dilated). A symmetrical formula for time dilation can be derived. Note that the contraction only takes place in the direction that the object is travelling:

$$l' = l\sqrt{1-\frac{v^2}{c^2}}$$

where

l length measured in the **<u>same</u>** frame of reference as the object

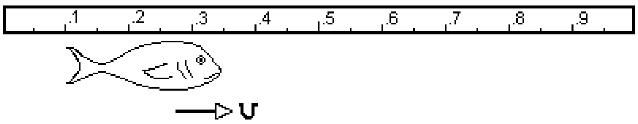
l' length measured in a frame of reference moving relative to the one containing the object.

LENGTH PARADOX

There is an apparent paradox thrown up by special relativity: consider a train that is just longer than a tunnel. If the train travels at high speed through the tunnel does length contraction mean that, from our stationary perspective, it fits inside the tunnel? How can this be reconciled with the fact that from the train's reference frame the tunnel appears even shorter as it moves towards the train? The key to this question is simultaneity, i.e. whether different reference frames can agree on the exact time of particular events. In order for the train to fit in the tunnel the front of the train must be inside at the same time as the back of the train. Due to time dilation, the stationary observer (you) and a moving observer on the train cannot agree on when the front of the train reaches the far end of the tunnel or the rear of the train reaches the entrance of the tunnel. If you work out the equations carefully then you can show that even when the train is contracted, the front of the train and the back of the train will not both be inside the tunnel at the same time!

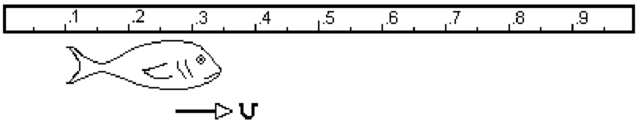
HOW DO YOU MEASURE A FISH?

Suppose an oceanographer wished to measure the length of a fish that swims by his metre stick. He would note, at some moment, where the head was and *at the same moment* he would notice where the tail was. The fish would then be said to be the same length as the distance between the two marks on the metre stick. If the oceanographer were to notice where the head was, wait a moment and then look where the tail was the fact that the fish was moving through the water would give a shorter measurement. Let's see that in pictures:



Look at both ends simultaneously:
head point: 0.35 metres
tail point: 0.1 metres
Length = 0.35 metres - 0.1 metres
=0.25 metres

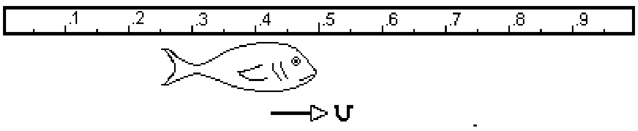
Now, the "wrong" way



Look at the head first:

head point: 0.35 metres.

(wait a moment)



Now look at the tail:

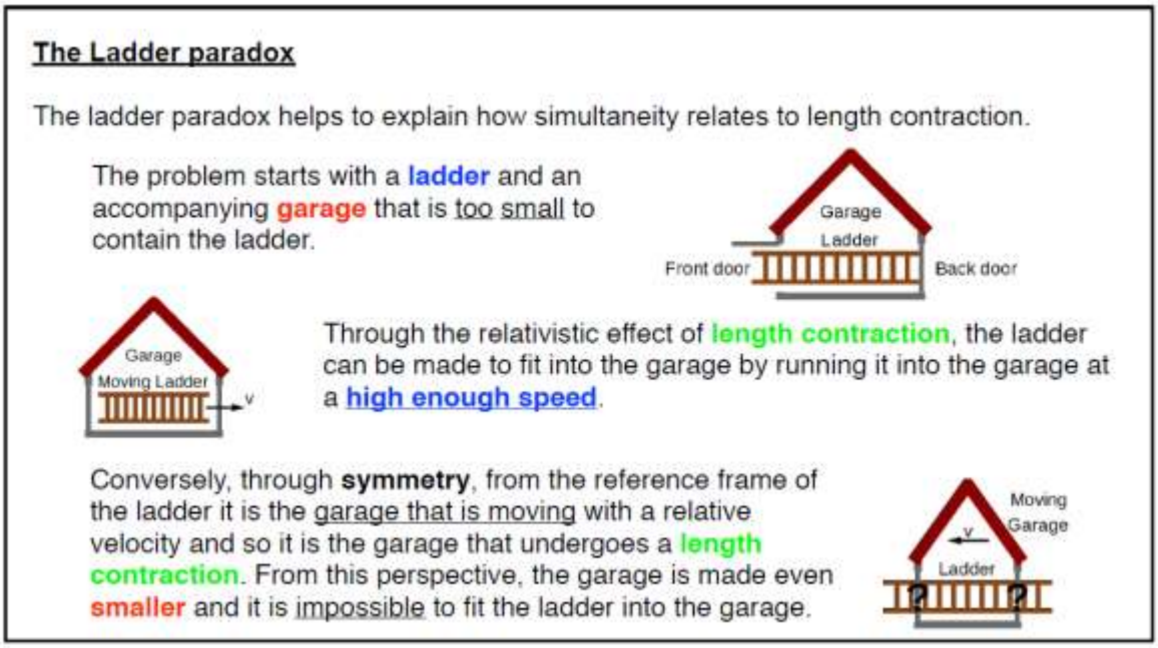
tail point: 0.25 metres.

length = 0.35 metres - 0.25 metres
= 0.1 metres.

For two objects to be the same length their endpoints must coincide **SIMULTANEOUSLY**. The measuring process for a moving object includes the concept of simultaneity. But, as we will see events deemed simultaneous in one inertial frame are not always simultaneous in another inertial frame. Thus the length of an object will be different when measured in different inertial frames. Since all these inertial frames are equally valid (postulate 1) each of their measurements are as correct as any other's.

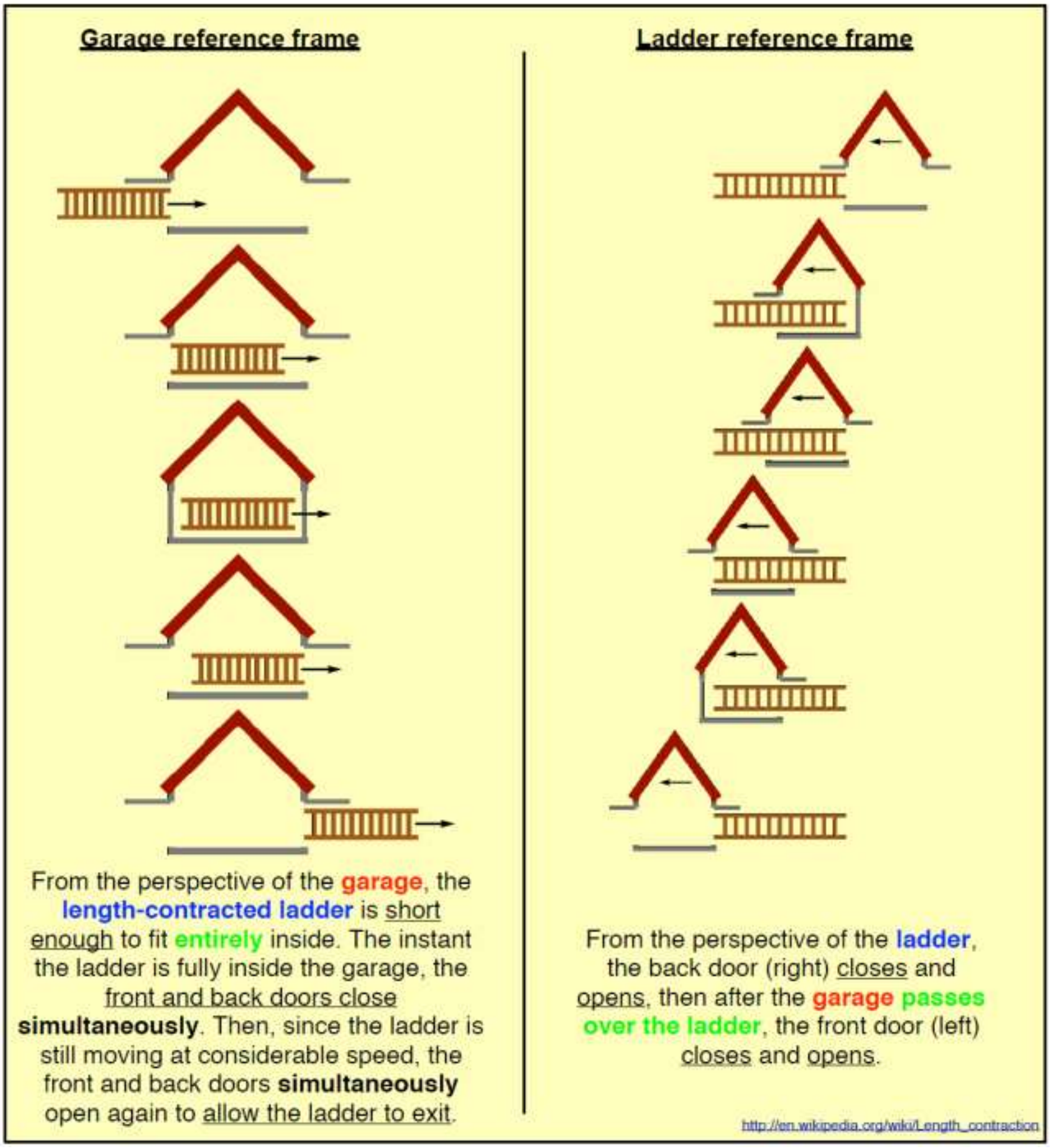
Another way of showing this is with the ladder paradox.

To gain an understanding of why an observer measures the l**ength of a fast moving object** as being **contracted** is related to the idea that the length of any object is found by knowing **where the two ends of the object are**, and determining the **distance between them**.
In relative motion, the **position** of both ends of the object **cannot** be determined **simultaneously**, which results in a **contracted length** measurement.



LADDER PARADOX - SOLUTION

Both the ladder and garage occupy their own inertial reference frames, and thus both frames are equally valid frames to view the problem. The solution to the apparent paradox lies in the fact that what one observer (e.g. the garage) considers as simultaneous does not correspond to what the other observer (e.g. the ladder) considers as simultaneous. A clear way of seeing this is to consider a garage with two doors that swing shut to contain the ladder and then open again to let the ladder out the other side.

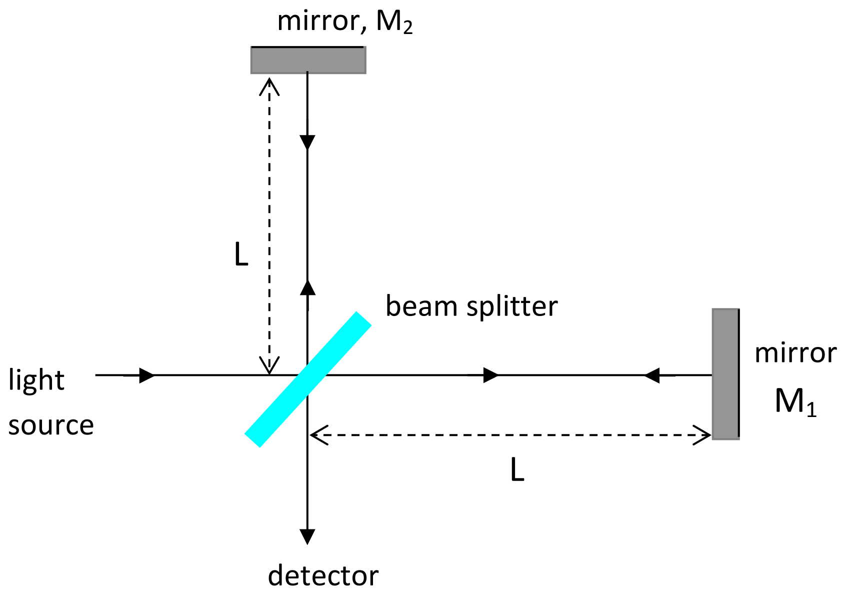


THE MICHELSON-MORLEY EXPERIMENT

The prevalent belief in the nineteenth century was that light is a wave, carried by a subtle medium, the aether, which is at rest in the universe. The sun is at rest, in the centre of the universe, and the earth moves through the aether and around the sun at about 30 km/s.

The Michelson-Morley (M&M) experiment was designed to verify this belief. If light is sent back and forth on earth, in the direction of the earth's movement, then the round trip should take longer than it would if there were no aether. It should also take a little longer, but not as much as in the parallel direction, for light moving back and forth in the direction perpendicular to the earth's movement. The M&M experiment used two identical rods perpendicular to each other along which the light moved back and forth.

The apparatus consists of a light source, a beam splitter (a half-silvered glass plate) and two mirrors, M_1 and M_2, each of which is equidistant from the beam splitter. The beam splitter is at 45^o to the incident beam and the return beams pass to the detector, a telescope.



Sometimes one light beam may be travelling in the same direction as the aether and the other beam at right angles to the aether. They will have slightly different speeds. The detector should show interference between the two beams travelling perpendicular to each other. The aim was to measure the speed of the Earth relative to the aether.

No discernible interference fringes were found, despite repeating the experiment at different times in the Earth's orbit and with different orientation. The world's physicists used these results to add on a few qualities to the ether. They assumed that the movement through the ether "compressed" the apparatus in the direction of the motion so that the "no movement" reading always resulted. On the prevailing theory and experimental accuracy, the small destructive interference should have been observed.

This experiment, first performed in 1881, repeated in 1887, and often thereafter, could find no indication of a difference. But the belief could not be shaken, and Hendrik Lorentz, in 1895 and again in 1904, thought the explanation could possibly be that the rod placed in the direction of the earth's movement might contract due to the earth's movement, just enough to make the round trip time equal to the case when there is no movement. The equation he developed is known as the Lorentz transformation (LT).

It was interesting. The Ether had been defined to be impossible to detect by any means. "If it was impossible to detect," one might ask, "how did they know it existed?"

This null result had great importance since it could not be explained.

It was against this background of theories that were fundamentally untestable that Einstein and others stated that any experimental science such as physics must have its fundamental entities defined in terms of some experiment. To avoid this was to invent ghosts. This is the basis of Operational Definition. An operational definition is simply a way of defining a physical quantity or quality in terms of an operation performed. Instead of saying length is "the amount of distance between two points" a modern physicist would give a procedure (an *operation*) for measuring the distance between two points. ("lay metre sticks end to end and count how many" or "send a light signal from point A to point B and reflect it back to A again; then multiply the elapsed time for the signal by ½ c.") The result of such a measurement would be the **definition** of the length. Physics is a way of predicting and explaining observations encountered under certain circumstances. All Physics should be based on observation and experiment. Any concept that could not, in principle, be connected with experiment was declared to be not physics but metaphysics.

http://www.youtube.com/watch?v=AKhvqO5UBsA

http://www.youtube.com/watch?v=SUf8zA-LwUk&feature=related

To summarise

1. Defining t, t', l and l' as shown below, equations for time dilation and length contraction can be derived. The equations (1) and (2) are provided on the SQA data sheet.

t and *l* time interval ('event') or length of object under discussion in a frame of reference.

t' and *l'* time interval or length of object 'measured' by travellers in a *different* frame of reference.

v relative velocity of the two frames of reference.

$$t' = \frac{t}{\sqrt{1-(v/c)^2}} \qquad (1)$$

$$l' = l\sqrt{1-(v/c)^2} \qquad (2)$$

2. No object can travel faster than the speed of light.

3. Relativistic effects are negligible when relative velocity is less than10% of the speed of light.

4. Experimental verification is provided by observing the life time of fast-moving muons.

5 There is not a separate conservation of mass, but a combined conservation of mass and energy. A greater energy than expected is required to increase the speed of an object as its speed approaches the speed of light.

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You will look at the various ways in which findings can be presented, and appreciate the possibility of using other media such as video clips, articles, papers, posters etc.

LEARNING OUTCOME

You will be more informed about the different ways in which one topic can be presented. You will begin to think about how to present your own work.

