

## 2018

## Higher

## Electricity Part 1


J. A. Hargreaves

Lockerbie Academy
June 2018

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## CHAPTER 1: BACKGROUND TO ELECTRICITY

## DEFINITIONS

## CURRENT

Current is the rate of flow of charge in a circuit and can be calculated using

$$
Q=I t \text { or } I=\frac{Q}{t}
$$

Where $Q$ =quantity of charge measured in Coulombs (C), I = current (A), $\mathrm{t}=$ time ( s )

In a complete circuit containing a cell, switch and a bulb the free electrons in the conductor will experience a force which will cause them to move drifting away from the negatively charged end towards the positively charged end.


## https://bit.ly/2M0x9sp

Electrons are negatively charged, 1 electron has a charge of $-1.6 \times 10^{-19} \mathrm{C}$.

## POTENTIAL DIFFERENCE

If one joule of work is done in moving one coulomb of charge between two points then the potential difference (p.d.) between the two points is 1 volt. (This means that work is done when moving a charge in an electric field)

$$
W=\boldsymbol{Q V}
$$

Where $W=$ the work done in moving quantity of charge between two points in an electric field (J), Q= quantity of charge (C ), V= potential difference between two points in an electric field in Joules per Coloumb ( $\mathrm{JC}^{-1}$ ) or volts (V). You can use $E_{w}$ for work done but the relationship sheet currently uses W.

| Component <br> Name | Function |
| :---: | :---: | :---: |
| Voltmeter | Measures potential difference. Must be Symbol <br> placed in parallel to measure the <br> difference in electrical potential <br> between two points. |
| Ammeter | Measures current. Must be placed in <br> series to measure the current flowing <br> in a circuit. |
| Ohmmeter | Measures resistance. Must be placed in <br> parallel with the component(s) which <br> are to be measured. |

These meters can be used in simple, or more complex circuits in order to investigate the size of any of these quantities.


The diagram shows how the voltmeter and ammeter must be connected.

The voltmeter must be connected in parallel because it is measuring the difference in electrical potential between two points. It has a high resistance which explains why it needs to be in parallel.

The ammeter must be connected in series because it is measuring the flow of current THROUGH a component or circuit. It has a low resistance which explains why it needs to be connected in series.

Resistance should be measured when a circuit is not 'on' or fully connected and uses the meters own power supply. To use an ohmmeter, you would place it 'across' a component.

SERIES AND PARALLEL: CIRCUIT RULES

| Measurement | Series Circuits | Parallel Circuits |
| :---: | :---: | :---: |
| Voltage | $V_{S}=V_{1}+V_{2}+V_{3} \cdots$ | $V_{S}=V_{1}=V_{2}=V_{3} \cdots$ |
| Current | $I_{1}=I_{2}=I_{3}=I_{4} \cdots$ | $I_{S}=I_{1}+I_{2}+I_{3} \cdots$ |
| Resistance | $R_{\text {total }}=R_{1}+R_{2}+R_{3}+\cdots$ | $\frac{1}{R_{\text {total }}}=\frac{\mathbf{1}}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}+\cdots$ |

RESISTANCE IN SERIES AND PARALLEL
From N5 you should know that resistance in series is equal to the sum of the individual resistors in the circuit, and don't forget that connecting wires can also have a small resistance. It is more complicated for resistance in parallel where adding resistors in parallel reduces the overall resistance. (Think of doors in series slowing the flow of students but doors in parallel would reduce the resistance to students leaving the building).

Worked examples

1. Calculate the resistance of the following circuit

|  |  |  |
| ---: | :--- | ---: |
| $R_{\text {total }}$ | $=?$ |  |
| $R_{1}=60 \Omega \Omega$ | $R_{\text {total }}=R_{1}+\mathrm{R}_{2}+\mathrm{R}_{3}$ |  |
| $\mathrm{R}_{2}=35 \Omega$ | $\mathrm{R}_{\text {total }}=60+35+22$ |  |
| $\mathrm{R}_{3}=22 \Omega$ |  | $\mathrm{R}_{\text {total }}=117 \Omega$ |

2. The total resistance of this circuit is $25 \mathrm{k} \Omega$. Calculate the value of Resistor


$$
\begin{array}{rlrl}
\mathrm{R}_{\text {total }} & =25 \mathrm{k} \Omega & \mathrm{R}_{\text {total }} & =\mathrm{R}_{1}+\mathrm{R}_{2}+\mathrm{R}_{3} \\
\mathrm{R}_{1} & =12 \mathrm{k} \Omega & 25000 & =12000+\mathrm{R}_{2}+500 \\
\mathrm{R}_{2} & =? & \mathrm{R}_{2} & =25000-12500 \\
\mathrm{R}_{3} & =500 \Omega & \mathrm{R}_{2} & =12500 \\
& \mathrm{R}_{2} & =\underline{12.5 \mathrm{k} \Omega}
\end{array}
$$

3. Calculate the resistance of the circuit shown below

|  | $\begin{array}{l}R_{\text {total }}=? \\ R_{1}=100 \Omega\end{array}$ |
| :---: | :--- |
| $200 \Omega$ | $R_{2}=200 \Omega$ |
| $R_{3}=400 \Omega$ |  |

$1 / R_{\text {total }}=1 / R_{1}+1 / R_{2}+1 / R_{3}$
$1 / R_{\text {total }}=1 / 100+1 / 200+1 / 400$
$1 / R_{\text {total }}=4 / 400+2 / 400+1 / 400$
$1 / R_{\text {total }}=7 / 400$
$R_{\text {total }}==400 / 7$
$R_{\text {total }}=\underline{57 \Omega}$
4. The total resistance of this circuit is $100 \Omega$. Calculate the value of Resistor 1

$$
\begin{array}{rlrl}
\mathrm{R}_{1} & \mathrm{R}_{\text {total }}=100 \Omega & 1 / \mathrm{R}_{\text {total }} & =1 / \mathrm{R}_{1}+1 / \mathrm{R}_{2} \\
\mathrm{R}_{1}=? & 1 / 100 & =1 / \mathrm{R}_{1}+1 / 200 \\
\mathrm{R}_{2}=200 \Omega & 1 / \mathrm{R}_{1} & =1 / 100-1 / 200 \\
1 / \mathrm{R}_{1} & =2 / 200-1 / 200 \\
1 / \mathrm{R}_{1} & =1 / 200 \\
& & \mathrm{R}_{1} & =200 \Omega
\end{array}
$$

At higher you can be given more complex problem with a mix of resistance in series and parallel. In these cases it is important to inspect each question and decide if the series or parallel part of the circuit needs to be calculated first.
5.


