##  <br> Higher Physics <br> Particles and Waves 2 Notes



## Teachers Booklet

# Learning Outcomes - Interference and diffraction 

## This builds on information from <br> N5 Waves and Radiation <br> H Particles and Waves book 1

## At the end of this section you should be able to

- State that coherent waves have
- a constant phase relationship
- the same frequency
- the same wavelength
- the same velocity
- Describe constructive and destructive interference in terms of phase between two waves.
- Calculate the path difference between two waves
- State that maxima are produced when the path difference between waves is a whole number of wavelengths
- State that minima are produced when the path difference between waves is an odd number of half wavelengths.
- Use path diff $=m \lambda$ OR path diff $=\left(m+\frac{1}{2}\right) \lambda$ where $m=0,1,2 \ldots$.. to solve problems involving the path difference between waves, wavelength and order number.
- Use the equation $m \lambda=d \sin \theta$ to solve problems involving grating spacing, wavelength, order number and angle to the maximum


## Waves - Background



All waves carry energy.
Mechanical waves are produced by moving particles - these include water waves, sound waves ad waves on a spring.

Electromagnetic waves are caused by varying electric and magnetic fields.

The energy of a wave depends on its amplitude.

When waves are created the frequency of the wave is the same as the frequency of the source and does not change once the wave is created.

## Equations

$$
f=\frac{N}{t} \quad v=f \lambda \quad T=\frac{1}{f}
$$

Where $f=$ frequency $(H z)$
$N$ = number of waves
$t=$ time ( $s$ )
$v=$ wavespeed $\left(\mathrm{ms}^{-1}\right)$
$\lambda=$ wavelength (m)
$T=$ period of wave (s)

## Wave Properties - Interference

All waves undergo the following four properties


Diffraction


Interference

Waves which have the same amplitude, frequency and have a fixed phase difference are described as COHERENT.

To produce coherent waves, use a single source


## Example 1

Which graph shows the relationship between frequency $f$ and wavelength $\lambda$ of photons of electromagnetic radiation?

C

E


D


A

## Wave Properties - Interference

- Two waves which have the same frequency, wavelength and velocity are described as being in phase if a crest coincides with a crest and a trough coincides with a trough.


This is called CONSTRUCTIVE INTERFERENCE.

Two waves which have the same frequency, wavelength and velocity are described as being out of phase if a crest coincides with a trough and a trough coincides with a crest.


This is called DESTRUCTIVE INTERFERENCE.

## Destructive interference is the test for a wave.

## Interference of Water waves

An interference pattern is produced when two sets of circular water waves overlap and combine.

- Constructive interference
- Crest



## Wave Properties - Interference of Sound

If we have two loudspeakers connected to the same source the sound produced is coherent because it has the same frequency, the same wavelength and is produced with a constant phase difference.

The sound waves produce an interference pattern like that produced by water waves, but instead of patterns of light and dark we get patterns of loud and quiet.
$L_{2} \begin{cases}\text { Loud Third maximum } & m=3 \\ \text { Quiet Third minimum } & m=2 \\ \text { Loud Second maximum } & m=2 \\ \text { Quiet Second minimum } & m=1 \\ \text { Loud First maximum } & m=1 \\ \text { Quiet First minimum } & m=0 \\ \text { Loud Central Maximum } & m=0 \\ \text { Quiet First minimum } & m=0 \\ \text { Loud First maximum } & m=1 \\ \text { Quiet Second minimum } & m=1 \\ \text { Loud Second maximum } & m=2 \\ \text { Quiet Third minimum } & m=2 \\ \text { Loud Third maximum } & m=3\end{cases}$

A maximum occurs when the path difference is a whole number of wavelengths. Path difference $=m \lambda$ (where $m=0,1,2,3 . .$. )
A minimum occurs when the path difference is multiple of half a wavelength. Path difference $=\left(m+\frac{1}{2}\right) \lambda$ (where $m=0,1,2,3 \ldots$ )


Both waves travel the same distance to arrive at the central maximum.

## Path difference - Sound waves



The distance $L_{1} Q$ is 47 cm and $L_{2} Q$ is 49 cm . Calculate the wavelength of the wave if $Q$ is the first order maximum.
Path difference $=49-47=2 \mathrm{~cm}$
First order maximum $=\Lambda=2 \mathrm{~cm}=0.02 \mathrm{~m}$

## Example 3



The distance $L_{1} P$ is 47 cm and the distance $L_{2} P$ is 45.5 cm . Calculate the wavelength of the wave if $P$ is the first minimum.
Path difference $=47-45.5=1.5 \mathrm{~cm}$
First minimum $=\lambda / 2 \rightarrow \lambda=2 \times 1.5=3 \mathrm{~cm}=0.03 \mathrm{~m}$


The distance $L_{1} R$ is 47 cm and the distance $L_{2} R$ is 41 cm . Calculate the wavelength of the wave if $P$ is the second order maximum.
Path difference $=47-41=6 \mathrm{~cm}$
Second order maximum $=2 \lambda \rightarrow \lambda=6 / 2=3 \mathrm{~cm}=0.03 \mathrm{~m}$

## Path difference - Sound waves

## Example 5

Two identical loudspeakers, $L_{1}$ and $L_{2}$, are connected to a signal generator as shown


An interference pattern is produced and a minimum is detected at point $T$. The wavelength of the sound is 40 mm . The distance from $L_{1}$ to $T$ is 500 mm . The distance from $L_{2}$ to $T$ is

A minimum is produced when the path difference is half a
A 450 mm wavelength.
If $\lambda=40 \mathrm{~mm}$ then $\lambda / 2=20 \mathrm{~mm}$.
B $\quad 460 \mathrm{~mm} \quad$ The distance from $L_{2}$ to $T$ must be either 20 mm or 60 mm less
C $\quad 470 \mathrm{~mm}$ than 500 mm .
D $\quad 480 \mathrm{~mm}$ The only option is D-480mm SQA 2011 Q 15
E $\quad 490 \mathrm{~mm}$

## Example 6

Two identical loudspeakers, $L_{1}$ and $L_{2}$, are operated at the same frequency and in phase with each other. An interference pattern is produced.