LEARNING ACTIVITY

You can work independently or in groups. There are three different resources:

1. A video clip entitled 'Two postulates' (http://www.youtube.com/watch?v=WdfnRWGgbd0).

2. A physicsworld article entitled 'Slowed Light Breaks Record' (http://physicsworld.com/cws/article/news/41246)

3. The paper

'On Velocities Beyond the Speed of Light c' (On Velocities beyond the speed of light c.pdf)

Students should examine and discuss the three resources. Teachers should point out that even though the physics content may not all be at the students' level of understanding, it is still possible to take information from it with their level of knowledge. This is emphasised by students completing the accompanying handout.

'TWO POSTULATES'

This clip discusses how to tell if an object is moving or not by way of an animation.

'SLOWED LIGHT BREAKS RECORD'

This is an article published in physicsworld in December 2009. It is not particularly long, although does contain a lot of information.

'ON VELOCITIES BEYOND THE SPEED OF LIGHT C'

This paper was published in 1998 from CERN. It has the more traditional scientific report structure and is a good example for you.

After completing the table on the sheet, you should find that all boxes are ticked – highlighting that even though the information is presented in different ways, all the resources contain what the students will have to put into their own reports.

There are many ways to present scientific findings. You might have written a report in the past but universities may ask you to present a poster of your work.

Here we will look at three different ways of presenting findings on special relativity.

On your own or in groups/pairs, have a look at the three examples of how findings on special relativity have been presented.

Copy and complete the table, either with a few notes or a tick or cross, to show if the example meets the criteria.

	'Two Postulates'	'Slowed Light Breaks Record'	'On Velocities Beyond the Speed of Light'
Is there mention of the objective for the investigation/experiment?			
Is there information given on the experiment/s conducted?			
Is there mention of the data (perhaps not all) and any analysis of the findings?			
Does the article discuss the conclusion for the experiment/investigation?			

Now you have looked at the three examples, ask yourself the following questions.

FIRST IMPRESSIONS

1. Was one resource more eye-catching than the others?

2. Does one look like it will be easier to read/understand than the others?

3. Which one looks most credible?

DOWN TO THE NITTY GRITTY

1. Which resource was the most interesting?
2. Which one was the best presented?
3. Which gave the most information?
4. Did you need to understand everything mentioned to gain an understanding of the experiment?

Which format might you consider for your Communicating Physics investigation?

SPECIAL RELATIVITY / TUTORIALS 1

1. A river flows at a constant speed of 0·5 ms^{-1} south. A canoeist is able to row at a constant speed of 1·5 ms^{-1}.

(a) Determine the velocity of the canoeist relative to the river bank when the canoeist is paddling north.

(b) Determine the velocity of the canoeist relative to the river bank when the canoeist is paddling south.

2. In an airport, passengers use a moving walkway. The moving walkway is travelling at a constant speed of 0·8ms^{-1} and is travelling east.

For the following people, determine the velocity of the person relative to the ground:

(a) a woman standing at rest on the walkway;

(b) a man walking at 2·0 ms^{-1} in the same direction as the walkway is moving;

(c) a boy running west at 3·0 ms^{-1} .

3. The steps of an escalator move upwards at a steady speed of 1·0 ms^{-1} relative to the stationary side of the escalator.

(a) A man walks up the steps of the escalator at 2·0 ms^{-1}. Determine the speed of the man relative to the side of the escalator.

(b) A boy runs down the steps of the escalator at 3·0 m s^{-1}. Determine the speed of the boy relative to the side of the escalator.

4. **Copy and complete** filling in the missing letters below:

In _A_ Theory of Special Relativity the laws of physics are the _B_for all observers, at rest or moving at constant velocity with respect to each other ie _C_ acceleration.

An observer, at rest or moving at constant _D_ has their own frame of reference.

In all frames of reference the _E_, *c*, remains the same regardless of whether the source or observer is in motion.

Einstein's principles that the laws of physics and the speed of light are the same for all observers leads to the conclusion that moving clocks run _F_(time dilation) and moving objects are _G_ (length contraction).

Match each letter with the correct word from the list below:

acceleration different Einstein's fast lengthened Newton's

same shortened slow speed of light velocity zero

5. An observer at rest on the Earth sees an aeroplane fly overhead at a constant speed of 2000 kmh^{-1}. At what speed, in kmh^{-1}, does the pilot of the aeroplane see the Earth moving?

6. A scientist is in a windowless lift. Can the scientist determine whether the lift is moving with a:

(a) uniform velocity

(b) uniform acceleration?

7. Spaceship A is moving at a speed of $2{\cdot}4 \times 10^8$ ms^{-1}. It sends out a light beam in the forwards direction. Meanwhile another spaceship B moves towards spaceship A at $2{\cdot}4 \times 10^8$ ms^{-1}. At what speed does spaceship B see the light beam from spaceship A pass?

8. A spacecraft is travelling at a constant speed of $7{\cdot}5 \times 10^7$ ms^{-1}. It emits a pulse of light when it is $3{\cdot}0 \times 10^{10}$ m from the Earth as measured by an observer on the Earth.

Calculate the time taken for the pulse of light to reach the Earth according to a clock on the Earth when the spacecraft is moving:

(a) away from the Earth

(b) towards the Earth.

9. A spaceship is travelling away from the Earth at a constant speed of $1{\cdot}5 \times 10^8$ ms^{-1}. A light pulse is emitted by a lamp on the Earth and travels towards the spaceship. Find the speed of the light pulse according to an observer on:

(a) the Earth

(b) the spaceship.

10. Convert the following fraction of the speed of light into a value in ms^{-1} :

(a) 0·1 c

(b) 0·5 c

(c) 0·6 c

(d) 0·8 c

11. Convert the following speeds into a fraction of the speed of light:

(a) $3{\cdot}0 \times 10^8$ m s^{-1}

(b) $2{\cdot}0 \times 10^8$ m s^{-1}

(c) $1{\cdot}5 \times 10^8$ m s^{-1}

(d) $1{\cdot}0 \times 10^8$ m s^{-1}

TIME DILATION / TUTORIAL 2

1. Write down the relationship involving the proper time t and dilated time t' between two events which are observed in two different frames of reference moving at a speed, *v*, relative to one another (where the proper time is the time measured by an observer at rest with respect to the two events and the dilated time is the time measured by another observer moving at a speed, *v*, relative to the two events).

2. In the table shown, use the relativity equation for time dilation to calculate the value of each missing quantity (a) to (f) for an object moving at a constant speed relative to the Earth.

Dilated time	*Proper time*	*Speed of object / m s^{-1}*
(a)	20 h	$1{\cdot}00 \times 10^{8}$
(b)	10 year	$2{\cdot}25 \times 10^{8}$
1400 s	(c)	$2{\cdot}00 \times 10^{8}$
1.40×10^{-4} s	(d)	$1{\cdot}00 \times 10^{8}$
84 s	60 s	(e)
21 minutes	20 minutes	(f)

3. Two observers P, on Earth, and Q, in a rocket, synchronise their watches at 11.00 am just as observer Q passes the Earth at a speed of 2×10^{8} ms $^{-1}$.

(a) At 11.15 a.m. according to observer P's watch, observer P looks at Q's watch through a telescope. Calculate the time, to the nearest minute, that observer P sees on Q's watch.

(b) At 11.15 a.m. according to observer Q's watch, observer Q looks at P's watch through a telescope. Calculate the time, to the nearest minute, that observer Q sees on P's watch.

4. The lifetime of a star is 10 billion years as measured by an observer at rest with respect to the star. The star is moving away from the Earth at a speed of 0·81 c.

Calculate the lifetime of the star according to an observer on the Earth.

5. A spacecraft moving with a constant speed of 0·75 c passes the Earth. An astronaut on the spacecraft measures the time taken for Usain Bolt to run a 100 m world record. The astronaut measures this time to be 14·48 s. Calculate Usain Bolt's winning time as measured on the Earth. (When and where on Earth was the spacecraft viewing?)

6. A scientist in the laboratory measures the time taken for a nuclear reaction to occur in an atom. When the atom is travelling at $8{\cdot}0 \times 10^{7}$ ms−1 the reaction takes $4{\cdot}0 \times 10^{-4}$ s.

Calculate the time for the reaction to occur when the atom is at rest.

7. The light beam from a lighthouse sweeps its beam of light around in a circle once every 10 s. To an astronaut on a spacecraft moving towards the Earth, the beam of light completes one complete circle every 14 s. Calculate the speed of the spacecraft relative to the Earth.

8. A rocket passes two beacons that are at rest relative to the Earth. An astronaut in the rocket measures the time taken for the rocket to travel from the first beacon to the second beacon to be 10·0 s. An observer on Earth measures the time taken for the rocket to travel from the first beacon to the second beacon to be 40·0 s. Calculate the speed of the rocket relative to the Earth.

9. A spacecraft travels to a distant planet at a constant speed relative to the Earth. A clock on the spacecraft records a time of 1 year for the journey while an observer on Earth measures a time of 2 years for the journey. Calculate the speed, in ms^{-1}, of the spacecraft relative to the Earth.

LENGTH DILATION / TUTORIAL 3

1. Write down the relationship involving the proper length l and contracted length l' of a moving object observed in two different frames of reference moving at a speed, v, relative to one another (where the proper length is the length measured by an observer at rest with respect to the object and the contracted length is the length measured by another observer moving at a speed, v, relative to the object).

2. In the table shown, use the relativity equation for length contraction to calculate the value of each missing quantity (a) to (f) for an object moving at a constant speed relative to the Earth.

Contracted length	*Proper length*	*Speed of object* / m s^{-1}
(a)	5·00 m	$1{\cdot}00 \times 10^{8}$
(b)	15.0 m	$2{\cdot}00 \times 10^{8}$
0·15 km	(c)	$2{\cdot}25 \times 10^{8}$
150 mm	(d)	$1{\cdot}04 \times 10^{8}$
30 m	35 m	(e)
10 m	11 m	(f)

3. A rocket has a length of 20 m when at rest on the Earth. An observer, at rest on the Earth, watches the rocket as it passes at a constant speed of $1{\cdot}8 \times 10^{8}$ ms^{-1}. Calculate the length of the rocket as measured by the observer.

4. A pi meson is moving at 0·90 c relative to a magnet. The magnet has a length of 2·00 m when at rest to the Earth.

Calculate the length of the magnet in the reference frame of the pi meson.

5. In the year 2050 a spacecraft flies over a base station on the Earth. The spacecraft has a speed of 0·8 c. The length of the moving spacecraft is measured as 160 m by a person on the Earth. The spacecraft later lands and the same person measures the length of the now stationary spacecraft.

Calculate the length of the stationary spacecraft.

6. A rocket is travelling at 0·50 c relative to a space station.
Astronauts on the rocket measure the length of the space station to be 0·80 km.
Calculate the length of the space station according to a technician on the space station.

7. A metre stick has a length of 1·00 m when at rest on the Earth. When in motion relative to an observer on the Earth the same metre stick has a length of 0·50 m. Calculate the speed, in ms^{-1}, of the metre stick.

8. A spaceship has a length of 220 m when measured at rest on the Earth. The spaceship moves away from the Earth at a constant speed and an observer, on the Earth, now measures its length to be 150 m.

Calculate the speed of the spaceship in ms^{-1}.

9. The length of a rocket is measured when at rest and also when moving at a constant speed by an observer at rest relative to the rocket. The observed length is 99·0 % of its length when at rest. Calculate the speed of the rocket.

RELATIVITY MISCELLANEOUS/ TUTORIAL 4

1. Two points A and B are separated by 240 m as measured by metre sticks at rest on the Earth. A rocket passes along the line connecting A and B at a constant speed. The time taken for the rocket to travel from A to B, as measured by an observer on the Earth, is $1{\cdot}00 \times 10^{-6}$s.

(a) Show that the speed of the rocket relative to the Earth is $2{\cdot}40 \times 10^{8}$ ms^{-1}.

(b) Calculate the time taken, as measured by a clock in the rocket, for the rocket to travel from A to B.

(c) What is the distance between points A and B as measured by metre sticks carried by an observer travelling in the rocket?

2. A spacecraft is travelling at a constant speed of 0·95 c. The spacecraft travels at this speed for 1 year, as measured by a clock on the Earth.

(a) Calculate the time elapsed, in years, as measured by a clock in the spacecraft.

(b) Show that the distance travelled by the spacecraft as measured by an observer on the spacecraft is $2{\cdot}8 \times 10^{15}$m.

(c) Calculate the distance, in m, the spacecraft will have travelled as measured by an observer on the Earth.

3. A pi meson has a mean lifetime of $2{\cdot}6 \times 10^{-8}$ s when at rest. A pi meson moves with a speed of 0·99 c towards the surface of the Earth.

(a) Calculate the mean lifetime of this pi meson as measured by an observer on the Earth.

(b) Calculate the mean distance travelled by the pi meson as measured by the observer on the Earth.

4. A spacecraft moving at $2{\cdot}4 \times 10^{8}$ m s^{-1} passes the Earth. An astronaut on the spacecraft finds that it takes $5{\cdot}0 \times 10^{-7}$ s for the spacecraft to pass a small marker which is at rest on the Earth.

(a) Calculate the length, in m, of the spacecraft as measured by the astronaut.

(b) Calculate the length of the spacecraft as measured by an observer at rest on the Earth.

5. A neon sign flashes with a frequency of 0·2 Hz.

(a) Calculate the time between flashes.

(b) An astronaut on a spacecraft passes the Earth at a speed of 0·84 c and sees the neon light flashing. Calculate the time between flashes as observed by the astronaut on the spacecraft.

6. When at rest, a subatomic particle has a lifetime of 0·15 ns. When in motion relative to the Earth the particle's lifetime is measured by an observer on the Earth as 0·25 ns. Calculate the speed of the particle.

7. A meson is 10·0 km above the Earth's surface and is moving towards the Earth at a speed of 0·999 c.

(a) Calculate the distance, according to the meson, travelled before it strikes the Earth.

(b) Calculate the time taken, according to the meson, for it to travel to the surface of the Earth.

8. The star Alpha Centauri is 4·2 light years away from the Earth. A spacecraft is sent from the Earth to Alpha Centauri. The distance travelled, as measured by the spacecraft, is 3·6 light years.

(a) Calculate the speed of the spacecraft relative to the Earth.

(b) Calculate the time taken, in seconds, for the spacecraft to reach Alpha Centauri as measured by an observer on the Earth.

(c) Calculate the time taken, in seconds, for the spacecraft to reach Alpha Centauri as measured by a clock on the spacecraft.

9. Muons, when at rest, have a mean lifetime of $2{\cdot}60 \times 10^{-8}$ s. Muons are produced 10 km above the Earth. They move with a speed of 0·995 c towards the surface of the Earth.

(a) Calculate the mean lifetime of the moving muons as measured by an observer on the Earth.

(b) Calculate the mean distance travelled by the muons as measured by an observer on the Earth.

(c) Calculate the mean distance travelled by the muons as measured by the muons.

EXAM QUESTIONS / SPECIAL RELATIVITY

1 OCR 10 JAN 494 4

1. A beam of charged particles is accelerated in particle accelerators to a speed of $2{\cdot}0 \times 10^{8}$ m s^{-1}.

(a) The particles are unstable and decay with a half-life of $8{\cdot}2 \times 10^{-7}$ s when at rest.