Parallel Section first:

$$
\begin{gathered}
\frac{1}{R_{p}}=\frac{1}{R_{1}}+\frac{1}{R_{2}} \\
\frac{1}{R_{p}}=\frac{1}{6}+\frac{1}{12}=\frac{2}{12}+\frac{1}{12} \\
\frac{1}{R_{p}}=\frac{3}{12} \quad R_{p}=\frac{12}{3} \\
=4 \Omega \\
R_{T}=R_{1}+R_{p}+R_{3} \\
R_{T}=4+4+16= \\
\underline{R_{T}}=24 \Omega
\end{gathered}
$$

6. Simplify the parallel branches


Branch 1
Branch 2

Calculate resistance of parallel section

$$
\begin{gathered}
R_{p 1}=R_{1}+R_{2} \\
R_{p 2}=R_{1}+R_{2} \\
R_{p 1}=3+6=9 \Omega \\
R_{p 2}=2+16=18 \Omega \\
\frac{1}{R_{p}}=\frac{1}{9}+\frac{1}{18} \\
\frac{1}{R_{p}}=\frac{2}{18}+\frac{1}{18} \\
\frac{1}{R_{p}}=\frac{3}{18} \\
=>R_{p}=18 / 3=6 \Omega \\
R T=R p_{1}+R p_{2} \\
R T=6+5=11 \Omega
\end{gathered}
$$

PRACTICAL 1 - RESISTORS IN SERIES PRACTICAL


Aim: To check the formula for resistors in series against our experience, set up the following circuit:-

1. Put 10 cm of resistance wire into the holder and note the current and voltage readings.
2. Feed out more wire so that 20 cm of the same wire lies between the terminals of the holder.
3. Adjust the variable resistor until the current is the same as for the 10 cm length of wire.
4. Note the voltmeter reading.

Predict the voltage reading after you have adjusted the current when 30 cm of wire stretches between the holder's terminals.

Discuss why the voltmeter reading increases as the resistance wire is lengthened.

PRACTICAL 2- RESISTANCE IN PARALLEL

1. Using the same basic apparatus as in Practical 1 put 10 cm of resistance wire into the holder.
2. Note the current and voltage readings.
3. Now put a second 10 cm length of resistance wire into the holder along with the first piece.


Two 10 cm lengths of resistance wire.
4. Use the variable resistor to adjust the voltage to the same as the previous value, then take note of the current.

PREDICT: What the current reading is after you have put 3 separate 10 cm lengths of wire into the holder and adjusted the voltage to its previous value.

DISCUSS: Why does the current reading increase as the experiment progresses.

## FIELDS

In Physics, a field means a region where an object experiences a force without being touched.

For example, there is a gravitational field around the Earth, attracting masses towards the Earth's centre. Magnets cause magnetic fields and electric charges have electric fields around them.

Electric Fields are quite difficult to visualise and hence understand. If however we compare it to something that we can understand, or at least something that is familiar then we have a better chance of visualising an abstract concept, in this case electric fields.

Firstly we need to know what a gravitational field is its cause. Anything that has a mass causes an attraction
 between it and other masses. You are currently attracted to this page, the bench, and your teacher. Don't worry though! The size of the attraction is tiny, that is because the masses involved are fairly small. However, if the mass is large (e.g. the Earth), then we feel this attraction and it causes an acceleration. We call this
the gravitational force, which pulls other masses towards it. Obviously when you are in the Southern Hemisphere you are still pulled towards the centre of the earth.

The area around the large mass in which another mass experiences this force is called a gravitational field. The strength of this field is called the gravitational field strength, remember this! On Earth it has an average value of $9.8 \mathrm{Nkg}^{-1}$ at the surface.

As you get further away from the mass the

strength of the field gets weaker. The strength of the field can be shown using field lines. These do not really exist (look out of the window!) but they do show a concept that is real. (This is also similar to magnetic field lines when you draw the direction that a compass will point around a magnet) These field lines are a convenient, fictional concept that represents a real field.

## NOTES ON FIELD LINES

$\checkmark$ The tangent to a field line gives the direction of the field at that point.
$\checkmark$ The no. of field lines per cross sectional area is proportional to the magnitude of the field at that point.
$\checkmark$ Field lines go from positive to negative, or North to South, towards the mass.
$\checkmark$ Field lines never cross.
$\checkmark$ If they are close together the field is strong
$\checkmark$ If they are far apart the field is weak
$\checkmark$ If they are parallel/ equally spaced the field is uniform.
$\checkmark$ As you move along a field line your potential is decreasing.

## GRAVITATIONAL FIELD

A gravitational field is the region around a mass where another mass experiences a force.

Or
A gravitational field is said to exist at a point if a force caused by a mass is exerted on a mass placed at that point.

The size of the field is given by:
$g=\frac{F_{w}}{m}$
where $g=$ gravitational field strength in $\mathrm{Nkg}^{-1}$
$F_{w}=$ Force on the mass in Newtons
$m=$ mass being attracted in kilograms

Now use the information above and predict the definition of an electric field.

## ELECTRIC FIELDS

In an electric field, a charged particle will experience a force. We use lines of force to show the strength and direction of the force. The closer the field lines the stronger the force. Field lines are continuous - they start on positive and finish on negative charge. The direction is taken as the same as the force on a positive "test" charge placed in the field.
The lines around a point charge are like the radii of a circle and are called radial fields. The strength of the field decreases as we move away from the charge.


Charges close to each other affect each other and hence the field pattern. Opposite charges attract and similar charges repel. The field lines between two parallel plates are equally spaced indicating the field strength is constant. and is hence called a uniform field.


## Definition:

A electric field is the region around a charge where another charge experiences a force.
Or
An electric field is said to exist at a point if a force of electrical origin is exerted on a test charge placed at that point.