At position $P$, which is the same distance from both loudspeakers, there is a maximum intensity.
The next maximum intensity is at position $R$, where $L_{1} R=5.6 \mathrm{~m}$ and
$L_{2} R=5.3 \mathrm{~m}$. The speed of sound $=340 \mathrm{~ms}^{-1}$.
The frequency of the sound emitted by the loudspeakers is given by
A $\frac{5.6-5.3}{340} \mathrm{~Hz}$

$$
f=v / \lambda 340 / \text { path difference }
$$

B $\frac{340}{5.6+5.3} \mathrm{~Hz}$
C $\frac{340}{5.6-5.3} \mathrm{~Hz}$
D $340 \times(5.6-5.3) \mathrm{Hz}$
SQA 2004 Q 14
E $340 \times(5.6+5.3) \mathrm{Hz}$

## $C$

## Path difference - electromagnetic waves

Path difference calculations can also be carried out with other types of wave.

## Example 7

$S_{1}$ and $S_{2}$ are sources of coherent waves. An interference pattern is obtained between $X$ and $Y$.


The first order maximum occurs at $P$, where $S_{1} P=200 \mathrm{~mm}$ and $S_{2} P=$ 180 mm . Calculate the path difference at $R$, the third order maximum. $\left(S_{1} P-S_{2} P\right)=200-180=20 \mathrm{~mm}$. This is the first order maximum so the path difference $=\lambda=20 \mathrm{~mm}$.
At the third order maximum the path difference $=3 \lambda$
so $\left(S_{1} R-S_{2} R\right)=3 \times 20=60 \mathrm{~mm}$
SQA 2012 Revised Q 9

## Example 8

A student is using different types of electromagnetic radiation to investigate interference. In the first experiment, two identical sources of microwaves, $S_{1}$ and $S_{2}$, are positioned a short distance apart as shown.

(i) The student moves a microwave detector from $X$ towards $Y$. The reading on the meter increases and decreases regularly. Explain. in terms of waves, what causes the minimum readings to occur.

Waves meet out of phase OR crests meet troughs OR appropriate diagram
(ii) The fourth maximum from the central maximum is located at $P$. The distance from $S_{1}$ to $P$ is 620 mm and the wavelength of the waves is 28 mm . Calculate the distance from $S_{2}$ to $P$.
Path diff $=\mathrm{m} \lambda$. So path diff $=4 \times 28 \mathrm{~mm}=112 \mathrm{~mm}$
$\mathrm{S}_{2} \mathrm{P}=620+112=732 \mathrm{~mm}$ SQA 2012 revised Q28 (a)

## Interference pattern - Light

## Example 9

In this experiment, microwaves of wavelength 30 mm pass through the gaps between metal plates as shown.

(a) The distances from each of the gaps to point $J$ are shown in the diagram. Use this information to determine whether point $J$ is a point of constructive or destructive interference.
You must justify your answer by calculation.
Path difference $=500-425=75 \mathrm{~mm}$
Number of wavelengths $=75 / 30=2.5$
Destructive interference.
(SQA did not accept 'a minimum' or 'deconstructive'
(b) The microwave detector is now moved to point $K$ which is a point of destructive interference.
Gap 1 is then covered with a sheet of metal.
Does the strength of the signal detected at $K$ increase, decrease or stay the same? You must justify your answer.

Increases
Destructive interference no longer occurs OR now only one sest of waves so they cannot cancel out
OR suitable diagram showing destructive interference before, then no interference.

SQA 2014 Q28 (b)

## Interference pattern - Light

To produce an interference pattern using light the light source is passed through a blackened slide which has two parallel slits cut into it to allow light through. A laser is used as the light source because it is a monochromatic light source (single frequency).


The screen shows a bright central point, with a pattern of light and dark patches on either side, showing constructive and destructive interference.


The path difference between $S_{1} P$ and $S_{2} P$ is one wavelength, $\lambda$.
The slit separation, $d$, is very small and is much smaller than the distance to the screen, $D$.
The triangle formed between the screen and the light sources and the triangle at the sources are similar triangles - they have the angle $\theta$ in common.

$$
\tan \theta=\frac{\Delta \mathrm{x}}{D} \quad \begin{aligned}
& \text { For small angles } \sin \theta \text { is } \\
& \text { approximately equal to } \tan \theta
\end{aligned} \quad \sin \theta=\frac{\lambda}{d}
$$

This is written as

$$
\begin{aligned}
m \lambda=d \sin \theta \text { where } m & =0,1,2,3 \ldots \text { for maxima } \\
m & =\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} \ldots \text { for minima }
\end{aligned}
$$

NOTE - old exam papers may use $n$ rather than $m$ as in $n \lambda=d \sin \theta$

## Interference pattern - Light

## Gratings

To make the pattern of light and dark areas (fringes) brighter and sharper we need more light. This is done using a grating which has multiple slits


The slit separation, $d$, is calculated from the number of lines per mm .
Example 10
If the number of lines on a diffraction grating is 200 lines per mm , calculate the slit separation, $d$.
3. find the number of lines per metre 200 lines per $\mathrm{mm}=200,000$ lines per m
4. Calculate the separation

$$
d=\frac{1}{200,000}=5 \times 10^{-6} \mathrm{~m}
$$

## Example 11

If the number of lines on a diffraction grating is 500 lines per mm , calculate the slit separation, $d$.

1. find the number of lines per metre 500 lines per $\mathrm{mm}=500,000$ lines per m
2. Calculate the separation

$$
d=\frac{1}{500,000}=2 \times 10^{-6} \mathrm{~m}
$$

Since $m \lambda=d \sin \theta$

- If we increase the number of lines per mm , d gets smaller, so the angle between the central fringe and the fringe being measured will increase.
- If we increase the wavelength of the light the angle between the central fringe and the fringe being measured will increase.


## Interference pattern - Light

## Example 12

In an experiment, laser light of wavelength 633 nm is incident on a grating. A series of bright spots are seen on a screen placed some distance from the grating. The distance between these spots and the central spot is shown.