Calculate the half-life of the particles in the beam as observed by a stationary observer. 2

(b) Calculate the mean distance travelled by a particle in the beam before it decays as observed by a stationary observer. 2 (4)

TUTORIAL ANSWERS

SPECIAL RELATIVITY

1. (a) 1·0 ms^{-1} north

(b) 2·0 ms^{-1} south

2. (a) 0·8 ms^{-1} east

(b) 2·8 ms^{-1} east

(c) 2·2 ms^{-1} west

3. (a) 3·0 ms^{-1}

(b) 2·0 ms^{-1}

4. A = Einstein's; B = same; C = zero; D = velocity; E = speed of light; F = slow; G = shortened

5. 2000 kmh^{-1}

6. (a) No

(b) Yes

7. 3×10^{8} ms^{-1}

8. (a) 100 s

(b) 100 s

9. (a) 3×10^{8} ms^{-1}

(b) 3×10^{8} ms^{-1}

10. (a) $0·3 \times 10^{8}$ ms^{-1}

(b) $1·5 \times 10^{8}$ ms^{-1}

(c) $1·8 \times 10^{8}$ m s^{-1}

(d) $2·4 \times 10^{8}$ m s^{-1}

11. (a) c

(b) 0·67 c

(c) 0·5 c

(d) 0·33 c

TIME DILATION

1. Teacher Check

2. (a) 21·2 h

(b) 15·1 year

(c) 1043 s

(d) $1·32 \times 10^{-4}$ s

(e) $2·10 \times 10^{8}$ m s^{-1}

(f) $9·15 \times 10^{7}$ m s^{-1}

3. (a) 11.20 am

(b) 11.20 am

4. 17·1 billion years

5. 9·58 s Berlin World Record 2009

6. $3·9 \times 10^{-4}$ s

7. $2·1 \times 10^{8}$ m s^{-1} or 0·70 c

8. $2·90 \times 10^{8}$ m s^{-1} or 0·97 c

9. $2·60 \times 10^{8}$ m s^{-1}

LENGTH DILATION

1. Teacher Check

2. (a) 4·71 m

(b) 11·2 m

(c) 0·227 km

(d) 160 mm

(e) $1·55 \times 10^{8}$ m s^{-1}

(f) $1·25 \times 10^{8}$ m s^{-1}

3. 16 m

4. 0·872 m

5. 267 m

6. 0·92 km

7. $2·60 \times 10^{8}$ m s^{-1}

8. $2·19 \times 10^{8}$ m s^{-1}

9. $4·23 \times 10^{7}$ m s^{-1} or 0.14 c

RELATIVITY MISCELLANEOUS

1. (a) Teacher Check

(b) $1{\cdot}67 \times 10^{-6}$ s

(c) 144 m

2. (a) 0·31 of a year

(b) Teacher Check

(c) $8{\cdot}97 \times 10^{15}$ m

3. (a) $1{\cdot}84 \times 10^{-7}$ s

(b) 54·6 m or 54·7 m

4. (a) 120 m

(b) 72 m

5. (a) 5 s

(b) 9·22 s

6. 0·8 c

7. (a) 447 m

(b) $1{\cdot}49 \times 10^{-6}$ s

8. (a) 0·52 c

(b) $2{\cdot}55 \times 10^{8}$ s

(c) $2{\cdot}18 \times 10^{8}$ s

9. (a) $2{\cdot}60 \times 10^{-7}$ s

(b) 77·6 m

(c) 7·75 m or 7·76 m

 Chapter 8: THE EXPANDING UNIVERSE

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6 THE EXPANDING UNIVERSE

$$f_{\text{observed}} = f_{source}\frac{v}{[v + v_{source}]}$$

$$v = H_o d$$

a) The Doppler Effect is observed in sound and light.

b) The Doppler Effect causes shifts in wavelengths of sound and light. The light from objects moving away from us is shifted to longer (more red) wavelengths

c) The redshift of a galaxy is the change in wavelength divided by the emitted wavelength. For slowly moving galaxies, redshift is the ratio of the velocity of the galaxy to the velocity of light.

d) Hubble's law shows the relationship between the recession velocity of a galaxy and its distance from us.

e) Hubble's law allows us to estimate the age of the Universe.

f) Evidence for the expanding Universe.

g) We can estimate the mass of a galaxy by the orbital speed of stars within it.

h) Evidence for dark matter from observations of the mass of galaxies.

i) Evidence for dark energy from the accelerating rate of expansion of the Universe.

j) The temperature of stellar objects is related to the distribution of emitted radiation over a wide range of wavelengths. The wavelength of the peak wavelength of this distribution is shorter for hotter objects than for cooler objects. Qualitative relationship between radiation emitted per unit surface area per unit time and the temperature of a star.

k) Cosmic microwave background radiation as evidence for the big bang and subsequent expansion of the universe.

SUGGESTED ACTIVITIES

- ✓ Doppler effect in terms of terrestrial sources, e.g. passing ambulances.
- ✓ For sound, the apparent change in frequency as a source moves towards or away from a stationary observer should be investigated.
- ✓ Investigating the apparent shift in frequency using a moving sound source and data logger. Applications include measurement of speed (radar), echocardiogram and flow measurement.
- ✓ (Note that the Doppler effect equations used for sound cannot be used with light from fast moving galaxies because relativistic effects need to be taken into account.)
- ✓ Measuring distances to distant objects. Parallax measurements and data analysis of apparent brightness of standard candles.
- ✓ The Unit 'Particles and Waves' includes an investigation of the inverse square law for light. Centres may wish to include this activity in this topic.
- ✓ In practice, the units used by astronomers include lightyears and parsecs rather than SI units.
- ✓ Data analysis of measurements of galactic velocity and distance.

 Chapter 8: THE EXPANDING UNIVERSE

- ✓ Measurements of the velocities of galaxies and their distance from us lead to the theory of the expanding Universe. Gravity is the force which slows down the expansion. The eventual fate of the Universe depends on its mass-energy density. .The orbital speed of the Sun and other stars gives a way of determining the mass of our galaxy.
- ✓ The Sun's orbital speed is determined almost entirely by the gravitational pull of matter inside its orbit. Measurements of the mass of our galaxy and others lead to the conclusion that there is significant mass which cannot be detected — dark matter.
- ✓ Measurements of the expansion rate of the Universe lead to the conclusion that it is increasing, suggesting that there is something that overcomes the force of gravity — dark energy.
- ✓ The revival of Einstein's cosmological constant in the context of the accelerating universe.
- ✓ Evolution of a star — Hertzsprung-Russell diagram.
- ✓ Remote sensing of temperature. Investigating the temperature of hot objects using infrared sensors.
- ✓ Change in colour of steel at high temperatures. Furnaces and kilns.
- ✓ History of cosmic microwave background (CMB) discovery and measurement.
- ✓ COBE satellite.
- ✓ Other evidence for the big bang includes The abundance of the elements hydrogen and helium and the darkness of the sky (Olber's paradox).
- ✓ The peak wavelength of cosmic microwave background. This temperature corresponds to that predicted after the big bang.

THE DOPPLER EFFECT AND REDSHIFT OF GALAXIES

THE DOPPLER EFFECT

Definition:

The Doppler Effect is the apparent change in frequency of a wave when the source and observer are moving relative to each other.

The effect is produced with all wave motions, including electromagnetic waves.

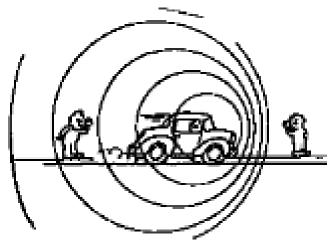
The Doppler Effect is applied in many different disciplines:

Police radar guns use the Doppler Effect to measure the speed of motorists. Police speed guns send out an electromagnetic wave (radar) and measure the Doppler shift of the reflected wave to measure the speed of an approaching car.

Doppler is used to measure the speed of blood flow in veins to check for deep vein thrombosis [DVT] in medicine.

An echocardiogram uses the Doppler Effect to measure the velocity of blood flow and cardiac tissue and is one of the most widely used diagnostic tests in cardiology.

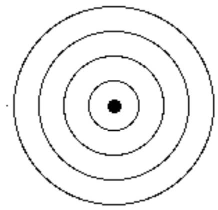
In this course we will concentrate on a wave source moving at constant speed relative to a stationary observer.



You have already experienced the Doppler Effect many times. You may have noticed a change in pitch as a car comes first towards you then passes and goes away from you. The most noticeable is when a police car, ambulance or fire engine passes you. You hear the pitch of their siren increase as they come towards you and then decrease as they move away. Another memorable example is the sound of a very fast moving vehicle, such as a Formula 1 car passing you (or passing a microphone on the television), the sound of the engine rises in frequency as it approaches and falls in frequency as it moves away.

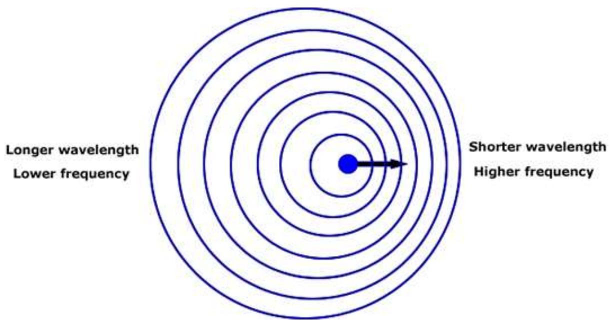
When the source moves towards you, more waves reach you per second and the frequency heard is increased. If the source moves away from you, fewer waves reach you each second and the frequency heard decreases.

The Doppler Effect applies to all waves, including light.



Why do we hear a change of frequency?

A stationary sound source produces sound waves at a constant frequency *f*, and the wavefronts propagate symmetrically out from the source at a constant speed, which is the speed of sound in the medium. The distance between wave-fronts is the wavelength. All observers will hear the same frequency, which will be equal to the actual frequency of the source:



$f_{source} = f_{observed}$.

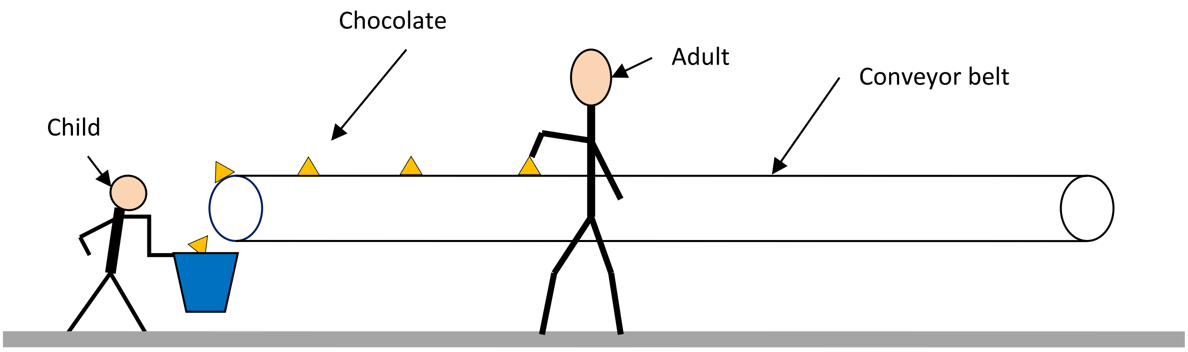
where f_{source} is the frequency produced by the source and $f_{observed}$ is the observed frequency.

The sound source now moves to the right with a speed v_s. The wavefronts are produced with the same frequency as before. **Therefore the period of each wave is the same as before.** However, in the time taken for the production of each new wave the source has moved some distance to the right. This means that the wavefronts on the left are created further apart and the wavefronts on the right are created closer together. This leads to the spreading out and bunching up of waves you can see above and hence the change in frequency.

https://www.youtube.com/watch?v=h4OnBYrbCjY

ANALOGY: SWEETS ON A CONVEYOR BELT

Imagine a long conveyor belt running at a steady speed. An adult standing about halfway along deposits sweets onto the belt at a regular rate, say one sweet per second (frequency).



Chocolates on conveyor belt

A child standing at the end of the conveyor belt collects the sweets in a bucket as they fall off the end. As long as they are both standing still, the child will be collecting the sweets at the same rate (frequency) as they are being deposited by the adult.

If the adult walks steadily away from the child, still depositing the sweets at the same rate, the child now receives the sweets at a lower rate (frequency). The sweets will be further spaced out on the conveyor belt (longer 'wavelength').

Conversely if the adult walks towards the child, the child will receive the sweets at a higher rate (frequency) and they will be spaced closer together (shorter 'wavelength').

EQUATIONS

There are equations that we can use to calculate the apparent change in frequency and we can derive these equations for the Doppler effect. For a *moving source* there is an equation for the source moving towards the observer and another for the source moving away from the observer.

Similarly for a moving observer there are two equations, one for moving away from and one for moving towards the source, but these last two are not covered in this course.

For this course, the only equations required are for a stationary observer with the source moving away and moving towards the observer.

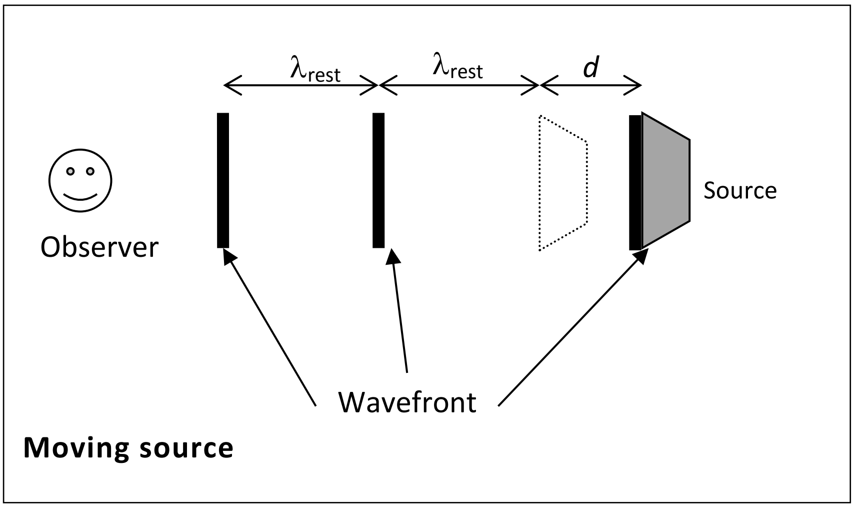
Derivation: Stationary observer and source moving away

Beware in these derivations the frequency of the source is the actual frequency generated by the object producing the sound waves. The speed of sound would have a value of 340 m/s. The sound is generated from a moving object which is moving with speed v_{source}. The speed of the source is the speed at which the vehicle is moving, not the speed of the sound. This generates an observed frequency and wavelength in a stationary observer. Other derivations use different symbols, you must define your terms.

These notes provide you with three methods for deriving the formula for observed frequency when the source is moving. Decide which method you find easiest to understand.

DERIVING THE DOPPLER EQUATIONS

METHOD 1



Wavelength of source $\lambda_{rest} = \frac{v}{f_{source}}$ .... [1] Where v is the speed of sound

When the source starts moving away, in the time between creating the first and the second wave (i.e the period T), the source will have moved away from the observer by a distance:

$$d = v_{source} \times t$$

for one wave

$$d = \lambda \text{ (wavelength) and } t = T(period) \text{ and } T = \frac{1}{f}$$

$$\lambda = v_{source} \times T$$

$$\lambda = \frac{v_{source}}{f_{source}}$$

ie the wavelength increases by $\frac{v_{source}}{f_{source}}$

$$\lambda_{observed} = \lambda_{rest} + \frac{v_{source}}{f_{source}}$$

Substituting from [1] gives:

$$\lambda_{observed} = \frac{v}{f_{source}} + \frac{v_{source}}{f_{source}}$$

$$\lambda_{observed} = \frac{[v + v_{source}]}{f_{source}}$$ ....[2]

From $v = f_{\text{observed}} \times \lambda_{\text{observed}}$

$$\lambda_{\text{observed}} = \frac{v}{f_{\text{observed}}}$$

Substituting from [2],

$$\frac{v}{f_{\text{observed}}} = \frac{[v + v_{source}]}{f_{source}}$$

ie for the source moving away from the observer:

$$f_{\text{observed}} = f_{source} \frac{v}{[v + v_{source}]}$$

where *v* is the speed of the waves, eg the speed of sound.