The size of the field is given by:

$$
E=\frac{F}{Q}
$$

Where $E=$ electrical field strength in NC-1,
$F=$ force on the charge in Newtons and
$Q=$ charge being attracted in Coloumbs.
So using the information about gravitational fields has helped us with our definition of an electric field. There are other similarities too which we will look at later.

| Field type | cause of <br> the field | definition of <br> field strength | Units | Inverse <br> square law |
| :---: | :---: | :---: | :---: | :---: |
| gravitational | $m$ | $g=\frac{F}{m}$ | g in Nkg-1 <br> F in newtons <br> $m$ in kilograms | $F=\frac{G m_{1} m_{2}}{r^{2}}$ |
| electric | $Q$ | $E=\frac{F}{Q}$ | $E$ in $N C^{-1}$ <br> Fe in Newtons <br> $Q$ in coloumbs | $F=k \frac{Q_{1} Q_{2}}{r^{2}}$ |

There is one special difference between electric fields and gravitational field which has led to much research. There are two types of charge so a charge can be attracted or repelled. The field lines are drawn in to indicate the direction a POSITIVE test charge would travel. If a negative charge were placed in the field then the negative charge would move in the opposite direction from the field line. Masses you are never repelled, always attracted. Hmm watch this space for the dark matter!

## Path of a charge in a uniform field



## NOTES ON ELECTRIC FIELDS

$\checkmark E$ is a vector quantity.
$\checkmark E$ is measured in $\mathrm{NC}^{-1}$ but can also be measured in $\mathrm{Vm}^{-1}$.
$\checkmark$ Q must be small enough so that it does not itself affect the field


## MOVING CHARGES

Two uncharged plates are shown in the first figure below. A charge is moved from one plate to the other. A small amount of work has to be done moving the charge from one plate to the other. This creates a very weak uniform electric field between the two plates.


A small amount of work has to be done moving the charge from one plate to the other. This creates a very weak electric field.

The more charges are moved the more work has to be done. The electric field becomes stronger.

Work is done moving the charge and this is stored in the system.

As one plate becomes positive and one plate becomes negative the potential difference between the plates increases.

The greater the number of charges moved the more work has to be done and the electric field between the plates becomes stronger.

Work is done moving the charge and this is stored in the system.
As one plate becomes positive and one plate becomes negative the potential difference between the plates increases.

If positively charge $Q$ is moved from the negative to the positive plate work has to be done against the field. The amount of work done is given by
work done $=$ potential difference between the plates $\times$ charge being moved

$$
W=V \boldsymbol{Q}
$$

As the field is uniform there is a constant force F on Q . The work done depends on the distance moved.

$$
\begin{gathered}
\text { work done }=\text { force } \times \text { distance } \\
E_{w}=F d
\end{gathered}
$$

but

$$
E_{w}=Q V
$$

so

$$
\begin{gathered}
Q V=F d \\
E=\frac{F}{Q}=\frac{V}{d}
\end{gathered}
$$

$\mathrm{F} / \mathrm{Q}$ is the electric field strength $\left(\mathrm{NC}^{-1}\right)$
$\mathrm{V} / \mathrm{d}$ is called the potential gradient $\left(\mathrm{Vm}^{-1}\right)$
Further to this the work done in moving a charge can be released as kinetic energy in a moving charge

$$
\begin{aligned}
E_{w} \text { lost } & =E_{K} \text { gained } \\
Q V & =\frac{1}{2} m v^{2}
\end{aligned}
$$

## Definition:

If one joule of work is done moving one coulomb of charge between two points, the p.d. between the two points is 1 V
or
the p.d.(voltage V ) between two points is a measure of the work done in moving one coulomb of charge between two points.

## Examples

1. The diagram below shows a cathode ray tube used in an oscilloscope.


Electrons released from the hot cathode are accelerated by a p.d. of 5.0 kV between the cathode and anode.
(a) i)Assuming that an electron starts from rest at the cathode, calculate its speed just before it reaches the anode. (You may have to refer to the Data sheet.)
ii) What is the effect on the speed of the electron just before it reaches the anode if the p.d. between the cathode and anode is halved? Show your reasoning.
(b) If the electron beam current is 15 mA , how many electrons leave the cathode each second? (You may have to refer to the Science Data Booklet.)
2. The electrons which are emitted from the cathode start from rest and reach the anode with a speed of $4.2 \times 10^{7} \mathrm{~ms}^{-1}$
a) i)Calculate the kinetic energy in joules of each electron just before it reaches the anode.
ii)Calculate the p.d. between the anode and the cathode.
b) Describe how the spot at the centre of the screen produced by the electrons can be moved to position $X$.

Your answer must make reference to the relative sizes and polarity (signs) of the voltages applied to plates P and Q .

Answers

| $E_{w}=q V$ | $I=15 \mathrm{~mA}$ |
| :--- | :--- |
| $E_{w}=1.6 \times 10^{-19} \times 5000$ | $Q=I t$ |
| $E_{w}=8 \times 10^{-16} \mathrm{~J}$ | $Q=15 \times 10^{-3} \times 1$ |
| Work done against the field = kinetic Energy gained | $Q=15 \times 10^{-3} \mathrm{C}$ |
| $E_{K}=\frac{1}{2} m v^{2}$ |  |
| $8 \times 10^{-16}=\frac{1}{2} \times 9.11 \times 10^{-31} \times v^{2}$ | No. $=\frac{Q_{T}}{Q_{e}}=\frac{15 \times 10^{-3}}{1.6 \times 10^{-19}}=9.4 \times 10^{16}$ |
| $\frac{8 \times 10^{-16} \times 2}{9.11 \times 10^{-31}}=v^{2}$ |  |
| $v=\sqrt{\frac{8 \times 10^{-16} \times 2}{9.11 \times 10^{-31}}}$ |  |
| $v=4.2 \times 10^{-7} \mathrm{~ms}^{-1}$ |  |

PRACTICAL 3- THE VAN DE GRAAFF


Aim: To familiarise yourself with the Van de Graaff generator.

Watch and participate in the experiments demonstrating the Van de Graaff generator.

Ensure you can explain how the Van de Graaff:
$\checkmark$ creates the charge
$\checkmark$ stores charge
$\checkmark$ creates the size and shape of the field pattern and force around it
$\checkmark$ distributes the charge.
Write up your experiences including the Physics behind the Van de Graaff.

## OHM'S LAW

Ohm's law forms the basis for understanding how electrons or charge flows through circuits. Ohm's Law deals with the relationship between voltage and current in an ideal conductor.

This relationship states that:
For a constant temperature for a given resistor:

V $\propto$ I or V/I = constant

$V=I R$
Where $V=$ potential difference in Volts, $I=$ current (A), $R=$ resistance ( $\Omega$ )
There are components which do not have a constant resistance causing the current through them to alter - e.g. a light bulb. Graphs of potential difference against current for this type of component will not be a straight line.

## REVISION TUTORIALS

TUTORIAL 1: ELECTRIC FIELDS
1.


Explain with the aid of a diagram, what effect we get when a positively charged object is brought near the right hand edge of these rods:-
2. State the size of angle $X$.