Calculate the number of lines per metre on the grating.
$\mathrm{m} \lambda=\mathrm{d} \sin \theta$
$1 \times 633 \times 10^{-9}=d \times \sin (37 / 2)$

$$
d=1.99 \times 10^{-6} \mathrm{~m}
$$

Number of lines per metre $=\frac{1}{2 \times 10^{-6}}=5.0 \times 10^{5}$

$$
\text { SQA } 2004 \text { Q } 28 \text { (b) }
$$

## Example 13

Light of wavelength $486 \times 10^{-9} \mathrm{~m}$ is viewed using a grating with a slit spacing of $2.16 \times 10^{-6} \mathrm{~m}$.
Calculate the angle between the central maximum and the second order maximum.
$m \lambda=d \sin \theta$
$2 \times 486 \times 10^{-9}=2.16 \times 10^{-6} \times \sin \theta$
$\sin \theta=0.45 \quad \theta=26.7^{\circ}$
SQA 2001 Q 27 (a)

## Example. 14

Monochromatic light is incident on a grating and the resulting interference pattern is viewed on a screen. The distance between neighbouring areas of constructive interference on the screen:
is increased when the screen is moved further away from the grating, is increased when light of a greater wavelength is used, is decreased when the distance between the slits is increased.

## Interference pattern - Light

## Example 15

A laser produces a narrow beam of monochromatic light.
(a) Red light from a laser passes through a grating as shown.


A series of maxima and minima is observed.
Explain in terms of waves how a minimum is produced.
Wave meet out of phase OR crest meets trough OR path differences $=\left(m+\frac{1}{2}\right) \lambda$
(b) The laser is now replaced by a second laser, which emits blue light.
Explain why the observed maxima are now closer together.
$\lambda$ blue light is shorter than $\lambda$ red light and $\mathrm{m} \lambda=\mathrm{d} \sin \theta$
Or $\sin \theta=m \lambda / d$
(c) The wavelength of the blue light from the second laser is $4.73 \times 10^{-7} \mathrm{~m}$. The spacing between the lines on the grating is $2.00 \times 10^{-6} \mathrm{~m}$
Calculate the angle between the central maximum and the second order maximum.
$\mathrm{m} \lambda=\mathrm{d} \sin \theta$
$2 \times 4.73 \times 10^{-7}=2.00 \times 10^{-6} \sin \theta$
$\theta=28.2^{\circ}$
SQA 2009 Q 27

## White Light



When white light passes through a prism the blue light is refracted most.

If monochromatic light is replaced by white light

$$
\begin{aligned}
& r \\
& 0 \\
& \begin{array}{l}
y \\
9
\end{array} \\
& \text { i } \\
& v
\end{aligned}
$$

$$
\begin{aligned}
& \text { White } \\
& \text { light } \\
& \begin{array}{ll}
v & \\
i & \\
b & \\
g & \\
y & v \\
o & i \\
r & b \\
& 9 \\
& y \\
& 0 \\
& r
\end{array}
\end{aligned}
$$

| Prism | Diffraction Grating |
| :--- | :--- |
| One spectrum produced | Multiple spectra produced |
| Produced by refraction | Produced by interference |
| Red deviated least | Red deviated most |
| Spectrum is bright | Spectra are less bright. |
|  | Central image white |

Spectra further out may overlap, making it difficult to decide what colours are seen.

| Colour | Wavelength $(\mathrm{nm})$ |
| :---: | :---: |
| Red | 700 |
| Green | 550 |
| Blue | 490 |

$$
1 \mathrm{~nm}=1 \times 10^{-9} \mathrm{~m}
$$

These wavelengths can be found in the data table so do not need to be memorised, but it is useful to have an idea of the range.

## White Light

Example 16
The spectrum of white light from a filament lamp may be viewed using a prism of a grating.
A student, asked to compare the spectra formed by the two methods made the following statements.
I. The prism produces a spectrum by refraction. The grating produces a spectrum by interference.
II. The spectrum formed by the prism shows all the wavelengths present in the white light. The spectrum formed by the grating shows only a few specific wavelengths.
III. The prism produces a single spectrum. The grating produces more than one spectrum
Which of the above statements is/are true?

I and III only

$$
\text { SQA } 2001 \text { Q } 16 \text { (adapted) }
$$

## Example 17

One of the most important debates in scientific history asked the question

## 'Is light a wave or a particle?'

Use your knowledge of physics to comment on our understanding of this issue.
Light demonstrates reflection, refraction, diffraction and interference. This suggests it is a wave.
Light of an appropriate frequency (ultraviolet) causes electrons to be emitted from the surface of a metal (zinc) that has an appropriate work function (is reactive enough). This suggests that light acts like a particle. This is wave particle duality.

SQA 2013 revised Q28

Example 18 SQA 2011 Q 28 (a)
The first demonstration of the interference of light was performed by Thomas Young in 1801.
What does the demonstration of interference prove about light?
Light travels as waves/is a wave or Energy in light is carried as a wave

## This builds on information from

Waves and Radiation-Booklet 2 - Light

## At the end of this section you should be able to

- State that the absolute refractive index of a material is the ratio of the speed of light in vacuum to the speed of light in the medium.
- Carry out calculations using the equation $n=\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {material }}}$
- Describe what happens when light travels from a more dense medium to a less dense medium.
- Carry out calculations using the equation

$$
n=\frac{\lambda_{\text {air }}}{\lambda_{\text {material }}}=\frac{v_{\text {air }}}{v_{\text {material }}}
$$

- Variation of refractive index with frequency.
- State what is meant by the critical angle for a material.
- Calculate the critical angle for a material using

$$
\sin \theta_{c}=\frac{1}{n}
$$

- Use the critical angle to evaluate whether total internal reflection or refraction takes place when light is incident on a material at a certain angle.


## Refraction of light - Revision

When a wave enters a new medium the velocity may change.


If the wave enters at an angle this can also cause a change in direction.

Angles are measured between the ray and the normal


The amount of refraction that takes place depends on

- The wavelength of the light
- The type of material it passes through


## Calculating Refractive Index



Monochromatic
light source


A laser is used to produce monochromatic light and is directed into a semicircular block as shown above. The angles $\theta_{\text {air }}$ and $\theta_{\text {substance }}$ are measured and the refractive index, $n$, is calculated using

$$
n=\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {substance }}}
$$

For a more accurate result, and to get an average, many values of $\theta_{\text {air }}$ are used. The results can be plotted to give the graph shown above.

## Refractive Index

$$
n=\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {substance }}} \quad \text { air } \quad \text { normal } \quad \text { substance }
$$

- To get the refractive index always put air on the top, substance on the bottom.
- The value for refractive index is always equal to or greater than one.
- Refractive index has NO UNITS.
- Frequency does not alter.
- Velocity is less in the more optically dense material.