Similarly, for the source moving towards the observer:

$$f_{\text{observed}} = f_{source} \frac{v}{[v - v_{source}]}$$

The first equation gives the expected reduction in frequency and the second an increase in frequency. If you are unsure hether to use the '+' or '–', you should check that your answer gives the expected increase or decrease. It is good practice for you to check your final answers are 'sensible', or have increased or decreased as predicted.

Frequency increases as they come towards you and frequency decreases as they move away.

We can also see from these equations that if the magnitude of v_{source} is very small compared to v there is little effect on $f_{observed}$. This is why for the effect to be noticeable, v_{source} should be reasonably large in comparison with the speed of the waves (v).

ALTERNATIVE DERIVATIONS- just choose one lot that you like but learn them!

METHOD 2

v = velocity of sound

v_s = velocity of the source

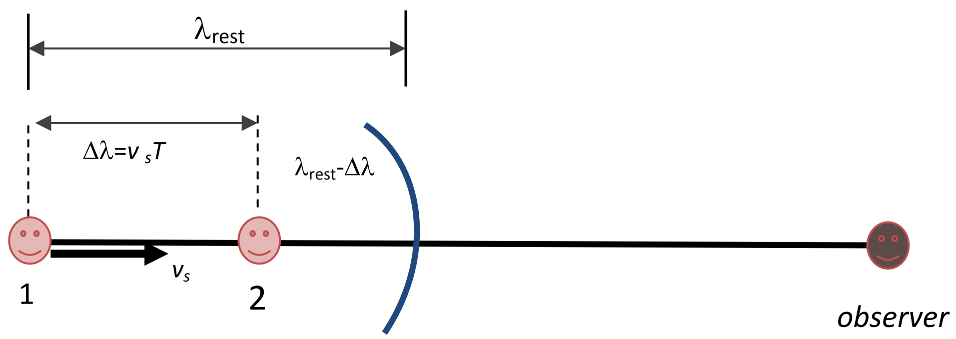
f_s = frequency of the source

f_o = observed frequency

λ_{rest} = original wavelength

For this derivation we first consider a stationary observer with the source moving towards the observer. The diagram shows the crest of the sound wave emitted from the moving source, which is about to emit the next crest. In the period (T, time for one wave) the source has moved from position 1 to position 2. To start you need to know the basic wave equation and the formula for speed distance and time.

$v = f\lambda$ and $v = \dfrac{d}{t}$



$$f_o = \frac{v}{\lambda_{rest} - \Delta\lambda}$$

but $\Delta\lambda = v_s \times T$

(velocity of the source $\times$ Period for one wave)

$$f_o = \frac{v}{\lambda_{rest} - v_s T} \quad \text{but } T = \frac{1}{f}$$

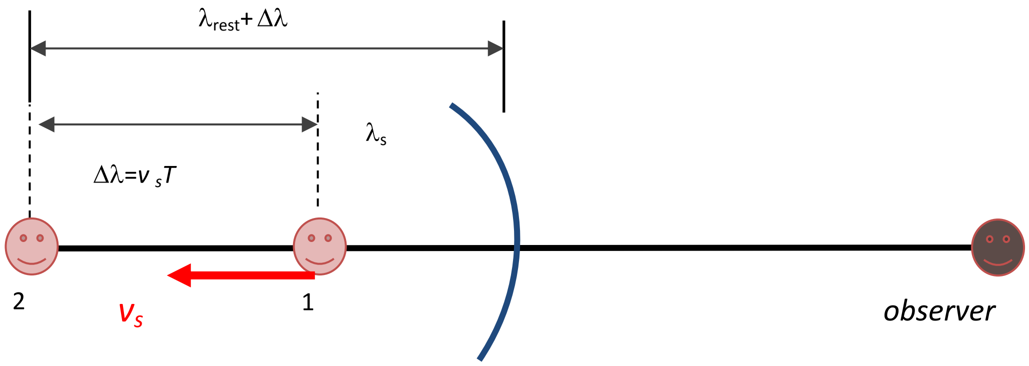
$$f_o = \frac{v}{\lambda_{rest} - v_s / f_s} = \frac{v}{\frac{\lambda_{rest} f_s}{f_s} - \frac{v_s}{f_s}} = \frac{v}{\frac{v}{f_s} - \frac{v_s}{f_s}}$$

$$f_o = f_s\left(\frac{v}{v - v_s}\right)$$

When the source is moving towards the observer the sign is negative

Next we consider a stationar y observer with the source moving **away** from the observer. The diagram shows the crest of the last sound wave emitted from the moving source, which is about to emit the next crest. To start you need to know the basic wave equation and the formula for speed distance and time.

$v = f\lambda$ and $v = \frac{d}{t}$



$$f_o = \frac{v}{\lambda_{rest} + \Delta\lambda}$$

but $\Delta\lambda = v_s \times T$

(velocity of the source $\times$ Period for one wave)

$$f_o = \frac{v}{\lambda_{rest} + v_s T} \quad \text{but } T = \frac{1}{f}$$

$$f_o = \frac{v}{\lambda_{rest} + v_s / f_s} = \frac{v}{\frac{\lambda_{rest} f_s}{f_s} + \frac{v_s}{f_s}} = \frac{v}{\frac{v}{f_s} + \frac{v_s}{f_s}}$$

$$f_o = f_s\left(\frac{v}{v + v_s}\right)$$

When the source is moving away the observer the sign is positive

METHOD 3

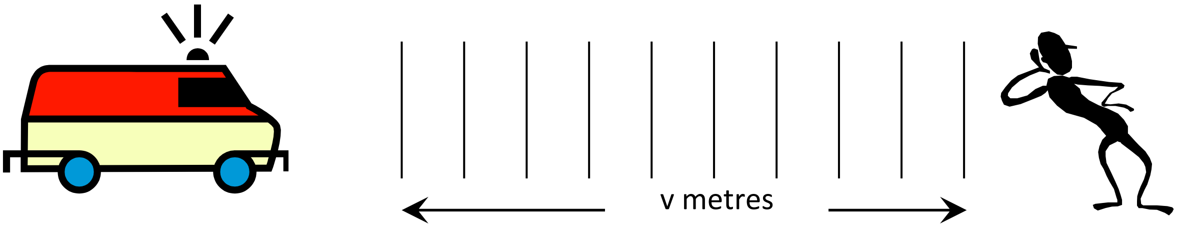
We can consider how this effect occurs. A source produces a sound of frequency f_s.

speed of sound = v speed of source = v_s

frequency source = f_s observed frequency = f_o

The frequency of the source will remain constant, it is the observed frequency that changes.

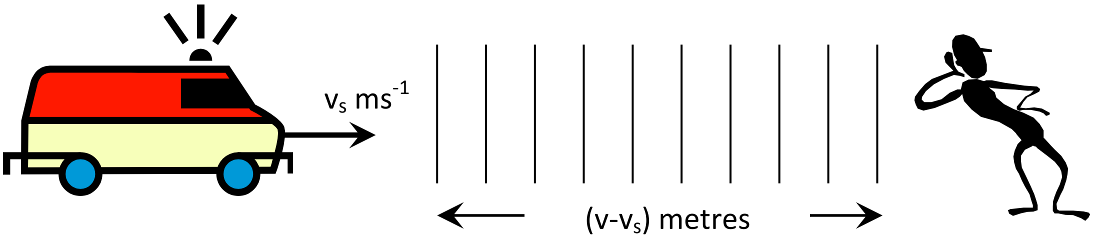
Movement towards observer: The waves will 'squash up'



Stationary: in 1 second there will be f_s waves produced. The sound will travel a distance of v metres in 1 second. This means there are f_s waves in a distance of v metres.

Moving at speed v_s: In the same way as above, in 1 second f_s waves will cover a **distance** of **(v - v_s)** metres.

As $v = \frac{d}{t}$ then numerically in one second the value of v is equal to the value of *d*



The observed wavelength of the waves will be

$$\lambda_{observed} = \frac{v}{f}$$

$$\lambda_{observed} = \frac{(v - v_s)}{f_s}$$

$$f_{observed} = \frac{v}{\lambda_{rest}}$$

$$f_o = \frac{v}{\frac{(v - v_s)}{f_s}}$$

$$f_o = f_s \frac{v}{(v - v_s)}$$

This gives an <u>observed</u> frequency that is **higher/greater** than that of the source.

If the source is moving away from the observer then the frequency observed will be **lower/smaller** than that of the source.

$$f_o = f_s \frac{v}{(v + v_s)}$$

Providing objects emitting or reflecting light are travelling at speeds less than 10%c then we can use this Doppler equation to find out fast objects are moving.

For a stationary observer with a light source moving **towards** them, the relationship between the original frequency, f_s, of the source and the observed frequency is:

$$f_o = f_s \left(\frac{c}{c - v_s} \right)$$

For a stationary observer with a light source moving **away** from them, the relationship between the original frequency, f_s, of the source and the observed frequency is:

$$f_o = f_s\left(\frac{c}{c+v_s}\right)$$

This second scenario is exactly what is observed when we look at the light from distant stars, galaxies and supernovae. These relationships also allow us to calculate the speed at which an exoplanet is orbiting its parent star, or the velocity of stars orbiting a galactic core, which has lead us to theorise the existence of dark matter.

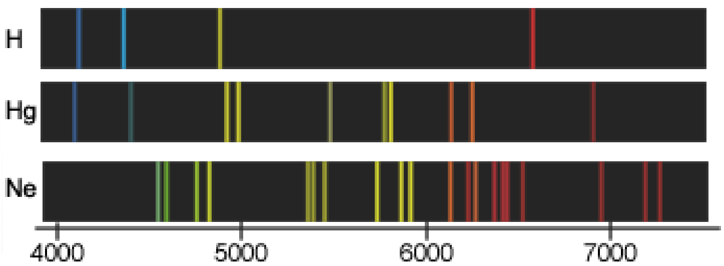
http://www.youtube.com/watch?v=Y5KaeCZ_AaY

BIG BANG THEORY

LINE SPECTRA

All atoms give off electromagnetic radiation when heated, although sometimes this is not in the visible part of the spectrum. Each element has its own distinctive line spectrum. A prism is an example of a device that can be used to split this light to form a spectrum. Spectroscopy is the name given to the experimental technique of spectroscopic methods; but also refers to the measurement of radiation intensity as a function of wavelength. Spectral measurement devices are referred to as spectrometers, spectrophotometers, spectrographs or spectral analyzers.

Some examples of what line spectra look like are shown below:



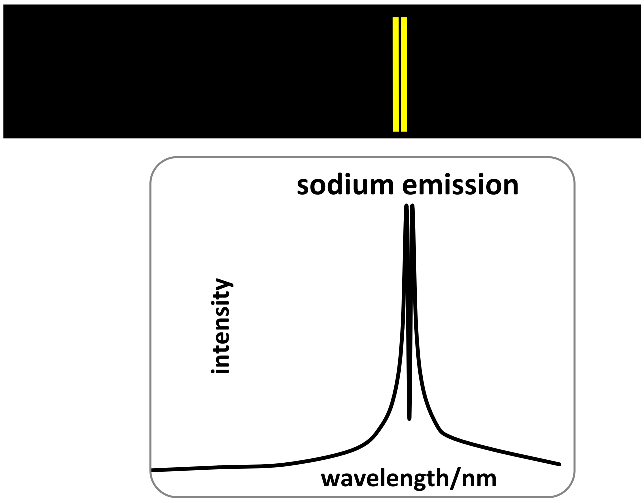
Scientists have used line spectra to discover new elements. In fact, the discovery of some elements, such as rubidium and caesium, was not possible until the development of spectroscopy. The element helium was discovered by studying line spectra emitted by the Sun.

Each element has its own characteristic line emission spectrum and the corresponding absorption spectrum.

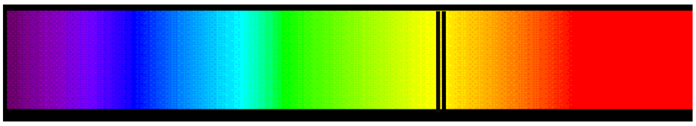
These can be indicated on a diagram as a coloured strip or as intensity versus wavelength graph.

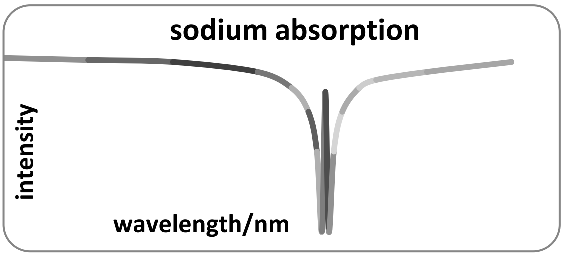
eg

Sodium emission spectrum



Sodium absorption spectrum





REDSHIFT, Z

Redshift is an example of the Doppler Effect. It is the term given to the change in frequency of the light emitted by stars, as observed from Earth, due to the stars moving away from us.

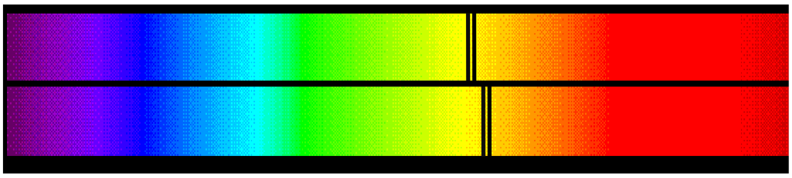
Redshift has always been present in the light reaching us from stars and galaxies but it was first noticed by astronomer Edwin Hubble, in the 1920's, when he observed that the light from distant galaxies was shifted to the red end of the spectrum.

The light emitted by a star is made up of the line spectra emitted by the different elements present in that star. Each of these line spectra is an identifying signature for an element and these spectra are constant throughout the universe. You will learn a lot more about spectra in the Particles and Waves unit of this course.

Sometimes the spectral lines are found to have been moved or shifted towards the red end of the spectrum because of an increase in wavelength. This effect is called redshift and, assuming it is due to the Doppler Effect, implies these bodies are moving away from us. The amount of the shift also allows us to calculate how fast they are moving away.

Normal

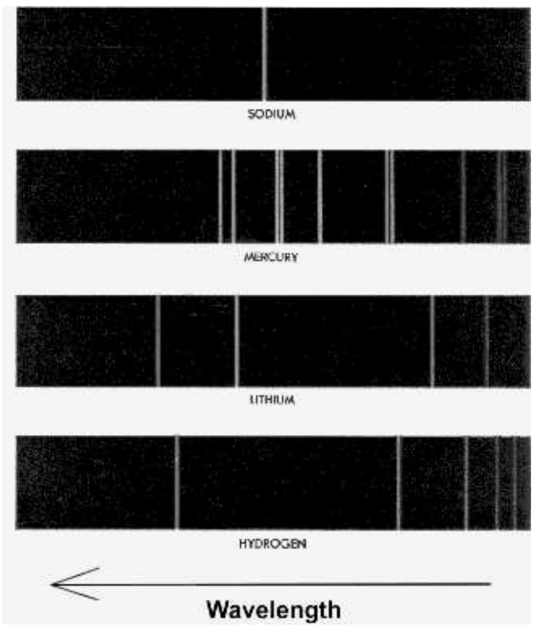
Redshifted

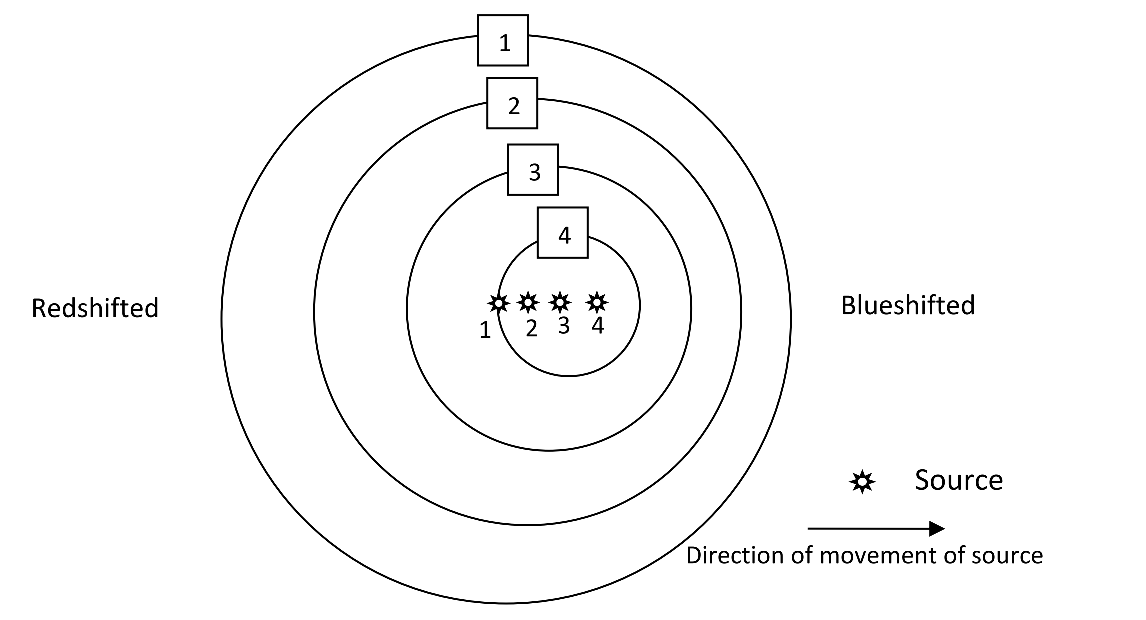


Redshift

An intensity versus wavelength graph also allows spectra outwith the visible region to be represented. These characteristic spectra allow elements in the stars and space to be identified.