3. An oil drop with a charge of 3 pC and a mass of $1.22 \times 10^{-8} \mathrm{~kg}$ is placed in a uniform electric field of $4 \times 10^{4} \mathrm{NC}^{-1}$. If the oil drop floats between the charged
plates, calculate its weight, (NB there is nothing in this question to state where the oil drop is held!)
4. A small polystyrene sphere with a positive charge of $5 p \mathrm{C}$ is placed in a uniform electric field at the 10 V level. If the sphere falls downwards to a place where the voltage level is 150 V , calculate the work done by the gravitational field.

## TUTORIAL 2 CIRCUITS

1. A lamp is rated at 48 W 12 V . When running normally,
a) Calculate the current through the lamp.
b) Calculate the lamp's resistance while it is operating.
2. If a $2.0 \mathrm{k} \Omega$ resistor has a p.d. of 9.6 V across it, determine the current in it?
3. A model power transmission line is 3.0 m long and carries a current of 4.0 A . Determine the p.d. across it if the wire used has a resistance of 125 milliohms per metre.

TUTORIAL 3- RESISTANCE

1. Calculate the total resistance of this arrangement?

2. If the equivalent resistance of the following arrangement is $250 \Omega$, calculate the resistance of the unknown resistor.

3. Calculate the equivalent resistance of this arrangement of resistors.

4. If the equivalent resistance of the following arrangement is $250 \Omega$, calculate the resistance of the unknown resistor.

5. Calculate the equivalent resistance of $200 \Omega, 300 \Omega$ and $600 \Omega$ when connected in parallel.
6. Calculate the equivalent resistance of three resistors, $4.0 \Omega, 4.0 \Omega$ and $2 \cdot 0 \Omega$ in parallel.
7. Calculate the resistance of a resistor which gives an equivalent resistance of $18.75 \Omega$ when connected in parallel with a $75.00 \Omega$ resistor.

PAST PAPER QUESTIONS

## SQA Revised Higher 2014 Q16 \& 17

16. Five resistors are connected as shown.

The resistance between X and Y is

A $\quad 12 \Omega$
B $\quad 20 \Omega$
C $\quad 30 \Omega$
D $60 \Omega$
E $\quad 180 \Omega$

17. A circuit is set up as shown. The internal resistance of the supply is negligible. Which row in the table shows the potential difference (p.d.) across the $12 \Omega$ resistor when switch S is open and when S is closed?


|  | p.d. across $12 \Omega$ <br> resistor when $S$ is <br> open/V | p.d. across $12 \Omega$ <br> resistor when $S$ <br> is closed/V |
| :---: | :---: | :---: |
| A | 30 | 18 |
| B | 45 | 45 |
| C | 60 | 45 |
| D | 60 | 72 |
| E | 72 | 60 |

## TUTORIAL ANSWERS

## TUTORIAL 1

1. 

copper $\qquad$ ㄱ
polys tyrene

2. $X=150$ since the field is uniform.
3. Since the drop is floating, the electric force, $1.2 \times 10^{-7} \mathrm{~N}$, is the same as its weight.
4. Since the motion is caused by the sphere's weight, the work done by the gravitational field is $7 \times 10^{-10} \mathrm{~J}$.

## TUTORIAL 2

1. a) 4 A
b) $3 \Omega$
2. 4.8 mA
3. 1.5 V

## TUTORIAL 3

1. $1210 \Omega$
2. $40 \Omega$
3. $300 \Omega$
4. $500 \Omega$
5. $100 \Omega$
6. $1 \Omega$
7. $25 \Omega$

SQA Revised Higher 2014
Q16 B
Q17 D

## CHAPTER 2: MONITORING AND MEASURING A.C

## SUMMARY OF CONTENT

## MONITORING AND MEASURING A.C.

| $T$ | eq | $T=\frac{1}{f} \quad V_{r m s}=\frac{V_{p e a k}}{\sqrt{2}} \quad I_{r m s}=\frac{I_{p e a k}}{\sqrt{2}}$ |
| :--- | :--- | :--- |
| a) | I know that an A.C. is a current which changes direction and <br> instantaneous value with time. |  |
| b) | I can use $\quad V_{r m s}=\frac{V_{p e a k}}{\sqrt{2}} \quad I_{r m s}=\frac{I_{p e a k}}{\sqrt{2}}$ to calculate involving peak <br> and r.m.s. values. |  |
| c)I can determine the frequency, peak voltage and r.m.s. values from <br> graphical data. |  |  |
| d) | I can use $T=\frac{1}{f}$ to determine the frequency. |  |

## MEASURING THE FREQUENCY

Frequency is the number of cycles of the waveform in one second. Frequency is calculated as the reciprocal of ('one divided by') the time for one cycle, $T$.

An oscilloscope plots a graph of changing voltage on the $y$-axis versus time on the $x$-axis. Just like a graph we need to read the scale, except the numbers aren't along the axis, they are on a dial underneath the screen. The voltage axis scale is usually called the volts/div or volts/cm and the time axis scale is called the timebase.

At Higher it is not enough to find the frequency by inspection (this means comparing your known trace to your unknown trace using a double beam oscilloscope and a signal generator) You need to be able to use the timebase! We first need to work out the period, $T$ or time for one wave.

1. Set up the oscilloscope to give a stationary trace.
2. Adjust the trace so that the (time) period can be found.
3. Period $T=$ time for one wave
4. Count the number of squares for 1 wave.

5. Multiply the number of squares for 1 wave by the time base setting, giving $T$.
6. Find $f=\frac{1}{T}$
7. (Time setting will be in ms or $\mu \mathrm{s}$. DON'T FORGET TO CHANGE IT!!)

NOW HOW TO MEASURE FREQUENCY USING AN OSCILLOSCOPE
e.g.

1. An oscilloscope is connected to the output terminals of a signal generator.

The trace displayed on the screen is shown below.
The timebase of the oscilloscope is set at $30 \mathrm{~ms} /$ div.
The frequency of the output signal from the signal generator is

A $2 \times 10^{-3} \mathrm{~Hz}$
B $8.3 \times 10^{-3} \mathrm{~Hz}$
C 0.28 Hz
D 4.2 Hz
E 8.3 Hz .