Since frequency is constant, $\lambda$ and velocity change

$$
f_{\text {air }}=\frac{v_{\text {air }}}{\lambda_{\text {air }}} \quad f_{\text {substance }}=\frac{v_{\text {substance }}}{\lambda_{\text {substance }}}
$$

Therefore

$$
n=\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {substance }}}=\frac{\lambda_{\text {air }}}{\lambda_{\text {substance }}}=\frac{v_{\text {air }}}{v_{\text {substance }}}
$$

## Example 19



Calculate the refractive index of the material this block is made from

$$
n=\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {substance }}}=\frac{\sin 27^{\circ}}{\sin 18^{\circ}}=1.5
$$

## Example 20

Calculate the value of the unknown angle $\theta$
 if the refractive index is 1.4.

$$
\begin{aligned}
n & =\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {substance }}}=\frac{\sin 35^{\circ}}{\sin \theta^{\circ}}=1.4 \\
& \Rightarrow \sin \theta_{\text {substance }}=\frac{\sin 35^{\circ}}{1.4} \Rightarrow \theta_{s}=24^{\circ}
\end{aligned}
$$

## Refractive Index

## Example 21

Calculate the velocity of light in a glass block which has a refractive index of 1.50 .
$n=\frac{v_{a}}{v_{s}} \Rightarrow 1.5=\frac{3 \times 10^{8}}{v_{s}} \Rightarrow v_{s}=2 \times 10^{8} \mathrm{~ms}^{-1}$
AHS notes

## Example 22

Red light ( $\lambda=700 \mathrm{~nm}$ in air) is passed into a plastic material of refractive index 1.47. Calculate the wavelength of light in the plastic.
$n=\frac{\lambda_{\text {air }}}{\lambda_{\text {substance }}}=>1.47=\frac{700}{\lambda_{\text {substance }}} \Rightarrow \lambda_{\text {substance }}=\frac{700}{1.47}$
$=476 \mathrm{~nm}$
AHS notes

## Example 23

Light travels from air into glass.
Which row in the table describes what happens to the speed, frequency and wavelength of the light?

|  | Speed | Frequency | Wavelength |
| :--- | :--- | :--- | :--- |
| A | increases | stays constant | increases |
| B | increases | decreases | stays constant |
| C | stays constant | decreases | decreases |
| D | decreases | decreases | stays constant |
| E | decreases | stays constant | decreases |
| SQA 2006 Q16 |  |  |  |

## Example 24

The diagram represents a ray of light passing from air into liquid. The refractive index of this liquid, relative to air is


$$
\begin{gathered}
n=\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {substance }}}=\frac{\sin 70^{\circ}}{\sin 40^{\circ}} \\
=1.46 \\
\quad \text { SQA } 2006 \text { Q15 adapted }
\end{gathered}
$$

## Refractive Index and Frequency



The refractive index of a medium depends on the frequency (colour) of the incident light.
This explains why white light splits up into the spectrum when it enters a prism. Each frequency within the white light is refracted by a different amount.

The diagram shows that

$$
n_{\text {violet }}>n_{\text {red }} \text { and } v_{\text {violet }}<v_{\text {red }}
$$

## Example 25

A decorative lamp has a transparent liquid in the space above a bulb. Light from the bulb passes through rotating coloured filters giving red or blue light in the liquid.
A ray of red light is incident on the liquid surface as shown.

(a) Calculate the refractive index of the liquid for the red light.
(b) A ray of blue light is incident on the liquid surface at the same angle as the ray of red light.
The refractive index of the liquid for blue light is greater than that for red light. Is the angle of refraction greater thanm equal to or less than $82^{\circ}$ for the blue light. Explain your answer.
(a) $n=\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {substance }}}=\frac{\sin 82^{\circ}}{\sin 45^{\circ}}=1.40$
(b) $n \sin \theta_{\text {liquid }}=\sin \theta_{\text {air }}$

But $\sin \theta_{\text {liquid }}=$ constant and $n_{\text {blue }}>n_{\text {red }}$
$\sin \theta_{\text {air }}$ is now greater, hence $\theta_{\text {air }}$ is greater than $82^{\circ}$

$$
\text { SQA } 2004 \text { Q27 (a) }
$$

## Critical Angle and Total Internal Reflection



If the angle of incidence is small the light is refracted out of the block.

If the angle of incidence is increased the angle of refraction increases until it passes along the back wall of the block.

$$
n=\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {substance }}}=\frac{\sin 90^{\circ}}{\sin \theta_{c}}=\frac{1}{\sin \theta_{c}}
$$

The angle at which this occurs is called the 'critical angle'.

If the angle of incidence is greater than the critical angle all of the light is reflected inside the material.
This is called total internal reflection.

| $\angle \boldsymbol{i}<\theta_{\boldsymbol{c}}$ | Refraction |
| :---: | :--- |
| $\angle \boldsymbol{i}=\boldsymbol{\theta}_{\boldsymbol{c}}$ | Light refracted at $90^{\circ}$ |
| $\angle \boldsymbol{i}>\theta_{\boldsymbol{c}}$ | Total internal reflection |



## Irradiance and the Inverse Square Law

Example 26
Calculate the critical angle of water if it has a refractive index of 1.33.

$$
\sin \theta_{c}=\frac{1}{n}=\frac{1}{1.33}=>\theta_{c}=48.8^{\circ}
$$

## Example 27

Calculate the refractive index of a substance which has a critical angle of $42.5^{\circ}$

$$
\sin \theta_{c}=\frac{1}{n}=>n=\frac{1}{\sin \theta_{c}}=\frac{1}{\sin 42.5^{\circ}}=1.48
$$

## Reflection

Example 28
A ray of red light is directed at a glass prism of side 80 mm as shown in the diagram below.

(a) Using information from this diagram, show that the refractive index of the glass for this red light is 1.52 .
$n=\frac{\sin \theta_{a}}{\sin \theta_{s}}=\frac{\sin 20^{\circ}}{\sin 13^{\circ}}=1.52$
(b) What is meant by the term critical angle?