Since these line spectra are so recognisable, we can compare the spectra produced by these elements, on Earth, with the spectra emitted by a distant star or galaxy.





Doppler Effect and redshift

Redshift, *z*, of a galaxy is defined as the change in wavelength divided by the original wavelength, and given the symbol *z*.

So, redshift

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

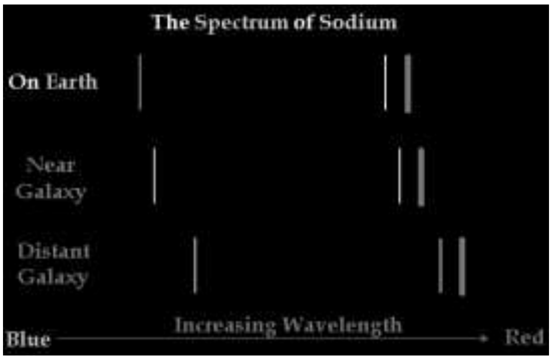
Where *z = redshift,* λ_{obs} *= observed wavelength*

λ_{rest} *= wavelength of the source*

Note: Redshift is a dimensionless quantity since it is a ratio of two lengths.

Redshift of galaxies, travelling at non-relativistic speeds, can also be shown to be the ratio of the velocity of the galaxy to the velocity of light:

$$z = \frac{v_{galaxy}}{c}$$



The redshift of a galaxy

http://www.youtube.com/watch?v=LIvVzJ6KZpk this even has a canal!

The Doppler Effect also applies to electromagnetic waves, but at high speeds relativistic effects have to be taken into account.

For bodies travelling at non-relativistic speeds (ie less than 10% of the speed of light) we can apply the Doppler equation for a stationary observer and a moving source. Using the Doppler equation for a source moving away from the observer and substituting *c* (the speed of light) *for v* , we get:

$$f_{observed} = f_{source}\frac{c}{[c + v_{source}]}$$

$$\frac{f_{source}}{f_{observed}} = \frac{[c + v_{source}]}{c} \quad \text{............. [1]}$$

Additionally $c = f\lambda$, then :

$$c = f_{source}\lambda_{rest} = f_{observed}\lambda_{observed}$$

$$\frac{f_{source}}{f_{observed}} = \frac{\lambda_{observed}}{\lambda_{rest}} \quad \text{.......... [2]}$$

From the definition of redshift,

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

$$z = \frac{\lambda_{observed}}{\lambda_{rest}} - \frac{\cancel{\lambda_{rest}}}{\cancel{\lambda_{rest}}}$$

$$z = \frac{\lambda_{observed}}{\lambda_{rest}} - 1$$

Substituting from [2] we get:

$$z = \frac{f_{source}}{f_{observed}} - 1$$

(But from the Doppler Equation) Substituting from [1] we get

$$z = \frac{[c + v_{source}]}{c} - 1$$

$$zc = \frac{\cancel{c}[c + v_{source}]}{\cancel{c}} - c$$

$$zc = [\cancel{c} + v_{source}] - \cancel{c}$$

$$zc = v_{source}$$

$$z = \frac{v_{source}}{c}$$

Note: This equation only applies to non-relativistic speeds, say less than 10% of the speed of light.

As an aside, which is not required for this current Higher Course: For electromagnetic radiation there is no fixed medium to provide a frame of reference and relativity has to be taken into account? For the source and observer moving rapidly apart when relativistic speeds are required the equation for observed frequency is:

$$f_{observed} = f_{source} \left\{ \frac{1 - {}^{v}/_{c}}{1 + {}^{v}/_{c}} \right\}^{1/2}$$

The exact relationship between redshift and velocity derived from the special theory of relativity is given by:

$$z = \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{1/2} - 1 \qquad or \qquad z = \sqrt{\left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)} - 1$$

Even this relationship ignores the expansion of the universe as the light propagates through it. This relationship is really the combined effect of the standard Doppler equation and a time dilation factor.

For a nice explanation, see: http://mb-soft.com/public/reltvty1.html.

Note: Radial speed

The redshift can only give us the radial speed of the object; it tells us nothing about its tangential speed

BLUESHIFT.

Note: If there is a decrease in wavelength, ie the line spectrum has moved towards the blue end of the spectrum, this makes z negative, which means the body is moving towards us. This is referred to as a blueshift.

See electronic version, it works so much better in colour!

ANECDOTE

This is the apocryphal tale of an astronomer in a hurry on his way to work. He goes through a red traffic light and is stopped by traffic police. He explains the light looked green as he passed it and explains this was perhaps due to the Doppler Effect and blueshift.

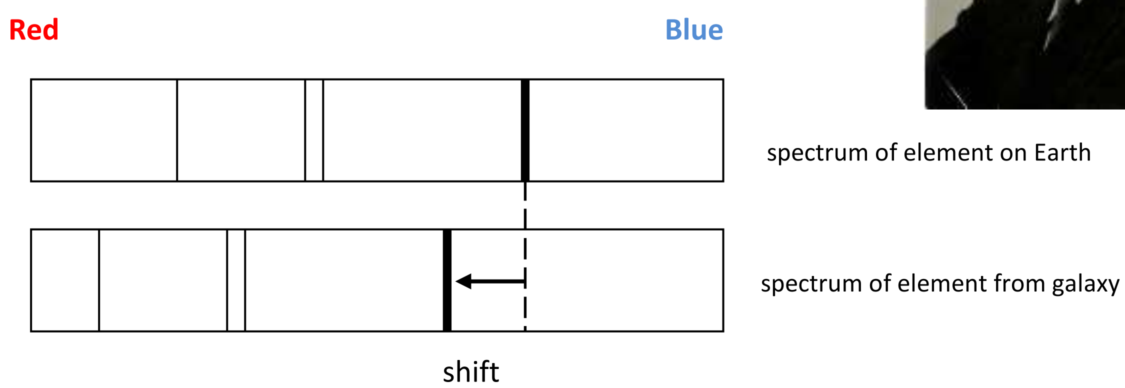
The policeman listens intently, accepts his explanation but then proceeds to write a speeding ticket.

'Why?' enquires the astronomer. The policeman explains that for such a blueshift to occur, he must have been travelling at approximately 2×10^8 kmh^{-1} !

HUBBLE'S LAW

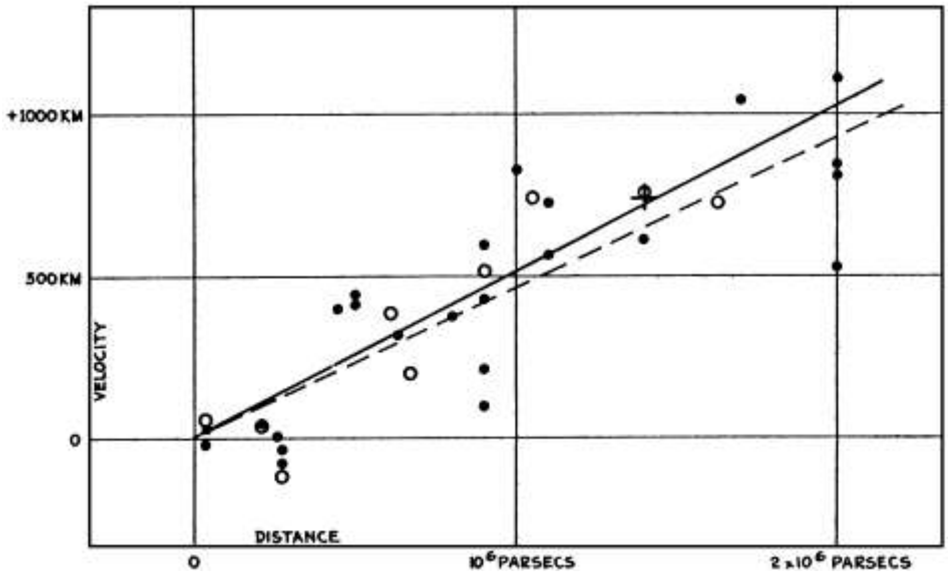
The astronomer Edwin Hubble noticed in the 1920's that the light from some distant galaxies was shifted towards the red end of the spectrum.

Hubble examined the spectral lines from various elements and found that each galaxy was shifted towards the red by a specific amount. This shift was due to the galaxy moving away from the Earth at speed, causing the Doppler Effect to be observed. The bigger the magnitude of the shift the faster the galaxy was moving.



Over the course of the few years Hubble examined the red shift of galaxies at varying distances from the Earth. He found that the further away a galaxy was the faster it was travelling. The relationship between distance and speed of galaxy is known as Hubble's Law.

The graph below shows the data collected by Hubble. It shows the relationship between the velocity of a galaxy, as it recedes (moves away) from us, and its distance, known as **Hubble's Law**.



x axis is the distance: *d* / Mpc (Mpc =megaparsec)

y axis is the recession velocity: *v* / km s^{-1}

Information from the graph above

The graph shows velocity against distance among extra-galactic nebulae. Radial velocities, corrected for solar motion (but labelled in the wrong units), are plotted against estimated distances. The black discs and full line represent the solution for solar motion by using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually. [Reproduced with permission from *Hubble, E. P. (1929) Proc. Natl. Acad. Sci. USA 15*, *168–173.* (Copyright 1929, The Huntington Library, Art Collections and Botanical Gardens).]

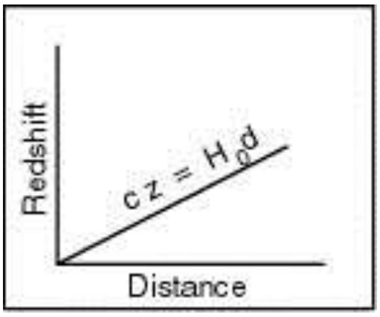
From his results, he concluded that more distant galaxies are receding faster than closer ones. In fact he found that the recession speed (*v*) varied directly with distance (*d*), ie:

$v \propto d$, so $v = H_o d$

$$\text{gradient} = \frac{v}{d}$$

This gradient is known as Hubble's constant, H_0:

$$H_o = \frac{v}{d}$$



$$z = \frac{v}{c}$$

$$\textit{thus} \qquad v = zc$$

$$\textit{therefore} \qquad H_o = \frac{zc}{d}$$

$$\textit{and} \qquad H_o d = zc$$

$$\textit{thus} \qquad H_o d = v$$

Where H_0 = the Hubble constant

c = speed of light

d = distance to galaxy

$v = H_0 d$

UNITS OF HUBBLE'S CONSTANT H_0:

The value of the Hubble constant is not known exactly, as the exact gradient of the line of best fit is subject to much debate. However, as more accurate measurements are made, especially for the distances to observable galaxies, the range of possible values has reduced. It is currently thought to lie between 50 – 80 kms^{-1} Mpc^{-1}, with the most recent data putting it at 70.4 ± 1.4 kms^{-1} Mpc^{-1}. The units for the Hubble constant are always in units of time.

1 Mpc = 3·2 x 10^6 light years = 3·1 x 10^{22}m

For this course the Hubble Constant is given as

Hubble's Constant Ho **2·3 x 10^{-18} s^{-1}**

EVIDENCE FOR THE EXPANDING UNIVERSE

From Hubble's graph of speed versus distance, we can obtain an estimate of how long it took for a galaxy to reach its current position. Assuming they have been moving away from us at a constant speed, the time taken for a particular galaxy to reach its current position can be found by dividing the distance by the speed.

Galaxies are moving away from the Earth and each other in all directions, which suggests that the universe is expanding. This means that in the past the galaxies were closer to each other than they are today. By working back in time it is possible to calculate a time where all the galaxies were in fact at the same point in space. This allows the age of the universe to be calculated. Currently NASA have a value of 13.7 billion years as the age of the universe from this method.

Example:

v = speed of galaxy receding from us

d = distance of galaxy from us

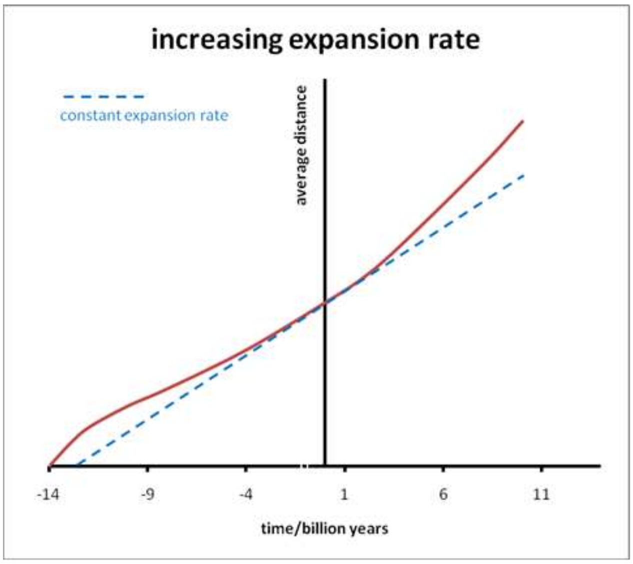
H_0 = Hubble's constant

t = time taken for galaxy to reach that distance, i.e. the age of the universe

$$t = \frac{d}{v} \qquad t = \frac{d}{H_o d} \qquad t = \frac{1}{H_o}$$

If we take the inverse of the Hubble constant we should be able to predict the age of the Universe i.e.

$$\begin{aligned} t &= \frac{1}{2.3\times10^{-18}} = 4.35\times10^{17}\ s \\ &= \frac{4.35\times10^{17}}{3600\times24\times365}\ years \\ &= 13.78\times10^{9}\ years \end{aligned}$$



Since Hubble's time, there have been other major breakthroughs in astronomy. All of these support the findings of Hubble, and allow the age of the universe to be calculated even more accurately. Such a discovery is Cosmic Microwave Background (CMB). Observations of the CMB also support the theory that the universe is expanding out from a single point, as Hubble postulated.

We have also discovered, however, that not only is the universe expanding but it is expanding at an increasing rate, i.e. the acceleration is increasing. This was the conclusion of astronomers in 1998 when observing distant supernovae. Their discovery was a great shock to the scientific community and was awarded the Nobel Prize in Physics in 2011.

THE EXPANDING UNIVERSE

WHY SUCH A SHOCK?

The force of gravity acts between all matter in the universe. Matter clumps together due to gravity, such as the contraction of hydrogen gas to create new stars, the grouping of stars to create galaxies and the grouping of galaxies to create local groups and superclusters. The Hubble telescope has been able to give us a glimpse of the universe on an even larger scale and images of many of the observable galactic clusters show them gravitating towards each other to form unimaginably large structures, known as filaments.