2. An alternating voltage is displayed on an oscilloscope screen. The Y-gain and the timebase settings are shown.

Which row in the table gives the values for the peak voltage and frequency of the signal?


|  | Peak <br> voltage/V | Frequency <br> $/ \mathrm{Hz}$ |
| :---: | :---: | :---: |
| A | 10 | 100 |
| B | 10 | 250 |
| C | 20 | 250 |
| D | 10 | 500 |
| E | 20 | 1000 |

## Example 1:

1 wave is 4 boxes across in the trace above. Each box is 30 ms .
$\therefore T=4 \times 30=120 \mathrm{~ms}=1.2 \times 10^{-1} \mathrm{~s}$

$$
f=\frac{1}{120 \times 10^{-3}}=8.3 \mathrm{~Hz}
$$

$\therefore$ Answer E
Example 2:
Peak $=8 / 2=4$ boxes each $5 \mathrm{~V}=5 \times 4=20 \mathrm{~V}$
1 wave is 4 boxes across in the trace above. Each box is 1 ms .
$\therefore T=4 \times 1=4.0 \mathrm{~ms}=4.0 \times 10^{-3} \mathrm{~s}$

$$
f=\frac{1}{4 \cdot 0 \times 10^{-3}}=250 \mathrm{~Hz}
$$

PEAK AND R.M.S. VOLTAGE
The definition of the r.m.s. voltage is " value of alternating voltage that produces the same power (e.g. heating or lighting) as the direct voltage". The r.m.s. value is what is quoted on a power supply so that a fair comparison between a.c. and d.c. can be made, e.g. a 6 V battery will produce the same brightness of light bulb as a 6 V r.m.s. a.c. supply. The r.m.s. current has a similar definition.

An a.c is one where the magnitude and direction of the current changes, usually many times per second. The mains is an example of an a.c.
But what do we take as the voltage to quote?

With an a.c trace the direction and magnitude keeps changing.



The average voltage is not a good indication of the effect of the voltage as the average voltage is always zero.

Peak voltage isn't a good indication of the voltage. It also cannot be compared with a d.c. supply. It will over- estimate the power dissipated in the circuit.


The solution is to do with the fact that half the time the charges move in one direction and half the time in the opposite direction. We can get rid of this problem by squaring everything.

Now we can average this squared voltage. When we square root the answer we get the the r.m.s (root mean square) value. Now a.c. and d.c. are comparable!


The pattern of the graph now changes and it is possible to work out an average for the new graph.

It turns out that the average is
$\frac{V_{p}{ }^{2}}{2}$.

To get back to the original we now need to take the square root of this value which will be $\frac{V_{p}}{\sqrt{2}}$ This is called the r.m.s. value $=\frac{\text { peak }}{\sqrt{2}}$

```
PEAK AND R.M.S. CURRENT
```

The same applies for current. a.c. is one in which electrons flow first in one direction then in the other.


NB. i) Peak is always the bigger than the value quoted as an a.c. supply. $r m s=\frac{\text { peak }}{\sqrt{2}}$
ii) $\sqrt{2}=1.414$

In a resistor the rate at which heat is produced is given by $P=I^{2} R$

With a.c. what value do you take as I?
a) average I $=0 \mathrm{~A}$
b) peak only occurs for a very short period therefore it is an overestimate.

We therefore want something in between.
The clue is in $1^{2}$


Squaring the current and plotting it on a graph shows that $\mathrm{I}^{2}$ is always positive. The graph is symmetrical and the average is half its maximum.

$$
\begin{aligned}
& \text { average of } I^{2}=\frac{1}{2}\left(I_{P}\right)^{2} \\
& \qquad I_{R M S}=\frac{I_{P}}{\sqrt{2}}
\end{aligned}
$$

Where

$$
\begin{aligned}
& I_{R M S}=\text { quoted or root mean squared current } \\
& \qquad I_{P}=\text { peak current }
\end{aligned}
$$

PRACTICAL 1: COMPARING BRIGHTNESS

COMPARING BULB BRIGHTNESS WITH AN A.C AND D.C SUPPLY.
Set up the circuit as shown below and take your own readings to shown that the r.m.s. of an a.c trace provides the same power to a circuit as a d.c trace.


## Power on d.c

Current $=1.42 \mathrm{~A}$
Voltage $=12.35 \mathrm{~V}$
$\mathrm{P}=\mathrm{IV}$
Power on d.c. $=17.5 \mathrm{~W}$

Power on a.c.
No. of squares from top of the trace to the bottom $=6.6$

Each square is $5.0 \mathrm{~V} /$ div

Therefore peak voltage $=3.3 \times 5 \cdot 0 \mathrm{~V}=16.5 \mathrm{~V}$
r.m.s value=
$V_{r m s}=\frac{V_{p}}{\sqrt{2}}=\frac{16.5}{\sqrt{2}}=11.7 \mathrm{~V}$
Current $=1.464 \mathrm{~A}$
Voltage $=11.7 \mathrm{~V}$
Power on a.c $=\underline{\underline{17.1} \mathrm{~W}}$

We can now calculate the average power delivered by this a.c. to the resistor.
$P=I^{2} R \quad P_{a c}=\frac{1}{2}\left(I_{P}\right)^{2} R$
where $I_{P}=$ peak current.
Compare with d.c. which provides a steady current:

$$
P_{d c}=I^{2} R
$$

If we arrange the equipment so that the same power is delivered by a.c. and d.c. supply then we can compare I and Ip.

$$
I=\sqrt{\frac{1}{2}\left(I_{P}\right)^{2}}
$$

$$
\sqrt{2}=1.4142
$$

$$
\therefore \quad I=\frac{I_{P}}{1.4142}
$$

$$
\begin{aligned}
& P_{a c}=P_{d c} \\
& \frac{1}{2}\left(I_{P}\right)^{2} R=I^{2} R \\
& \therefore \quad \frac{1}{2}\left(I_{P}\right)^{2}=I^{2} \\
& \therefore \quad I^{2}=\frac{I_{P}^{2}}{2} \\
& \therefore \quad I=\frac{I_{P}}{\sqrt{2}}
\end{aligned}
$$

REMEMBER THAT $I_{P}$ IS ALWAYS $>I_{\text {rms }}$
In a circuit containing a resistor the current is completely independent of frequency. i.e. you change the frequency and the current stays the same. This would not be the same for a capacitor.