This is the angle of incidence (inside medium) which causes light to emerge into a vacuum at $90^{\circ}$ to the normal or along surface.
OR Incident angle at which total internal reflection takes place.
(c) Calculate the critical angle for the red light in the prism. $\sin \theta_{c}=\frac{1}{n}=\frac{1}{1.52}=>\theta_{c}=41^{\circ}$
(d) Sketch a diagram showing the path of the ray of red light until after it leaves the prism. Mark on your diagram the values of all relevant angles.


> Angle $A=90^{\circ}$
> Angle $B=(180-60-77)=43^{\circ}$
> Angle $C=90-43=47^{\circ}$

Since $47^{\circ}$ is greater than the critical angle total internal reflection occurs, so $D$ is $47^{\circ}$ and $E$ is $43^{\circ}$.
Angle $F=(180-60-43)=77^{\circ}$
Angle $G=90-77=13^{\circ}$
Light refracts out at same angle as it
came in at - Angle $\mathrm{H}=20^{\circ}$

## Reflection

## Example 29

Monochromatic light is shone into a triangular prism of flint glass.
The graph shows how the refractive index of flint glass varies with the wavelength of light in air.

(a) A ray of monochromatic light of wavelength 660 nm in air passes through the prism as shown.


Calculate the angle of refraction $\theta$.
(b) The ray of light is now replaced with one of shorter wavelength. Is the speed of this now ray in the prism less than, the same as or greater than the speed of the 660 nm ray in the prism? Justify your answer.
(a) $n=\frac{\sin \theta_{a}}{\sin \theta_{s}}=>\sin \theta_{a}=1.615$ (from graph) $\times \sin 38^{\circ} \Rightarrow \theta a=83.9^{\circ}$
(b) Refractive index is larger (from graph - since wavelength is shorter)
$n=\frac{v_{a}}{v_{g}}=>v_{g}=\frac{v_{a}}{n}=>$ velocity in glass is smaller

## Reflection

## Example 30

A gardener observes a spectrum when sunlight illuminates the drops of water in a spray. This is because each drop of water is acting like a prism. The diagram below shows the path taken by light of wavelength 650 nm through a drop of water.

(a) What happens to the frequency of this light when it enters the drop of water?
Unchanged
(b) Using information from the diagram, calculate the refractive index of the water for this wavelength of light.

$$
n=\frac{\sin \theta_{a}}{\sin \theta_{s}}=\frac{\sin 60}{\sin 41}=1.32
$$

(c) Calculate the critical angle for this wavelength of light in the water

$$
\sin \theta_{c}=\frac{1}{n}=\frac{1}{1.32}=>\theta_{c}=49
$$

(d) Light of shorter wavelength also passes through the drop of water. Will the critical angle for this light be less than, equal to or greater than for light of wavelength 650nm Justify your answer.

The critical angle will be less because the refractive index is larger.

$$
\text { SQA } 2010 \text { Q } 28
$$

## Reflection

## Example 31

A grating is placed in a colourless liquid in a container. Laser light is incident on the grating along the normal. The spacing between the linds on the grating is $5 \times 10^{-6} \mathrm{~m}$. Interference occurs and the maxima produced are shown in the diagram.

second order maximum first order maximum central maximum first order maximum second order maximum
container filled with a colourless liquid
(a) Calculate the wavelength of the laser light in the liquid

$$
m \lambda=d \sin \theta=>2 \lambda=5 \times 10^{-6} x \sin 11=>\lambda=480 \mathrm{~nm}
$$

(b) The refractive index of the colourless liquid decreases as the temperature of the liquid increases.
The liquid is now heated.
What effect does this have on the spacing between the maxima? You must justify your answer.

The spacing of the maxima increases. $n$ decreases as liquid is heated.
Since $n=\frac{\lambda_{a}}{\lambda_{s}}=>\lambda_{s}=\frac{\lambda_{a}}{n}$, so wavelength in liquid increases
$m \lambda=d \sin \theta$
$\sin \theta=\frac{m \lambda}{d}$
$\theta$ increases since $\lambda$ increases

# Learning Outcomes - Spectra 

## This builds on information from

Dynamics and Space - Space
Waves and Radiation - Wave characteristics and parameters

- Electromagnetic Spectrum and Light


## At the end of this section you should be able to

- State that irradiance is power per unit area incident on a surface.
- Use $I=P / A$ to solve problems involving irradiance, the power of radiation incident on a surface area and the area of the surface.
- State that irradiance is inversely proportional to the square of the distance from a point source
- Use $I=k / d^{2}$ to solve problems involving irradiance and distance from a point light source.
- Explain what is meant by a point light source
- Explain the difference between what would be observed from a point light source in comparison to light from a laser.
- Describe the Bohr model of the atom.
- Illustrate what is meant by the terms
- Ground state
- Energy levels
- Ionisation level
- Zero potential energy for the Bohr model of the atom
- Describe what is observed in a line emission spectrum
- Describe what is observed in a continuous emission spectrum.
- Describe what is observed in an absorption spectrum
- Explain the mechanism of production of line emission spectra, continuous emission spectra and absorption spectra.
- Describe electron energy level transitions for both absorption and emission of radiation
- Use $E_{2}-E_{1}=h f$ and $E=h f$ to solve problems involving energy levels and the frequency of the radiation emitted/absorbed.
- State that absorption lines in the spectrum of sunlight (Fraunhofer lines) provide evidence for the composition of the Sun's upper atmosphere


## Irradiance

$$
\begin{aligned}
I=\frac{P}{A} \quad \text { Where } \quad I & =\operatorname{Irradiance}\left(\mathrm{Wm}^{-2}\right) \\
P & =\operatorname{Power}(\mathrm{W}) \\
A & =\operatorname{Area}\left(\mathrm{m}^{2}\right)
\end{aligned}
$$

Since Power $=\frac{\text { energy }}{\text { time }}$ Irradiance could also be expressed as the amount of light energy incident on each square metre of a surface per second.

Example 32
A satellite is orbiting the Earth where the irradiance of the Sun's radiation is $1.4 \mathrm{kWm}^{-2}$. Calculate the power received by the satellite's solar panels if they have an area of $15 \mathrm{~m}^{2}$
$I=\frac{P}{A} \Rightarrow P=I A=1.4 \times 10^{3} \times 15=2.1 \times 10^{4} \mathrm{~W} \quad$ AHS H notes

## Example 33

The diagram shows a light sensor connected to a voltmeter. A small lamp is placed in front of the sensor.