The force of gravity *should* be an unbalanced force acting to slow the expansion down. A universe like this, which eventually collapses back in on itself is known as a **closed universe**. The force of gravity, *determined by the mass of the universe*, eventually overcomes any expansion and all matter accelerates back towards a central point. The alternative scenario is an open universe where the universe continues to expand. Which of these two scenarios is more likely is purely down to the total mass of the universe.

Hubble's Law and subsequent observations, shows that the rate of expansion of the universe is ***increasing***. This suggests that there is a force acting against the force of gravity, pushing matter apart. This force is causing a significant acceleration and so it is much greater in magnitude than the force of gravity. As yet, astronomers and cosmologists have not been able to determine a source of energy capable of producing this force. For lack of a better term it is, for now, simply referred to as **dark energy**.

Try this experiment

http://www.schoolsobservatory.org.uk/astro/cosmos/uniball

Summary

1. Closed universe: the universe will slow its expansion and eventually begin to contract.

2. Open universe: the universe will continue to expand forever.

EINSTEIN'S GREATEST BLUNDER

http://www.jb.man.ac.uk/~jpl/cosmo/blunder.html#[1]

THE COSMOLOGICAL CONSTANT

Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder of his life.

-- George Gamow, My World Line, 1970

Einstein's remark has become part of the folklore of physics, but was he right? He certainly had cause to feel rueful about the cosmological constant; he had introduced it into his general theory of relativity in 1917, as a last resort, to force the equations to yield a static universe. Even at the time, he apologized for doing so, because it spoiled the elegant simplicity of the field equations that he had struggled so hard to find. Of course the universe is not static, just as his original equations were trying to tell him; his blindness lost him the chance to make one of the great predictions in physics. Even worse, a little more analysis would have shown that his static universe was not stable, and would have started to expand or contract if its perfect equilibrium was disturbed in any way.

A universe which does not collapse in due to gravity but continues to expand indefinitely is called an **open universe**. This type of universe would undergo an end known as 'heat death'. This refers to the fact that, eventually, all energy becomes heat energy and as time goes on all matter would be so far apart that the heat energy of the universe would be spread too far apart to allow any further production of stars and galaxies.

As the magnitude of the force of gravity in the universe is dependent on the mass within that universe, **it is mass which determines whether a universe is open or closed and therefore the eventual fate of that universe**:

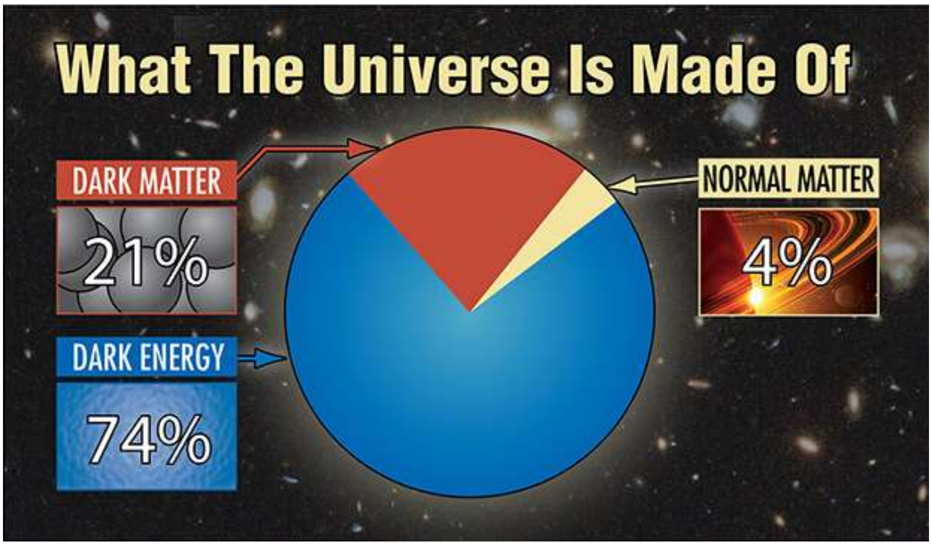
if a universe has enough mass then the force of gravity will be greater than that produced by dark energy; expansion will decrease and the universe will be closed.

If a universe does not have enough mass then the force produced by dark energy will be greater than gravity; expansion will continue indefinitely and the universe will be open.

DARK ENERGY VS. DARK MATTER

Our universe may contain 100 billion galaxies, each with billions of stars, great clouds of gas and dust, planets and moons. The stars produce an abundance of energy, from radio waves to X-rays, which streak across the universe at the speed of light.

Yet everything that we can see only accounts for only about 4% of the **total mass and energy** in the universe.



About one-quarter of the universe consists of dark matter, which releases no detectable energy, but which exerts a gravitational pull on all the visible matter in the universe. What dark matter consists of is uncertain at present, though it is believed not to be the type of matter with which we are familiar – electrons, neutrons, and protons**.**

While dark energy repels, dark matter attracts. And **dark matter's influence shows up even in individual galaxies, while dark energy acts only on the scale of the entire universe**

Are dark matter and dark energy related? No one knows. The leading theory says that dark matter consists of a type of subatomic particle that has not yet been detected, although upcoming experiments with the world's most powerful particle accelerator may reveal its presence. Dark energy may have its own particle, although there is little evidence of one.

Instead, dark matter and dark energy appear to be competing forces in our universe. The only things they seem to have in common is that both were forged in the Big Bang, and both remain mysterious.

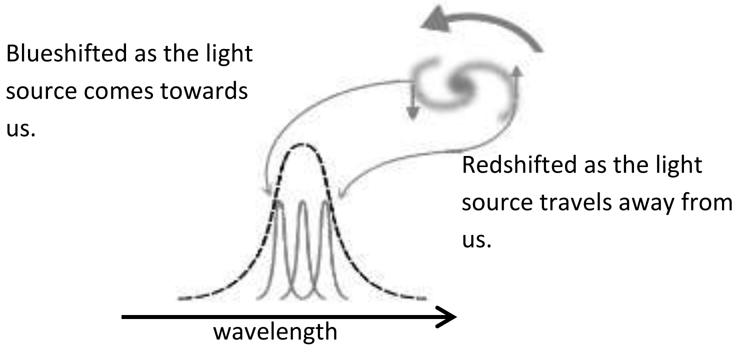
Because of the names, it's easy to confuse dark matter and dark energy. And while they may be related, their effects are quite different. **In brief, dark matter attracts, dark energy repels**. While dark matter pulls matter inward, dark energy pushes it outward. Also, while dark energy shows itself only on the largest cosmic scale, dark matter exerts its influence on individual galaxies as well as the universe at large.

MORE ON DARK MATTER

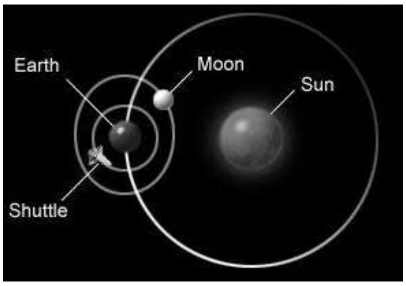
Observations of the Doppler Effect being evident in light observed from space has led to the development of another, equally perplexing theory; dark matter. This one comes from observing the light from individual stars within galaxies.

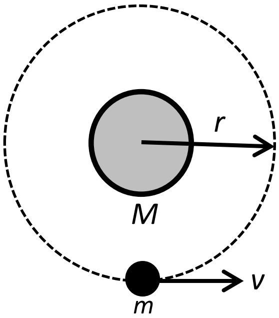
We know that galaxies rotate about their cores as the light observed on one side of a galaxy will be blue shifted, indicating that the source of that light is moving towards us, in comparison to the other side which will be redshifted, by the same amount.

Measuring the amount which the light is shifted by allows us to calculate the exact rotational velocity of that galaxy and thus the velocity of the stars within it.



From the section on gravitation you know that orbits are a careful balance between the gravitational field strength, created by a mass, and the velocity of a projectile.





If you give an object a great enough velocity it can escape the gravitational field it is in and escape from the orbit. An example of the same kind of action, occurring on Earth, is being on a roundabout in a park.

When on a roundabout you must hold on to stay on. If you did not provide this force your body would continue in a straight line and you would come of at a tangent to the circle. In the case of a roundabout, the force causing you to go in a circle is a force of friction. As the roundabout spins faster, so do you and you must provide a larger and larger force, by holding on more tightly? If you cannot provide a force big enough you will come off.

http://www.youtube.com/watch?v=m0bSJkrDYH8 (idiots! NEVER TRY THIS IT HAS LED TO SERIOUS INJURY!)

It is exactly the same with an orbit in space. Only here, the force causing the object to continue in a circle is gravity. As the magnitude of this force is determined by the mass of the planet, star, or galaxy, it is fixed and cannot increase. If the object travels too fast, the force of gravity will not be able to keep it on a circular path and it will escape its orbit and travel off in a straight line.

If we know:

- the mass of the orbiting object, m
- the velocity of the orbiting object, v
- the radius of its orbit, r

Then we can calculate the mass of the central body, M, using the Universal Law of Gravitation.

Stars on the on the outer arms of the galaxy should travel slower than those towards the galactic core as they are further from the central mass and therefore experience a smaller gravitational force

$$F\alpha\frac{1}{r^2}$$

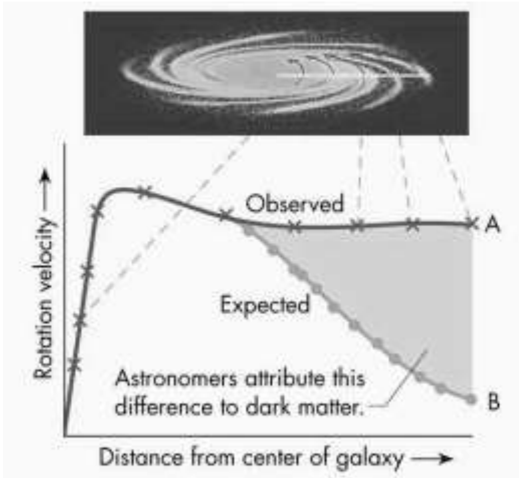
We can directly observe the distribution of matter, and therefore mass, within a galaxy from its brightness. Matter in a galaxy is either producing light,(stars), or reflecting it, (nebulae and dust).

WHAT DO WE OBSERVE?

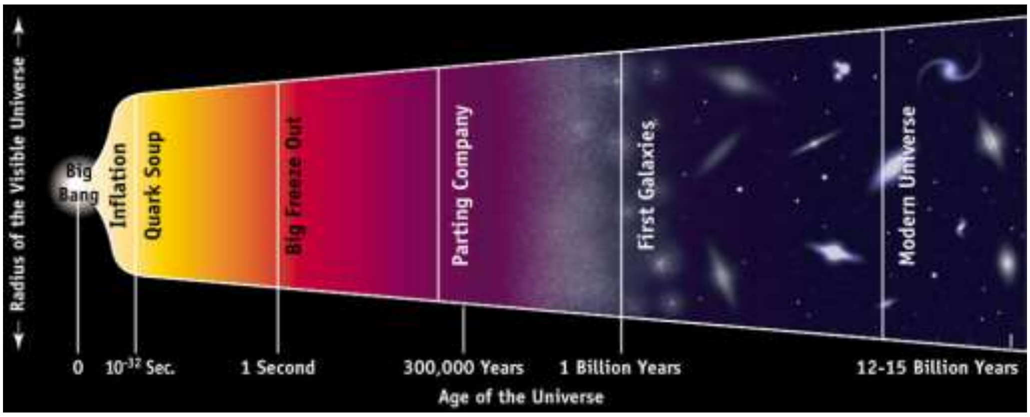
The stars are travelling too fast.

As you can see from the graph above, the velocity of stars does not drop off as expected, at greater orbital radii. At these high velocities the observed mass of the galaxy should not be enough to hold on to many of its stars and we should see them fly off into intergalactic space.

The only logical conclusion that astronomers have to explain this consistent observation is that there must be a significant amount of mass that we cannot see. Hence the name: **dark matter**.



THE BIG BANG THEORY



http://archive.ncsa.illinois.edu/Cyberia/Cosmos/Images/CosmicTimeline_gr.jpg

http://www.schoolsobservatory.org.uk/astro/cosmos/bb_history

Scientists have gathered a lot of evidence and information about the universe. They have used their observations to develop a theory called the Big Bang. The theory states that originally all the matter in the universe was concentrated into a single incredibly tiny point, called a singularity. This began to enlarge rapidly in a hot explosion, and it is still expanding today. This explosion is called the Big Bang, and happened about 13.7 billion years ago (that's 13,700,000,000 years using the scientific definition of 1 billion = 1,000 million).

The universe started with a sudden appearance of energy which consequently became matter and is now everything around us. There were two theories regarding the universe

The Steady State Universe: where the universe had always been and would always continue to be in existence.

The Created Universe: where at some time in the past the universe was created.

Ironically the term ‘Big Bang’ was coined by Fred Hoyle a British astronomer who was the leading supporter of the Steady State theory and who was vehemently opposed to the, currently named, Big Bang theory. He used the phrase ‘Big Bang theory’ during a broadcast on the BBC radio station The Third Programme, broadcast on 28 March 1949.

WHAT WAS THE EVIDENCE THAT FINALLY SWUNG THE BALANCE TOWARDS THE BIG BANG THEORY?

DETERMINING THE TEMPERATURE OF DISTANCE STARS & GALAXIES.

From everyday experience we know that substances glow (incandesce) when heated to very high temperatures. The colour of light emitted depends on exactly how high the temperature is.

When an object is heated it does not initially glow, but radiates large amounts of energy as infrared radiation. We can feel this if we place our hand near, but not touching, a hot object.

As an object becomes hotter it starts to glow a dull red, followed by bright red, then orange, yellow and finally white (white hot). At extremely high temperatures it becomes a bright blue-white colour.

We first need to consider how it is possible to determine the temperature of distant stars and galaxies. You will have seen what happens to a piece of iron as it is heated, as it gets hotter its colour changes from dull red to bright red to orange then yellow.

The observant amongst you may realize that these are the first colours in the visible spectrum. The temperature of an object determines the frequency of light it emits. This idea has been with us for a long time; Jožef Stefan proposed in 1879 that the power irradiated from an object was proportional to its temperature in Kelvin to the fourth power.

$$P = \sigma T^4$$

Where σ is Stefan's (also referred to as the Stefan-Boltzmann) constant and

$$\sigma = 5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}.$$

What this means is that by examining the spectrum of a distant star, its temperature can effectively be determined.

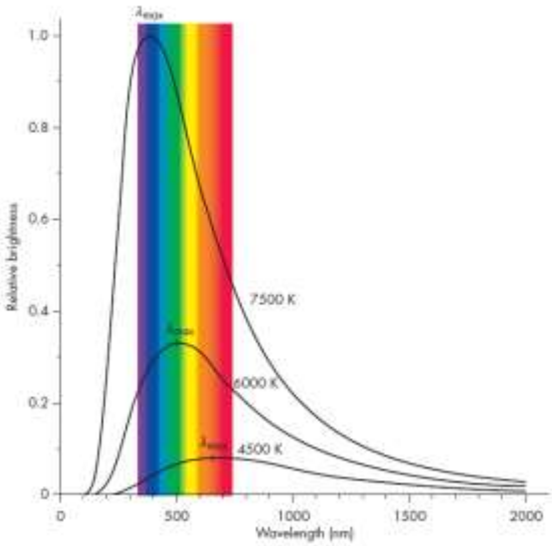


Figure 3 Courtesy of John D Fix Astronomy Journey to the Cosmic Frontier 5th Ed.