PRACTICAL 2: CHECKING EQUATION: I IMS :


Check the equation above by finding the connection between the peak value of an a.c. supply and the value of a direct current which lights a bulb with the same brightness. The circuit needed is shown here: -


- With the two-way switch at A, the bulb lights on a.c.
- With the two-way switch at $\mathbf{B}$, the bulb lights on d.c. whose value can be adjusted by using the variable resistor as a voltage divider.
- Adjust the d.c. output until movement of the switch between $A$ and $B$ shows no difference in the brightness of the bulb. This ensures that both the a.c. and d.c. supplies are providing the same power.
- Record the value of the direct current.
- Use the C.R.O. screen and Ohm's law to find the alternating current's peak value.
- Adjust the a.c. power supply, repeat the d.c. adjustments and get another pair of alternating and direct current values.
- Use your table of results to plot a graph of Peak a.c. value against d.c. value
- Use this graph to find the average of the ratio $\frac{\text { d.c. }}{\text { peak value of a.c. }}$


## TUTORIAL 1: ALTERNATING CURRENT \& VOLTAGE

1. Calculate the peak value of our 230 V mains supply.
2. An oscilloscope shows the peak voltage from a laboratory power supply as 141 Volts, calculate the r.m.s. voltage it supplies.
3. A motor for use with an alternating current supply is rated at $250 \mathrm{~V}, 500 \mathrm{~W}$. Calculate the peak voltage and current the motor is designed to handle?
4. Determine the frequency of the signal shown on this oscilloscope screen.

5. A mass of 10 kg falls steadily through a height of 5 m while attached by a wire to an electric generator. If the mass takes 10 seconds to fall and all of its energy goes into the generator,
a) Calculate the output power from the generator.
b) Calculate the peak voltage output if the generator produces a steady 3.5A.
6. A mains electric clock stores enough energy in 10 s to run itself for 10 hours. If the current during the storage phase has a peak value of 10.6 A , calculate the power used by the clock mechanism.
7. An electric heater has a heating element of resistance $50 \Omega$ with an alternating current of peak value 8.5 A in it. Determine the raise in temperature of 9.7 kg of water in 15 minutes? (Take the specific heat capacity of water as $4.19 \times 10^{3} \mathrm{Jkg}^{-10} \mathrm{C}^{-1}$.

PRACTICAL 3 - ELECTRON GUN
Examine the electron gun from a cathode ray oscilloscope and note the subtle shapes employed to alter the uniformity of the fields that guide electrons into the desired beam shape.

The deflection plates are positioned above and below the screen, which has its own graduated scale. So the effect of the deflecting voltage can be measured on the scale.

Using the graticule, it is possible to show that the path is parabolic in an electric field and circular in a magnetic field.

http://www.practicalphysics.org/ types-electron-tube.html

PRACTICAL 4- ENERGY PER UNIT CHARGE IS VOLTAGE
Aim: To show that the energy delivered by the current is measured in joules per coulomb which is volts.

(i) Use the ammeter and stopwatch together (Q=It) to calculate the number of coulombs passed through the immersion heater.
(ii) Use the relationship $\mathrm{E}_{\mathrm{h}}=\mathrm{mc} \Delta \mathrm{T}$ to calculate the number of joules delivered by the heater.
(iii) Use the answers to parts (i) and (ii) to calculate the number of joules delivered by each coulomb and compare it with the reading shown on the voltmeter.

TUTORIAL 2- PAST PAPER QUESTIONS

## 1997 Paper 1 Question 34.

The output from a signal generator is connected to the input terminals of an oscilloscope. A trace is obtained on the oscilloscope screen. The oscilloscope control settings and the trace on the oscilloscope screen are shown in the diagram below.
a) Calculate the frequency of the output from the signal generator
b) The frequency and amplitude of the output from the signal generator are kept constant. The time base control setting is changed to $5 \mathrm{~ms} /$ division. What will be the effect on the trace shown on the oscilloscope?

## SQA Higher 1992 Paper 1 Q17

The diagram shows the screen and the settings of an oscilloscope, which is being used to measure the output frequency of a

 signal generator.

What is the frequency of the signal applied to the input of the oscilloscope?

A $\quad 2.5 \mathrm{~Hz}$
B $\quad 12.5 \mathrm{~Hz}$
C $\quad 40 \mathrm{~Hz}$
D $\quad 250 \mathrm{~Hz}$
E $\quad 500 \mathrm{~Hz}$

## SQA Higher 1992 Paper 1 Q17

The corresponding r.m.s. voltage is
A 5V

B $\quad \frac{10}{\sqrt{2}}$
C $\quad 10 \mathrm{~V}$
D $\quad 10 \sqrt{2} \mathrm{~V}$
E $\quad 20 \mathrm{~V}$


## SQA Higher 1993 Paper 1 Q34

An oscilloscope is connected across a resistor in a circuit. The trace obtained is shown below.

The peak voltage shown on the oscilloscope is 10 volts and the time base setting is $0.2 \mathrm{~ms} \mathrm{~cm}-1$. Calculate
(a) The r.m.s voltage across the resistor
(b) The frequency of the a.c. voltage

## SQA Higher Physics Paper 2013 Q 26

The circuit shown is used to compare the voltage from a battery and the voltage produced by a signal generator.


The switch is connected to X and the voltage across the lamp is $2 \cdot 30 \mathrm{~V}$. The reading on the light meter is recorded.

The switch is now connected to Y . The resistance of $\mathrm{R}_{\mathrm{V}}$ is adjusted until the light meter reading is the same as before. The trace on the oscilloscope screen is shown.
(a) The timebase setting is $0.01 \mathrm{~s} / \mathrm{div}$. Calculate the frequency of the output voltage of the signal generator.
(b) Calculate the peak value of the voltage displayed on the oscilloscope.
(c) With the switch still connected to Y , the signal generator
 frequency is now doubled without altering the output voltage. State what happens to the reading on the light meter. Justify your answer.

## SQA Higher Physics Paper 2010 Q 26 part a and b

A signal generator is connected to a lamp, a resistor and an ammeter in series. An oscilloscope is connected across the output terminals of the signal generator.


The oscilloscope control settings and the trace displayed on its screen are shown.

(a) For this signal calculate:
(i) the peak voltage;
(ii) the frequency.
(b) The frequency is now doubled. The peak voltage of the signal is kept constant.

State what happens to the reading on the ammeter.

## SQA Higher 2011 Q11

11. The output of a 50 Hz a.c. supply is connected to the input of an oscilloscope. The trace produced on the screen of the oscilloscope is shown.


1 div

The time-base control of the oscilloscope is set at

A $1 \mathrm{~ms} / \mathrm{div}$
B $10 \mathrm{~ms} / \mathrm{div}$
C $20 \mathrm{~ms} / \mathrm{div}$
D $100 \mathrm{~ms} / \mathrm{div}$
E $200 \mathrm{~ms} /$ div.