The reading on the voltmeter is 20 mV for each 1.0 mW of power incident on the sensor.
The reading on the voltmeter is 40.0 mV .
The area of the light sensor is $8.0 \times 10^{-5} \mathrm{~m}^{2}$.
Calculate the irradiance of light on the sensor.

40 mV is a power of 2 mW .

$$
I=\frac{P}{A}=\frac{2 \times 10^{-3}}{8.0 \times 10^{-5}}=25 \mathrm{Wm}^{-2}
$$

A light is considered a point source if it emits light equally in all directions in three dimensions, creating a sphere around the light source. The source does not have to be small, but its size must be small in comparison to the distance from the observer to the source. This is why stars can be considered point sources.

## Irradiance and the Inverse Square Law

Light radiates out in all directions to make a sphere.


Example 34
A student uses a lamp to investigate how irradiance varies with distance. The results are shown in the table below.

| Distance (m) | 0.5 | 0.7 | 0.9 |
| :--- | :--- | :--- | :--- |
| Irradiance $\left(\mathrm{Wm}^{-2}\right)$ | 1.1 | 0.8 | 0.6 |

Can this new source be considered to be a point source of light?
Use all the data to justify your answer.
For a point source $I_{1} d_{1}^{2}=I_{2} d_{2}^{2}$

| Distance $(\mathrm{m})$ | 0.5 | 0.7 | 0.9 |
| :--- | :--- | :--- | :--- |
| Irradiance $\left(\mathrm{Wm}^{-2}\right)$ | 1.1 | 0.8 | 0.6 |
| $\mathrm{Id}^{2}$ | 0.28 | 0.39 | 0.49 |

Since the values are not equal the light is not a point source.

## Example 35

The irradiance on a wall from a light 0.4 m away is $6 \mathrm{Wm}^{-2}$. What will the irradiance be if the distance is increased to 2 m .

$$
\begin{aligned}
I_{1} d_{1}^{2} & =I_{2} d_{2}^{2} \\
6 \times 0.4^{2} & =I \times 2^{2} \\
0.96 & =4 I \\
I & =0.24 \mathrm{Wm}^{-2}
\end{aligned}
$$

AHS H notes

## Example 36

The irradiance of light from a point source is $20 \mathrm{Wm}^{-2}$ at a distance of 5.0 m from the source.

What is the irradiance of the light at a distance of 25 m from the source?

$$
\begin{aligned}
I_{1} d_{1}^{2} & =I_{2} d_{2}^{2} \\
20 \times 5^{2} & =I \times 25^{2} \\
500 & =625 I \\
I & =0.80 \mathrm{Wm}^{-2}
\end{aligned}
$$

## Example 37

Calculate the irradiance of a 100 W light bulb at a distance of 2 m .
The light spreads out in all directions.
Surface area of a sphere $=4 \pi r^{2}=4 \times \pi \times 2^{2}=16 \pi=50.27 \mathrm{~m}^{2}$

$$
I=\frac{P}{A}=\frac{100}{50.27}=1.99 \mathrm{Wm}^{-2}
$$

## AHS H notes

## Example 38

The irradiance of light from a point source is 160 units at a distance of 0.50 m from the source.

At a distance of 2.0 m from this source, the irradiance is
A 160 units
B 80 units
C $\quad 40$ units
D 10 units
E $\quad 5$ units

## Irradiance and the Inverse Square Law

## Example 39

A student carries out an experiment to investigate how irradiance on a surface varies with distance from a small lamp.
Irradiance is measured with a light meter.
The distance between the small lamp and the light level meter is measured with a metre stick.
The apparatus is set up as shown in a darkened laboratory.


The following results are obtained.

| Distance from source (m) | 0.20 | 0.30 | 0.40 | 0.50 |
| :--- | :--- | :--- | :--- | :--- |
| Irradiance (units) | 675 | 302 | 170 | 108 |

(a) What is meant by the term irradiance?
(b) Use all the data to find the relationship between irradiance I and distance $d$ from the sourse.
(c) What is the purpose of the black cloth on top of the bench?
(d) A small lamp is replaced by a laser.

Light from the laser is shone onto the light meter.
A reading is taken from the light meter when the distance between it and the laser is 0.50 m
The distance is now increased to 1.00 m .
State how the new reading on the light meter compares with the one taken at 0.50 m
Justify your answer.
(a) energy per second/power incident per square metre/on unit area
(b)

| Distance from source $(\mathrm{m})$ | 0.20 | 0.30 | 0.40 | 0.50 |
| :--- | :--- | :--- | :--- | :--- |
| Irradiance (units) | 675 | 302 | 170 | 108 |
| $I d^{2}$ | 27.0 | 27.2 | 27.2 | 27.0 |

(c) Black cloth absorbs light/Black cloth prevents reflections from the bench top.OR Meter receives light only from the bulb
(d) (approximately) the same reading. Laser beam does not spread out/ diverge/ is parallel. SQA 2006 Q 28

## Irradiance of Photons and Radiation

Since $E=h f$ then $E_{\text {total }}=N h f$
Since $P=\frac{E}{t}$ then $P=\frac{N h f}{t}$
Since $t=1 \mathrm{~s}, \mathrm{P}=\mathrm{Nhf}$
Since $I=\frac{P}{A}$ then $I=\frac{N h f}{A}$
$P=\operatorname{power}(W)$
$E=$ Energy ( $J$ )
$t=$ time ( $s$ )
$h=$ Planck's constant $6.63 \times 10^{-34} \mathrm{Js}$
$f=$ frequency $(\mathrm{Hz})$
$N$ = number of photons per second per $m^{2}$

Since $A=1 \mathrm{~m}^{2}, I=\mathrm{Nhf}$

## Example 40

A laser beam is shone onto a screen which is marked with a grid.
The beam produces a uniformly lit spot of radius $5.00 \times 10^{-4} \mathrm{~m}$ as shown.