A "black-body" is an ideal object whose temperature is the same at every point and constant in time. It is useful for study because such an object produces a spectrum which is dependent only on the temperature. The spectrum of black-body radiation is continuous from zero energy at zero wavelength, rising to a maximum at some wavelength determined by Wien's law (AH Physics), and then falling off to zero at infinite wavelength. The hotter the black body, the more energy is produced at EVERY wavelength. Black-bodies are useful models because many stars radiate approximately as black-body.

The black-body spectrum has three main features:

1. **The basic shape is more or less the same (apart from a scaling factor) at all temperatures; it is like a skewed bell curve, falling off gently on the long wavelength side of the peak, and much more sharply on the shorter wavelength side.**
2. **As the temperature of the object is increased, the peak of the intensity spectrum shifts towards the shorter wavelengths.**
3. **As the temperature of the object is increased, the intensity increases for all wavelengths.**

The properties of the spectrum are characterised by a single parameter, temperature, hence it is sometimes referred to as a thermal radiation spectrum.

When the temperature is raised the peak moves towards the short wavelength end (blue), which gives the visible effect of changing from red to orange to yellow to white to blue-white, in that order.

BANG?

The Big Bang was not like a conventional explosion in which matter is thrown apart from a single point into a pre-existing empty space.

Most cosmologists think space, time and matter originated at the Big Bang and before that there was no space, time or matter in the sense we think of them today.

Rather than thinking of galaxies as flying away from each other through space, it is much better to think of them as being essentially at rest in an expanding space.

EVIDENCE FOR THE BIG BANG THEORY

REDSHIFT (SEE PREVIOUS NOTES) / EVIDENCE FROM THE EXPANDING UNIVERSE

From the expanding universe idea we can extrapolate our figures back (we consider running time backwards) and come to the conclusion that all matter in the Universe was all collected at a single point called the initial singularity.

COSMIC MICROWAVE BACKGROUND (CMB) RADIATION

CMB is predicted by the hot Big Bang theory.

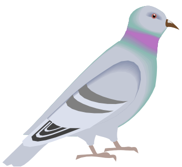
During the very early stages of the universe, say when it was one millionth its present size, the temperature would have been around 3,000,000 K. If an electron became bound with a nucleus, high energy radiation would immediately strip it off. The universe was in a plasma state.

The universe expanded and cooled due to the redshifting effect. As space expanded, the wavelength of photons became longer, so each photon had less energy.

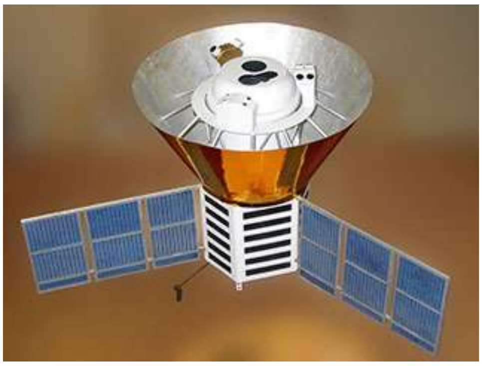
Gamow, Alpher and Herman, three physicists, had produced a paper in 1948 that if the Big Bang had actually taken place then there would be a residual background EM radiation, in the microwave region, in every direction in the sky representing a temperature of around 2.7K.

This value for the wavelength of the light and it's consequent equivalent temperature was arrived at by considering how the light produced at the Big Bang would have changed as the universe expanded.

The discovery of this background radiation was another example of scientists finding something they weren't looking for.



Arnold Penzias and Robert Wilson were working for Bell Labs in the USA. They were working with a special radio telescope [shown in the picture above] experimenting with satellite communication.



They were getting a residual signal that seemed to come from outside the galaxy. At first they though it was actually due to pigeon droppings from the pigeons that roosted in the horn. Finally they realised that they had found the echo of the Big Bang.

As one of their colleagues commented,

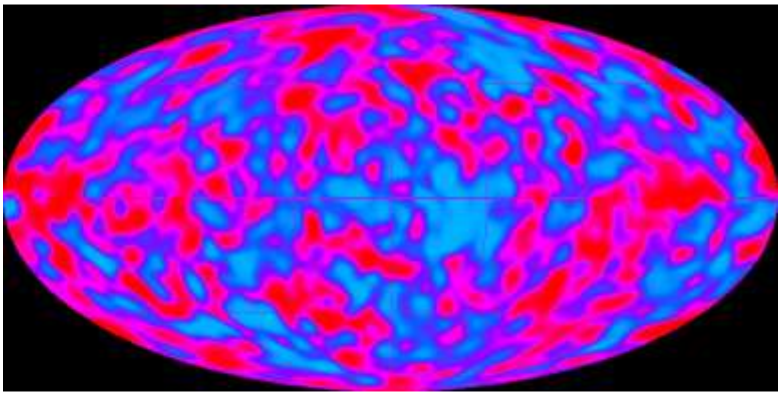
"Thus, they looked for dung but found gold, which is just opposite of the experience of most of us."

He may not have actually said the word dung but I'm not going to write in the actual word as I find it too rude!

In 1989 a satellite was launched to study the background radiation, it was called the Cosmic Background Explorer [COBE].

In 1992 it was announced that COBE had managed to **measure fluctuations in the background radiation**. This was further evidence to support the Big Bang theory.

An image of the fluctuations is shown below.



BLACK BODY RADIATION

Stars are black bodies in that they absorb all the radiation. The existence of black-body (thermal) radiation, at a temperature of about 3000 K, filling the universe at the time of recombination (when electrons combined with nuclei to form atoms), is consistent with a universe which has expanded and cooled from a much hotter and denser original state at much earlier times – as the Big Bang theory predicts.

THE HELIUM PROBLEM

Other evidence to support the Big Bang theory includes the relative abundances of hydrogen and helium in the universe. Scientists predicted that there should be a significantly greater proportion of hydrogen in the universe. The next most abundant should be helium. The most abundant elements in the universe are hydrogen (70–75%) and helium (25–30%). However, the abundance of helium in the universe cannot be explained by the formation inside stars as most of it remains locked up in their cores. However, it can be accounted for by being formed during the dense phase of the early universe, ie shortly after the Big Bang.

The elements present in the universe can be determined by spectroscopy, which you will study later in Particles and Waves.

The ability to determine the elements in space brought an updated version of an old song

Twinkle, twinkle little star,
I don't wonder what you are;
For by spectroscopic ken,
I know that you are hydrogen;
Twinkle, twinkle little star,
I don't wonder what you are.

The latest proportions are given in the table shown. These observations conform to the predicted proportions.

Element	relative abundance
Hydrogen	10 000
Helium	1 000
Oxygen	6
Carbon	1
All others	1

OLBERS' PARADOX

WHY IS THE NIGHT SKY DARK?

This question can be traced back to around 1576 and Thomas Digges, but it was first stated formally by the Prussian astronomer Heinrich Olbers in 1823, hence the name. It was commonly assumed, prior to the expansion of the universe being demonstrated by Hubble in 1929 that the universe was:

1. infinite
2. eternal
3. static.

If this was true, no matter which direction you looked, your line of sight would eventually intersect with a star. The entire sky would be virtually as bright as the Sun!

You may think that cannot be true, since distant stars are fainter than near ones. However, surface brightness is independent of distance. This is because although the flux received decreases with the square of the distance, so does the apparent size of the star, so the flux per unit area stays the same. That means the further you look into space, a greater number of stars will appear in your line of sight, compensating for any dimming due to the inverse square law.

Olbers' own explanation – that invisible interstellar dust absorbed the light – would make the intensity of starlight decrease exponentially with distance.

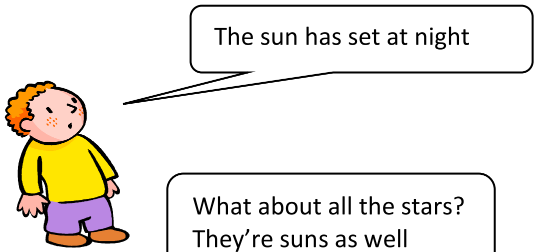
It can be shown that the amount of dust needed to do this would be so great as to block out the Sun! Also the radiation would heat up the dust so much that it would start to glow, becoming visible in the infrared region.

One solution to the paradox comes from considering the finite time light takes to reach us. Consider a galaxy very distant from our own. We only become aware of its existence when light from it reaches us. So if the universe is not infinitely old, galaxies must exist that are so distant that their light has not reached us yet. If the universe is not infinitely old, that implies that it was created some time in the past, which is consistent with the Big Bang theory.

Another is the explanation for Olber's paradox was in answer to the question, "why is the sky dark at night?"

This is not as obvious as you first might imagine.

Tam says:



Wullie replies:

The Big Bang theory gives a finite age to the universe, and only stars within the observable universe can be seen. This means that only stars within the distance of 15,000 light years will be observed. Not all stars will be within that range and so the dark sky can be explained. So the main reason why the night sky is dark is due to the finite age of the galaxies, not the expansion of the universe. This is consistent with the hot Big Bang model, but not with a steady-state universe.

A SUMMARY OF SOME OF THE EVIDENCE OF THE BIG BANG AND ITS INTERPRETATION

Evidence	Interpretation
The light from other galaxies is red-shifted.	The other galaxies are moving away from us.
The further away the galaxy, the more its light is red-shifted.	The most likely explanation is that the whole universe is expanding. This supports the theory that the start of the universe could have been from a single explosion.
Cosmic Microwave Background	The relatively uniform background radiation is the remains of energy created just after the Big Bang.

FURTHER QUESTIONS ON THE UNIVERSE

WHERE IN THE UNIVERSE WAS THE BIG BANG?

Everywhere! It’s a bit like asking ‘Where in your body were you born?’

WHAT DOES THE UNIVERSE EXPAND INTO?

The basic problem lies with the nature of spacetime. In order for the universe to expand into some larger space, that large space would need to exist in another universe. After all, what does ‘space’ even mean outside of the universe?

If you try to imagine yourself watching our universe from the outside, doesn’t that mean you are in a universe surrounding ours? The key is to think of the universe as a collection of information rather than a collection of objects. Think of it more like the flash memory in an MP3 player. When you record more tracks of music, the memory chip itself doesn’t become any larger.

For more information read the document

The Expanding Universe and Big Bang Theory Teacher’s Notes by Nathan Benson

ExpandingUniverseBigBangTheoryTeachersNote_tcm4=649504(1).doc

DEFINITIONS FOR THE EXPANDING UNIVERSE:

Black Body Radiation

A "black-body" is an ideal object whose temperature is the same at every point and constant in time. It is useful for study because such an object produces a spectrum which is dependent only on the temperature. The spectrum of black-body radiation is continuous from zero energy at zero wavelength, rising to a maximum at some wavelength determined by Wien's law, and then falling off to zero at infinite wavelength. The hotter the black body, the more energy is produced at EVERY wavelength. Black-bodies are useful models because many stars radiate approximately as black-body.

Heliocentric Universe

A model of the universe that puts the sun at its center.

Hubble Expansion

Used to describe the expansion of the universe based on the redshifted light of distant galaxies. The relationship between recession velocity and distance is described by Hubble's Law, where [recessional velocity of an object]=[Hubble constant]*[distance of object from Earth]

Infinite

When we talk about the universe being infinite in relation to steady state theory, we can talk about several things. First, steady state theory predicts the universe to be infinitely big, without limits. Secondly, steady state theory says the universe is infinitely old, as old as time itself.

Isotropic

An isotropic universe has no preferred direction. It acts the same in every direction. For example, the redshift of distant galaxy clusters looks the same from our location as it does from another distant cluster in the universe.

Not Expanding

A universe that is not expanding is what we call static. According to steady state theory, the universe just is. It cannot be expanding because steady state theorists think this is the way the universe has always looked. A universe that is expanding would have looked very different at a time billions of years ago.

Primordial Soup

The name for the time in the evolution of the universe where the universe was to hot to form ordinary matter. The universe was opaque and glowed due to photons that were being continuously scattered by electrons.

Redshift

When a radiating object moves away from us, we observe a *redshift* in its light, or the light waves it emits are getting longer (shifting to the red part of the spectrum).

Steady State Theory

Theory of the creation of the universe that says the universe has been and always will be like it is today. It assumes that the universe is uniform, infinite, and not expanding.
For more about steady state theory go to: http://cfpa.berkeley.edu/Education/IUP/Big_Bang_Primer.html.

Uniform

When we talk about a homogeneous universe, we are making the assumption that the universe is *uniform*, or has the same makeup throughout. So, we figure that the matter density of our local region, or lets say the amount of galaxies, stars, gas and dust per a certain volume is pretty much the same anywhere in the universe.

Visible Spectrum

This is the part of the Electromagnetic Spectrum that contains the light we can see. The colors in the *visible Spectrum* from longest wavelength to shortest are: red, orange, yellow, green, blue, and violet.

A closed universe: if the mean density exceeds a critical value then the universe will expand to a finite size and then begin to collapse back in on itself, slowly at first, and then at an ever-increasing rate until all the galaxies collide and the universe ends in a Big Crunch.

An open universe: if the mean density is less than the critical value, gravity will slow the rate of expansion towards a steady value and the expansion will continue forever. If it is assumed that effectively all matter in the universe is luminous, then the mean density turns out to be little more than 1% of the critical value. If that was the case the universe would certainly be 'open' and destined to expand forever.

A flat universe: if the mean density equals the critical value, the universe will be able to expand forever with the recession velocities tending towards zero.

Space expanding

It is generally considered that a better way to think of redshift is that it is due to space itself expanding. As space expands, the waves become stretched, ie their wavelength increases.

In the time it takes light to travel from one galaxy to another, space has expanded so the distance between the galaxies and the wavelength of light have both been stretched by the same factor.

EXPANSION NOT EXPLOSION

Although we talk about the Big Bang, it is important to emphasise that the universe, ie space, is expanding. There are a number of characteristics that indicate it is an expansion and not the result of an explosion.

Explosion	Expansion
Different bits fly off at different speeds	Expansion explains the large-scale symmetry we see in the distribution of galaxies
Fast parts overtake slow parts	Expanding space explains the redshifts and the Hubble law
Difficult to imagine a suitable mechanism to produce the range of velocities from 100 kms^{-1} to almost the speed of light	Expansion also explains redshifts and the Hubble law even if we are not at the centre of the universe
Seems likely velocity would be related to some physical property, eg if given the same energy, less massive galaxies would be moving faster	Balloon analogy – every galaxy moves away from every other as the space expands
If this was the case a definite correlation between mass and velocity would be expected – this is not observed	No galaxy is located at the centre
Hubble's Law works well even if we only plot data for galaxies of similar mass	Not only are we not at the centre of the universe, it doesn't even need to have a centre
Faster galaxies would leave slower ones behind, resulting in those near the centre (start) being more closely packed than those on the periphery (finish), like runners in a marathon, but this is not observed	

THE EXPANDING UNIVERSE/ TUTORIAL 1

In the following questions, when required, use the approximation for speed of sound in air = 340 m s^{-1}.

1. In the following sentences the words represented by the letters A, B, C and D are missing:

A moving source emits a sound with frequency f_s. When the source is moving towards a stationary observer, the observer hears a __**A**_ frequency f_o. When the source is moving away from a stationary observer, the observer hears a _**B**_ frequency f_o . This is known as the _**C**_ _**D**_.

Match each letter with the correct word from the list below:

Doppler effect	higher	louder
lower	quieter	softer

2. Write down the expression for the observed frequency f_o, detected when a source of sound waves in air of frequency f_s moves:

(a) towards a stationary observer at a constant speed, v_s

(b) away from a stationary observer at a constant speed, v_s .

3. In the table shown, calculate the value of each missing quantity (a) to (f), for a source of sound moving in air relative to a stationary observer.

Frequency heard by stationary observer / Hz	*Frequency of source* / Hz	*Speed of source moving towards observer* / ms^{-1}	*Speed of source moving away from observer* / ms^{-1}
(a)	400	10	–
(b)	400	–	10
850	(c)	20	–
1020	(d)	–	5
2125	2000	(e)	–
170	200	–	(f)

4. A girl tries out an experiment to illustrate the Doppler effect by spinning a battery-operated siren around her head. The siren emits sound waves with a frequency of 1200 Hz.