## A.C \& VOLTAGE/ TUTORIAL ANSWERS

## TUTORIAL 1

1. The peak value is 325 V
2. The r.m.s. voltage is 100 V
3. The motor is designed to handle a maximum voltage of 354 V and a maximum current of 2.82 A
4. Frequency $=1 / 0.001=1000 \mathrm{~Hz}$
5. a) The power given out is 49 W
b) The peak voltage output is 19.8 V
6. The power used by the clock is 0.48 W
7. The water temperature is raised by $40^{\circ} \mathrm{C}$.

TUTORIAL 2: SOLUTIONS TO PAST PAPER QUESTIONS
1997 34.a. Wave period $(T)=4 \times 2.5 \mathrm{~ms} \quad \mathrm{~T}=10 \times 10^{-3} \mathrm{~s}$

$$
\begin{aligned}
& f=1 / T \\
& f=1 / 10 \times 10^{-3} S \\
& f=100 \mathrm{~Hz}
\end{aligned}
$$

b. The amplitude of the waves displayed on the oscilloscope will be unchanged, but, five complete waves will now appear on the screen.
1992 P1 Q14
D 250 Hz

1992 P1 Q17
B $10 / \sqrt{2}$
1993 P1 34.a.

$$
\begin{aligned}
& V_{\text {rms }}=V_{\text {peak }} / \sqrt{2} \quad V_{\text {RMS }}= \\
& \text { b. } \quad \mathrm{f}=1 / \mathrm{T} \\
& \mathrm{~T}=8 \times 0.2 \mathrm{~ms}=1.6 \mathrm{~ms} \\
& \mathrm{~T}=1.6 \times 10^{-3} \\
& \mathrm{f}=1 / 1.6 \times 10^{-3} \\
& \mathrm{f}=625 \mathrm{~Hz}
\end{aligned}
$$

$$
V_{\text {RMS }}=10 / 1.414
$$

$$
\mathrm{V}_{\text {RMS }}=7.07 \mathrm{~V}
$$

## 2013 SQA Higher Paper Q26

26a Total time $=$ no. of division $\times$ time base setting

$$
\begin{aligned}
=\quad 10 \times 0.01 & =0.1(\mathrm{~s}) \\
& f=\frac{\text { No.of waves }}{\text { total time }} \\
f & =\frac{2.5}{0.1}=2.5 \mathrm{~Hz}
\end{aligned}
$$

b)

$$
\begin{gathered}
V_{p}=\sqrt{2} V_{r m s} \\
V_{p}=\sqrt{2} \times 2.3=3.25 \mathrm{~V}
\end{gathered}
$$

c) Stays constant / no change/ nothing happens. Current is independent of supply frequency

## 2010 SQA Higher Paper Q26

26 a) i) $\quad V p=2.0 \mathrm{~V}$
ii) $f=\frac{1}{T}=\frac{1}{0.01}=100 \mathrm{~Hz}$
b) Stays the same/ constant/ no change / nothing
c) Increases / double

## 2011 SQA Higher Paper Q11

11. B

CHAPTER 3: I, V, P \& R

## SUMMARY OF CONTENT

Current, potential difference, power and resistance

| 雷 | $V=I R \quad P=I V=I^{2} R=\frac{V^{2}}{R} \quad R_{T}=R_{1}+R_{2}+\ldots$ |
| :---: | :---: |
| eq | $\frac{1}{R_{T}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\ldots \quad V_{2}=\left(\frac{R_{2}}{R_{1}+R_{2}}\right) V_{S} \quad \frac{V_{1}}{V_{2}}=\frac{R_{1}}{R_{2}}$ |
| a) | I can use relationships involving potential difference, current, resistance and power to analyse circuits even those that may involve several steps in the calculations. |
| T ${ }^{\text {T }}$ b) | I can correctly use calculations involving potential dividers circuits. |

The power of a circuit component (such as a resistor) tells us how much electrical potential energy the component transforms (changes into other forms of energy) every second: The following formulae are also used to calculate power (P):

$$
\begin{gathered}
P=\frac{E}{t} \\
P=I V \\
P=I^{2} R, \\
P=\frac{V^{2}}{R}
\end{gathered}
$$

Where $\mathrm{P}=\operatorname{Power}(\mathrm{W}), \quad E=$ energy (J), $\quad \mathrm{t}=$ time ( s ), $\quad \mathrm{V}=$ voltage (V),
$\mathrm{I}=$ current $(\mathrm{A}), \quad \mathrm{R}=$ resistance $(\Omega)$
These bottom two equation arise as a combination of $P=I V$ and $V=I R$. Try to do these substitutions to remind you how they are obtained.

## POTENTIAL DIVIDER CIRCUITS

Any circuit that contains more than one component can be described as a potential divider circuit. In its simplest form a potential divider is 2 resistors connected across a power supply. If another component is placed in parallel with a part of the potential divider circuit, the operating potential difference of this component can be controlled.


Formula for a series circuit

$$
\begin{array}{lr}
V_{s}=V_{1}+V_{2}+V_{3} & \text { etc } \\
I_{T}=I_{1}=I_{2}=I_{3} & \text { etc } \\
V=I \times R & \\
R_{T}=R_{1}+R_{2}+R_{3} & \text { etc }
\end{array}
$$

In a SERIES circuit the current through each resistor is the same. To find the current use the formula:
$\frac{V_{s}}{R_{T}}=I_{T} \quad$ Where
$I_{T}$ is the current,
$R_{T}=R_{1}+R_{2}+R_{3}$,
$V_{s}=$ supply voltage


We already know that $I_{T}$ is the same as the current going through $R_{1}, R_{2}, R_{3}$ etc.

$$
\begin{aligned}
& V_{s}=I_{T} \times R_{T} \\
& V_{1}=I_{T} \times R_{1} \\
& V_{2}=I_{T} \times R_{2} \\
& V_{3}=I_{T} \times R_{3}
\end{aligned}
$$

So to find $V_{1}, V_{2}, V_{3}$, use:

$$
I_{T}=\frac{V_{1}}{R_{1}}=\frac{V_{2}}{R_{2}}=\frac{V_{3}}{R_{3}}=\frac{V_{s}}{R_{T}}
$$

As $I_{T}$ is the same
To find the voltage across resistors you do not need to work out the current.

$$
I_{T}=\frac{V_{1}}{R_{1}}=\frac{V_{2}}{R_{2}}=\frac{V_{s}}{R_{T}}
$$



Either work out by ratios (quick if you can do it but costly if it goes wrong!)