(a) The intensity of the spot of light on the screen is $1020 \mathrm{Wm}^{-2}$. Calculate the power of the laser beam.
(b) The distance between the screen and the laser beam is now doubled. State how the radius of the spot now compares with the one shown in the diagram. You must justify your answer.
(a) Area $=\pi r^{2}=3.14 \times\left(5 \times 10^{-4}\right)^{2}=7.85 \times 10^{-7} \mathrm{~m}^{2}$ $P=I A=1020 \times 7.85 \times 10^{-7}=8 \times 10^{-4} \mathrm{~W}$
(b) The radius is approximately the same because the laser light is nearly parallel. (it is not a point source) OR
The radius is slightly greater since the beam diverges a little.

SQA 2008 Q23

## Line Spectra

Each element produces an individual line spectrum. This can be observed using a spectrometer and can be used to identify the element's presence.


## Emission Line spectrum for Helium

The Bohr model of the atom proposes that electrons occupy orbits at particular radii corresponding to particular energy states.


Ground state $\left(\mathrm{E}_{0}\right)$ is the orbit nearest the nucleus. This is where the electron has its lowest energy.

Excited states - electrons in higher energy levels
Ionisation level - this is where electrons gain enough energy to escape from the atom completely. By convention the electron is said to have zero energy when this happens meaning that the other levels have negative energy levels.

To move up an energy level - the electron needs energy.
To move down an energy level - the electron gives off energy as a photon.


When electrons drop down from a higher energy state to a lower energy state a photon of light is emitted.
There are six possible jumps corresponding to six lines in the line spectrum.

## Line Spectra



If many electrons make the same jump the spectral line corresponding to that frequency will be bright because many photons are being emitted. The more excitation levels, the more possible jumps.

## NOTE - transitions occur from one level to another - partial values arenot possible

## Example 41

An atom has the energy levels shown.

$\longrightarrow \mathrm{E}_{0}$
Electron transitions occur between all of these levels to produce emissions lines in the spectrum of this atom.
How many emission lines are produced by transitions between these energy levels?

3 emission lines.
SQA 2004 Q16 (adapted)

Since the lines represent an energy state we can predict the energy needed to move up between levels or the energy released as a photon when an electron moves down between levels.


The move between $E_{2}$ and $E_{1}$ releases energy. We can predict the frequency of the light associated with this photon using $E=h f$

$$
\begin{gathered}
E_{2}-E_{1}=h f \\
\left(-4.29 \times 10^{-19}\right)-\left(-6.47 \times 10^{-19}\right)=6.63 \times 10^{-34} \times f \\
f=\frac{2.18 \times 10^{-19}}{6.63 \times 10^{-34}}=3.29 \times 10^{14} \mathrm{~Hz}
\end{gathered}
$$

Note - to calculate the energy this is the same as [6.47-4.29]

## Line Spectra

## Example 42

Part of the energy level diagram for an atom is shown.

$X$ and $Y$ represent two possible electron transitions.
Which of the following statements is/are correct?
I. Transition $Y$ produces photons of higher frequency than transition $X$.
II. Transition $X$ produces photons of longer wavelength than transition $Y$.
III. When an electron is in the energy level $E_{0}$, the atom is ionised.

A I only
B I and II only
C I and III only
B
D II and III only
E I, II and III only
SQA 2008 Q 17

## Example 43

The diagram represents some electron transitions between energy levels in an atom.


The radiation emitted with the shortest wavelength is produced by an electron making the transition
A $\quad E_{1}$ to $E_{0}$
B $\quad E_{2}$ to $E_{1}$
$C \quad E_{3}$ to $E_{2}$
$E$
$D \quad E_{3}$ to $E_{1}$
E $\quad E_{3}$ to $E_{0}$

## Line Spectra

## Example 44

The following diagrams represent some of the energy levels for two difference atoms $P$ and $Q$. The diagrams are drawn to the same scale.


Electrons are continuously excited to levels $P_{2}$ and $Q_{3}$. When electrons make transitions to lower energy levels, photons of light are emitted. This light is observed as various lines in the emission spectrum of each atom.
(a) For atom $Q$, determine the number of lines in the emission spectrum for the energy levels shown. 6
(b) Considering both atoms, identify the transition that produces radiation of the lowest frequency.
$Q_{3}$ to $Q_{2}\left(N O T\right.$ between $Q_{3}$ and $Q_{2}$ )
(c) The table shows information about the energy levels in atom $P$.

| Energy Level | Energy (J) |
| :---: | :---: |
| $\mathrm{P}_{2}$ | $-2.4 \times 10^{-19}$ |
| $\mathrm{P}_{1}$ | $-5.4 \times 10^{-19}$ |
| $\mathrm{P}_{0}$ | $-21.8 \times 10^{-19}$ |

Calculate the shortest wavelength of radiation emitted from atom P.
Shortest wavelength corresponds to highest frequency.
$\mathrm{P}_{2}$ to $\mathrm{P}_{0}$
$\Delta E=(21.8-2.4) \times 10^{-19}$

$$
=1.94 \times 10^{-18} \mathrm{~J}
$$

$E=h f \quad$ and $\quad v=f \lambda$
$E=h \frac{v}{\lambda} \quad$ and $\quad \lambda=\frac{h v}{E}$
$\lambda=\frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.94 \times 10^{-18}}$

$$
\lambda=1.03 \times 10^{-7} \mathrm{~m}
$$

## Line Spectra

Example 44 (continued)
(d) (i) The emission line due to the transition from $P_{2}$ to $P_{1}$ is the same colour as the emission line from $Q_{2}+Q_{1}$. Explain this observation

Energy gap same size (not similar) Frequency of light emitted is same
(ii) The emission line due to the transition $P_{2}$ to $P_{1}$ is brighter than the emission line from $Q_{2}+Q_{1}$. Explain this observation
( $P_{2}$ to $P_{1}$ is brighter because) more electrons make this transition per second and so more photons are emitted per second. NOTE - a description of rate needs to be mentioned at least once SQA 2014 Q31

## Example 45

The following diagram shows some of the energy levels for a particular atom.

$$
\begin{aligned}
& \mathrm{E}_{3} \longrightarrow-5.2 \times 10^{-19} \mathrm{~J} \\
& \mathrm{E}_{2} \longrightarrow-9.0 \times 10^{-19} \mathrm{~J} \\
& \mathrm{E}_{1} \longrightarrow-16.2 \times 10^{-19} \mathrm{~J} \\
& \mathrm{E}_{0} \longrightarrow-24.6 \times 10^{-19} \mathrm{~J}
\end{aligned}
$$

An electron is excited from energy levels $E_{2}$ to $E_{3}$ by absorbing light energy.
What frequency of light is used to excite this electron?

$$
\Delta E=h f \quad \text { or } \quad W_{2}-W_{1}=h f
$$

$$
\begin{gathered}
-5.2 \times 10^{-19}-\left(-9.0 \times 10^{-19}\right)=6.63 \times 10^{-34} \times f \\
f=5.7 \times 10^{14} \mathrm{~Hz}
\end{gathered}
$$

## Absorption Spectra

White light causes a continuous spectrum when observed through a spectroscope or spectrometer.