Describe what would be heard by a stationary observer standing a few metres away.

5. A police car emits sound waves with a frequency of 1000 Hz from its siren. The car is travelling at 20 ms^{-1}.

(a) Calculate the frequency heard by a stationary observer as the police car moves towards her.

(b) Calculate the frequency heard by the same observer as the police car moves away from her.

6. A student is standing on a station platform. A train approaching the station sounds its horn as it passes through the station. The train is travelling at a speed of 25 ms^{-1}. The horn has a frequency of 200 Hz.

(a) Calculate the frequency heard as the train is approaching the student.

(b) Calculate the frequency heard as the train is moving away from the student.

7. A man standing at the side of the road hears the horn of an approaching car. He hears a frequency of 470 Hz. The horn on the car has a frequency of 450 Hz.

Calculate the speed of the car.

8. A source of sound emits waves of frequency 500 Hz. This is detected as 540 Hz by a stationary observer as the source of sound approaches.

Calculate the frequency of the sound detected as the source moves away from the stationary observer.

9. A whistle of frequency 540 vibrations per second rotates in a circle of radius 0·75 m with a speed of 10 ms^{-1}.

Calculate the lowest and highest frequency heard by a listener some distance away at rest with respect to the centre of the circle.

10. A woman is standing at the side of a road. A lorry, moving at 20 ms^{-1}, sounds its horn as it is passing her. The lorry is moving at 20 ms^{-1} and the horn has a frequency of 300 Hz.

(a) Calculate the wavelength heard by the woman when the lorry is approaching her.

(b) Calculate the wavelength heard by the woman when the lorry is moving away from her.

11. A siren emitting a sound of frequency 1000 vibrations per second moves away from you towards the base of a vertical cliff at a speed of 10 $m\ s^{-1}$.

(a) Calculate the frequency of the sound you hear coming directly from the siren.

(b) Calculate the frequency of the sound you hear reflected from the cliff.

12. A sound source moves away from a stationary listener. The listener hears a frequency that is 10% lower than the source frequency. Calculate the speed of the source.

13. A bat flies towards a tree at a speed of 3·60 m s $^{-1}$ while emitting sound of frequency 350 kHz. A moth is resting on the tree directly in front of the bat.

(a) Calculate the frequency of sound heard by the bat.

(b) The bat decreases its speed towards the tree. Does the frequency of sound heard by the moth increase, decrease or stays the same? Justify your answer.

(c) The bat now flies directly away from the tree with a speed of 4·50 m s $^{-1}$ while emitting the same frequency of sound. Calculate the new frequency of sound heard by the moth.

14. The siren on a police car has a frequency of 1500 Hz. The police car is moving at a constant speed of 54 km h^{-1}.

(a) Show that the police car is moving at 15 m s^{-1}.

(b) Calculate the frequency heard when the car is moving towards a stationary observer.

(c) Calculate the frequency heard when the car is moving away from a stationary observer.

15. A source of sound emits a signal at 600 Hz. This is observed as 640 Hz by a stationary observer as the source approaches.

Calculate the speed of the moving source.

16. A battery-operated siren emits a constant note of 2200 Hz. It is rotated in a circle of radius 0·8 m at 3·0 revolutions per second. A stationary observer, standing some distance away, listens to the note made by the siren.

(a) Show that the siren has a constant speed of 15·1 m s^{-1}.

(b) Calculate the minimum frequency heard by the observer.

(c) Calculate the maximum frequency heard by the observer.

17. You are standing at the side of the road. An ambulance approaches you with its siren on. As the ambulance approaches, you hear a frequency of 460 Hz and as the ambulance moves away from you, a frequency of 410 Hz. The nearest hospital is 3 km from where you are standing.

Estimate the time for the ambulance to reach the hospital. Assume that the ambulance maintains a constant speed during its journey to the hospital.

18. On the planet Lots, a poobah moves towards a stationary glonk at 10 m s $^{-1}$. The poobah emits sound waves of frequency 1100 Hz. The stationary glonk hears a frequency of 1200 Hz.

Calculate the speed of sound on the planet Lots.

19. Copy and complete the following paragraph in your notes replacing the letters with the correct word from the word list:

A hydrogen source gives out a number of emission lines. λ for one of these lines is measured. When the light source is at rest, the value of this wavelength is λ_{rest}. When the same emission line is observed in light coming from a distant star the value of the wavelength is $\lambda_{observed}$.

When a star is moving away from the Earth $\lambda_{observed}$ is _**A**_ than λ_{rest}. This is known as the _**B**_ shift.

When the distant star is moving towards the Earth $\lambda_{observed}$ is __**C**__ than λ_{rest}. This is known as the __**D**__ shift.

Measurements on many stars indicate that most stars are moving _**E**__ from the Earth.

Match each letter with the correct word from the list below:

away	blue	longer
red	shorter	towards.

20. In the table shown, calculate the value of each missing quantity.

Fractional change in wavelength, z	*Wavelength of light on Earth λ_{rest}/ nm*	*Wavelength of light observed from star, $\lambda_{observed}$ / nm*
(a)	365	402
(b)	434	456
$8{\cdot}00 \times 10^{-2}$	486	(c)
$4{\cdot}00 \times 10^{-2}$	656	(d)
$5{\cdot}00 \times 10^{-2}$	(e)	456
$1{\cdot}00 \times 10^{-1}$	(f)	402

HUBBLES LAW / TUTORIAL 2

In the following questions, when required, use the approximation for $H_o = 2{\cdot}4 \times 10^{-18}\ s^{-1}$

1. Convert the following distances in light years into distances in metres.

(a) 1 light year

(b) 50 light years

(c) 100, 000 light years

(d) 16, 000, 000, 000 light years

2. Convert the following distances in metres into distances in light years.

(a) Earth to our Sun = $1{\cdot}44 \times 10^{11}$m.

(b) Earth to next nearest star Alpha Centauri = 3.97×10^{16} m.

(c) Earth to a galaxy in the constellation of Virgo = $4{\cdot}91 \times 10^{23}$ m.

3. In the table shown, calculate the value of each missing quantity.

Speed of galaxy relative to Earth / m s^{-1}	*Approximate distance from Earth to galaxy / m*	*Fractional change in wavelength, z*
(a)	7.10×10^{22}	(b)
(c)	1.89×10^{24}	(d)
$1{\cdot}70 \times 10^{6}$	(e)	(f)
$2{\cdot}21 \times 10^{6}$	(g)	(h)

4. Light from a distant galaxy is found to contain the spectral lines of hydrogen. The light causing one of these lines has a measured wavelength of 466 nm. When the same line is observed from a hydrogen source on Earth it has a wavelength of 434 nm.

(a) Calculate the Doppler shift, *z*, for this galaxy.

(b) Calculate the speed at which the galaxy is moving relative to the Earth.

(c) In which direction, towards or away from the Earth, is the galaxy moving?

5. Light of wavelength 505 nm forms a line in the spectrum of an element on Earth. The same spectrum from light from a galaxy in Ursa Major shows this line shifted to correspond to light of wavelength 530 nm.

(a) Calculate the speed that the galaxy is moving relative to the Earth.

(b) Calculate the approximate distance, in metres, the galaxy is from the Earth.

6. A galaxy is moving away from the Earth at a speed of 0·074 c.

(a) Convert 0·074 c into a speed in ms^{-1}.

(b) Calculate the approximate distance, in metres, of the galaxy from the Earth.

7. A distant star is travelling directly away from the Earth at a speed of $2{\cdot}4 \times 10^7\ ms^{-1}$.

(a) Calculate the value of z for this star.

(b) A hydrogen line in the spectrum of light from this star is measured to be 443 nm. Calculate the wavelength of this line when it observed from a hydrogen source on the Earth.

8. A line in the spectrum from a hydrogen atom has a wavelength of 489 nm on the Earth. The same line is observed in the spectrum of a distant star but with a longer wavelength of 538 nm.

(a) Calculate the speed, in $m\ s^{-1}$, at which the star is moving away from the Earth.

(b) Calculate the approximate distance, in metres and in light years, of the star from the Earth.

9. The galaxy Corona Borealis is approximately 1 000 million light years away from the Earth. Calculate the speed at which Corona Borealis is moving away from the Earth.

10. A galaxy is moving away from the Earth at $3{\cdot}0 \times 10^7\ ms^{-1}$. The frequency of an emission line coming from the galaxy is measured. The light forming the same emission line, from a source on Earth, is observed to have a frequency of $5{\cdot}00 \times 10^{14}$ Hz.

(a) Show that λ for the light of the emission line from the source on the Earth is $6{\cdot}00 \times 10^{-7}$m.

(b) Calculate the frequency of the light forming the emission line coming from the galaxy.

11. A distant quasar is moving away from the Earth. Hydrogen lines are observed coming from this quasar. One of these lines is measured to be 20 nm longer than the same line, of wavelength 486 nm from a source on Earth.

(a) Calculate the speed at which the quasar is moving away from the Earth.

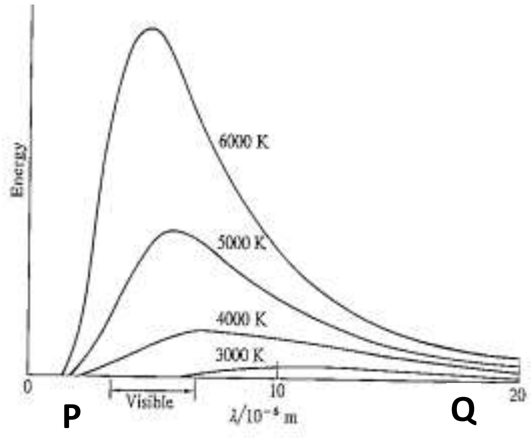
(b) Calculate the approximate distance, in millions of light years, that the quasar is from the Earth.

12. A hydrogen source, when viewed on the Earth, emits a red emission line of wavelength 656 nm. Observations, for the same line in the spectrum of light from a distant star, give a wavelength of 660 nm. Calculate the speed of the star relative to the Earth.

13. Due to the rotation of the Sun, light waves received from opposite ends of a diameter on the Sun show equal but opposite Doppler shifts. The relative speed of rotation of a point on the end of a diameter of the Sun relative to the Earth is 2 kms^{-1}. Calculate the wavelength shift for a hydrogen line of wavelength 486·1 nm on the Earth.

BIG BANG THEORY/ TUTORIAL 3

1. The graphs below are obtained by measuring the energy emitted at different wavelengths from an object at different temperatures.



(a) Which part of the *x*-axis, P or Q, corresponds to ultraviolet radiation?

(b) What do the graphs show happens to the amount of energy emitted at a *certain* wavelength as the temperature of the object increases?

(c) What do the graphs show happens to the *total* energy radiated by the object as its temperature increases?

(d) Each graph shows that there is a wavelength λ_{max} at which the maximum amount of energy is emitted.

(i) Explain why the value of λ_{max} decreases as the temperature of the object increases.

The table shows the values of λ_{max} at different temperatures of the object.

Temperature /K	λ_{max} / m
6000	$4{\cdot}8 \times 10^{-7}$
5000	$5{\cdot}8 \times 10^{-7}$
4000	$7\ 3 \times 10^{-7}$
3000	$9{\cdot}7 \times 10^{-7}$

(ii) Use this data to determine the relationship between temperature T and λ_{max}.

(e) Use your answer to (d) (ii) to calculate:

(i) the temperature of the star Sirius where λ_{max} is $2{\cdot}7 \times 10^{-7}$ m

(ii) the value of λ_{max} for the star Alpha Crucis which has a temperature of 23,000 K

(iii) the temperature of the present universe when λ_{max} for the cosmic microwave radiation is measured as $1{\cdot}1 \times 10^{-3}$ m.

(iv) the approximate wavelength and type of the radiation emitted by your skin, assumed to be at a temperature of 33^{o} C.

THE EXPANDING UNIVERSE/ EXAM QUESTIONS

Exam questions are found in the L.A. Homework Booklet.

THE EXPANDING UNIVERSE TUTORIAL ANSWERS

1. A = higher; B= lower; C = Doppler; D = effect

2. (a) and (b) Teacher Check

3. (a) 412 Hz

(b) 389 Hz

(c) 800 Hz

(d) 1035 Hz

(e) 20 m s^{-1}

(f) 60 m s^{-1}

4. Teacher Check

5. (a) 1063 Hz

(b) 944 Hz

6. (a) 216 Hz

(b) 186 Hz

7. 14·5 m s^{-1}

8. 466 Hz

9. 556 Hz, 525 Hz

10. (a) 1·07 m

(b) 1·2 m

11. (a) 971 Hz

(b) 1030 Hz

12. 37·8 m s^{-1}

13. (a) 354 kHz

(b) Decrease – denominator is larger

(c) 345 kHz

14. (a) Teacher Check

(b) 1569 Hz

(c) 1437 Hz

15. 21·3 m s^{-1}

16. (a) Teacher Check

(b) 2106 Hz

(c) 2302 Hz

17. 154 s

18. 120 m s^{-1}

19. A = longer; B = red; C = shorter; D = blue; E = away

20. (a) $1{\cdot}01 \times 10^{-1}$

(b) $5{\cdot}07 \times 10^{-2}$

(c) 525 nm

(d) 682 nm

(e) 434 nm

(f) 365 nm

Hubbles Law

1. (a) $9{\cdot}46 \times 10^{15}$m

(b) $4{\cdot}73 \times 10^{17}$m

(c) $9{\cdot}46 \times 10^{20}$m

(d) $1{\cdot}51 \times 10^{26}$ m

2. (a) $1{\cdot}52 \times 10^{-5}$light years

(b) 4·2 light years

(c) $5{\cdot}19 \times 10^{7}$ light years

3.

v / m s^{-1}	d / m	z
$1{\cdot}70 \times 10^{5}$	$7{\cdot}10 \times 10^{22}$	$5{\cdot}67 \times 10^{-4}$
$4{\cdot}54 \times 10^{6}$	$1{\cdot}89 \times 10^{24}$	$1{\cdot}51 \times 10^{-2}$
$1{\cdot}70 \times 10^{6}$	$7{\cdot}08 \times 10^{23}$	$5{\cdot}667 \times 10^{-2}$
$2{\cdot}21 \times 10^{6}$	$9{\cdot}21 \times 10^{23}$	$7{\cdot}37 \times 10^{-3}$

4. (a) $7{\cdot}37 \times 10^{-2}$

(b) $2{\cdot}21 \times 10^{7}$ ms

(c) Away

5. (a) $1{\cdot}49 \times 10^{7}$ ms^{-1}

(b) $6{\cdot}21 \times 10^{24}$ m

6. (a) $2{\cdot}22 \times 10^{7}$ ms^{-1}

(b) $9{\cdot}25 \times 10^{24}$ m

7. (a) 8×10^{-2}

(b) 410 nm

8. (a) $3{\cdot}0 \times 10^{7}$ ms^{-1}

(b) $1{\cdot}25 \times 10^{25}$ m, $1{\cdot}32 \times 10^{9}$ light years

9. $2{\cdot}27 \times 10^{7}$ ms^{-1}

10. (a) Teacher Check

(b) $4{\cdot}55 \times 10^{14}$ Hz

11. (a) $1{\cdot}23 \times 10^{7}$ ms^{-1}

(b) 542 million light years

12. $1{\cdot}83 \times 10^{6}$ ms^{-1}

13. $3{\cdot}24 \times 10^{-12}$ m

<u>The Big Bang Theory</u>

1. (a) P

(b) Energy emitted increases

(c) Increases

(d) (ii) $T \lambda_{max} = 2{\cdot}9 \times 10^{-3}$ m K

(e) (i) T =11, 000 K

(ii) $\lambda_{max} = 1{\cdot}3 \times 10^{-7}$ m

(iii) T = 2·6 K

(iv) $\lambda = 9{\cdot}5 \times 10^{-6}$ m, infrared