When white light is passed through a coloured filter or a vapour the frequencies of light corresponding to the energy level transitions for that material are absorbed. This causes black lines on the continuous spectrum.


## Fraunhofer Lines

When the sun is observed through a spectroscope and absorption spectrum is produced. Gases present in the outer part of the sun absorb photons of particular frequency and this is what produces the absorption lines. They correspond to the emission lines of particular elements. As a result the elements present in the atmosphere of the sun can be identified.
(Named after the German physicist who identified over 500 of these lines)

## Absorption Spectra

## Example 46

The Sun emits a continuous spectrum of visible light. When this light passes through hydrogen atoms in the Sun's outer atmosphere, certain wavelengths are absorbed.
The diagram shows some of the energy levels for the hydrogen atom.

(a) One of the wavelengths absorbed by the hydrogen atoms results in an electron transition from energy level $E_{1}$ to $E_{3}$. Calculate this wavelength.

Energy absorbed $=E_{3}-E_{1}$
$=-1.360 \times 10^{-19}-\left(-5.424 \times 10^{-19}\right)$
$=4.064 \times 10^{-19} \mathrm{~J}$
$\mathrm{E}=\mathrm{hf}$
$4.064 \times 10^{-19}=6.63 \times 10^{-34} \times f$
$f=6.13 \times 10^{14} \mathrm{~Hz}$
$\lambda=\frac{c}{f}=\frac{3 \times 10^{8}}{6.13 \times 10^{14}}=489 \mathrm{~nm}$
(b) The absorption of this wavelength produces a faint dark line in the continuous spectrum from the Sun.
In which colour of the spectrum is this dark line observed?

The light is blue or blue-green (NOT green).

OR consistent with (a) as long as in the visible spectrum.
(Data sheet $480 \mathrm{~nm}=$ blue, 486 nm blue-green, 509 nm green)

## Absorption Spectra

## Example 47

A binary star is a star system consisting of two stars orbiting around each other.
One of the techniques astronomers use to detect binary stars is to examine the spectrum of light emitted by the stars. In particular they look for the changes in wavelength of a specific line called the hydrogen alpha line, over a period of time.
Accurate measurements of the wavelength of the hydrogen alpha line on Earth have determined it to be 656.28 nm .
The following diagram shows some of the energy levels for the hydrogen atom.


Radiation is emitted when electrons make transitions from higher to lower energy levels.
Which transition, between these energy levels, produces the hydrogen alpha line?
Justify your answer by calculation.
$f=\frac{v}{\lambda}=\frac{3 \times 10^{8}}{656.28 \times 10^{-9}}=4.57 \times 10^{14} \mathrm{~Hz}$
$E=h f=4.57 \times 10^{14} \times 6.63 \times 10^{-34}=3.03 \times 10^{-19} \mathrm{~J}$

This corresponds to the transition from $E_{3}$ to $E_{2} O R\left[E_{3} \rightarrow E_{2}\right]$

NOTE
[Mark not awarded if written $E_{2}$ to $E_{3}$ or as $E_{3}-E_{2}$ ]

## Laser

Electrons at a higher energy level are described as 'excited'. When they drop down to a lower energy level a photon is emitted. This can happen spontaneously, but can also be made to happen in a process called 'stimulated emission of radiation'.

Light Amplification (by the) Stimulated Emission (of) Radiation or LASER is an example of a device which uses this process.

Energy supplied to the material the laser is made from causes electrons to become excited. A photon of light stimulates these electrons at a higher energy level to drop down to a lower energy level, releasing more photons. These photons have the same frequency, same direction and are in phase with the photon causing their release, so can be described as coherent. More and more photons are emitted, creating the laser beam. Part of the beam is reflected back inside the laser.
Energy is supplied to the laser to cause electrons to go back to a higher energy level. The reflected photons are used to keep the process going.

The beam created by a laser is monochromatic, intense and does not spread out in the same way as a point source.

## Example 48

In an atom, a photon of radiation is emitted when an electron makes a transition from a higher energy level to a lower energy level as shown


The wavelength of the radiation emitted due to an electron transition between the two energy levels is.
A $\quad 1.2 \times 10^{-7} \mathrm{~m}$
B $\quad 7.3 \times 10^{-8} \mathrm{~m}$
C $\quad 8.2 \times 10^{6} \mathrm{~m}$
D $\quad 1.4 \times 10^{7} \mathrm{~m}$
E $\quad 2.5 \times 10^{15} \mathrm{~m}$
SQA 2012 Q 12

## Example 49

In a laser, a photon of radiation is emitted when an electron makes a transition from a higher energy level to a lower level, as shown below.


The energy in each pulse of radiation from the laser is 10 J . How many photons are there in each pulse?

Energy difference $=-2.2 \times 10^{-19}-\left(-3.3 \times 10^{-19}\right)=1.1 \times 10^{-19} \mathrm{~J}$
Number of photons $=\frac{10}{1.1 \times 10^{-19}}=9.1 \times 10^{19}$
SQA 2002 Q 17 (adapted)

## Example 50

A laser produces a beam of light with a frequency of $4.74 \times 10^{14} \mathrm{~Hz}$.
(a) The laser has a power of 0.10 mW . Explain why light from this laser can cause eye damage.

Very small area/diameter/radius
So
High irradiance because $I=P / A$
(b) Calculate the energy of each photon in the laser beam.
$E=h f$
$=6.63 \times 10^{-34} \times 4.74 \times 10^{14}$
$=3.14 \times 10^{-19} \mathrm{~J}$

$$
\text { SQA } 2010 \text { Q } 29 \text { (a), (b) }
$$