## EITHER:

1. Summarise
$\mathrm{V}_{\mathrm{S}}=12 \mathrm{~V}, \mathrm{R}_{1}=90 \Omega, \mathrm{R}_{2}=30 \Omega$
$\mathrm{V}_{1}=$ ?, $\mathrm{V}_{2}=$ ?

## 2. Find $\mathrm{R}_{\mathrm{I}}$

Find $R_{T}=R_{1}+R_{2}$
$R_{T}=90+30=120 \Omega$
3. Find $\mathrm{V}_{1}$
$\frac{V_{S}}{R_{T}}=\frac{V_{1}}{R_{1}}$
$\frac{12}{120}=\frac{V_{1}}{90}$
$V_{1}=\frac{12 \times 90}{120}$
$V_{1}=9 \mathrm{~V}$
4. Find $\mathrm{V}_{2}$
$\frac{V_{S}}{R_{T}}=\frac{V_{2}}{R_{2}} \quad \frac{12}{120}=\frac{V_{2}}{30}$
$V_{2}=\frac{12 \times 30}{120} \quad V_{2}=3 \mathrm{~V}$
5. Check
$V_{s}=V_{1}+V_{2}=9+3=12 \mathrm{~V}$

OR:

1. Summarise
$\mathrm{V}_{\mathrm{S}}=12 \mathrm{~V}, \mathrm{R}_{1}=90 \Omega, \mathrm{R}_{2}=30 \Omega$
$\mathrm{V}_{1}=?, \mathrm{~V}_{2}=$ ?
2. .Find $R_{I}$

Find $R_{T}=R_{1}+R_{2}$
$R_{T}=90+30=120 \Omega$
3. Find IT
$\frac{V_{S}}{R_{T}}=I_{T}=\frac{12}{120}=0.1 \mathrm{~A}$
4. Find $\mathrm{V}_{1}$
$V_{1}=I_{T} R_{1}=0.1 \times 90=9 \mathrm{~V}$
5. Find $\mathrm{V}_{2}$
$\underline{\underline{V_{2}}=I_{T} R_{2}=0.1 \times 30=3 \mathrm{~V}}$
6. 6.Check
$\mathrm{V}_{\mathrm{s}}=\mathrm{V}_{1}+\mathrm{V}_{2}=9+3=12 \mathrm{~V}$ ()

OR USE

$$
\begin{gathered}
V_{1}=\frac{R_{1}}{\left(R_{1}+R_{2}\right)} \times V_{s} \\
V_{2}=\frac{R_{2}}{\left(R_{1}+R_{2}\right)} \times V_{s} \\
V_{2}=V_{s}-V_{1}
\end{gathered}
$$

POTENTIAL DIVIDER EXAMPLES


The voltage across the branch of the circuit must be equal to the terminal p.d (potential difference). NB I am not using $\varepsilon$ in this case, although the examiners could be horrible and combine the question!
$\frac{V_{s}}{R_{T}}=\frac{V_{1}}{R_{1}}=\frac{V_{2}}{R_{2}}=I$
or use

$$
\frac{V_{s}}{\left(R_{1}+R_{2}\right)}=\frac{V_{1}}{R_{1}}=I
$$

rearrange
e.g.

$$
V_{1}=\frac{R_{1}}{\left(R_{1}+R_{2}\right)} \times V_{s} \quad \text { or } \quad V_{1}=\frac{R_{1}}{\left(R_{T}\right)} \times V
$$

$15 \Omega$ and $7 \Omega$ in series in a potentialdivider with a 12 V supply
$V_{1}=\frac{V_{s} \times R_{1}}{R_{t}}$
$V_{1}=\frac{12 \times 15}{22}=8.2 \mathrm{~V}$
$V_{2}=\frac{12 \times 7}{22}=3.8 \mathrm{~V}$ or $V_{s}=V_{1}+V_{2}, V_{2}=12-8.2=3.8 \mathrm{~V}$
If part of the circuit has a resistor in parallel with one of the resistors then this section has to be worked out first. Don't forget that if the two resistors are identical then the total resistance in the parallel section will be half the resistance of one of the resistors in parallel.


For example keeping the 12 V supply a $10 \Omega$ resistor is placed in series with two $15 \Omega$ resistors in parallel, what is the voltage across each part of the circuit?

Well the $15 \Omega$ resistors in parallel would have a total resistance of $7.5 \Omega$, if you don't believe me work it out!

$$
\begin{aligned}
\frac{1}{R_{t}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}} \\
\frac{1}{R_{t}} & =\frac{1}{15}+\frac{1}{15} \\
\frac{1}{R_{t}} & =\frac{2}{15} \\
\frac{R_{t}}{1} & =\frac{15}{2}=7.5 \Omega
\end{aligned}
$$

Then work out the voltage divider part as previously, but using the total of the two parallel resistors instead of the top resistor.
$10 \Omega$ and $7.5 \Omega$ in series in a potential divider with a 12 V supply

$$
V_{1}=\frac{V_{s} \times R_{1}}{R_{t}}
$$

$$
V_{1}=\frac{12 \times 10}{17.5}=6.9 \mathrm{~V}
$$

$$
V_{2}=\frac{12 \times 7.5}{17.5}=5.1 \mathrm{~V} \text { or } V_{s}=V_{1}+V_{2}, V_{2}=12-6.9=5.1 \mathrm{~V}
$$



You are likely to feel quite confident about this after practice; after all it was in the National 5 Physics Course.

Now consider adding another branch to the circuit. The voltage drop across the bottom branch of the circuit must be equal to the voltage drop across the top branch.

The voltage across each of the branches of the circuit must be equal to the terminal p.d (potential difference) as voltages in parallel are equal to the terminal p.d

The voltage drop across the resistors in the top branch is completely independent of the voltage dropped across each resistor in the bottom branch. If you don't believe me try it! Imagine that the top resistors are both $15 \Omega$, if the voltage supply is 12 V the voltage across each of the top resistors is 6 V . If the resistors in the bottom were $5 \Omega$ and $10 \Omega$, then 4 V will be dropped across the $5 \Omega$ resistor and 8 V will be dropped across the $10 \Omega$ resistor. Or if the top branch still contains our two $15 \Omega$ resistors and our bottom branch now has two $150 \Omega$ resistors then the voltage across each resistor will be 6 V . However, the current in the top branch will now be 10 times that of the bottom branch. Use $V=I R$ to prove this using $V$ as the terminal p.d and $R$ as the total resistance of each branch.

But there is more....
If we put a wire connecting these two sets of resistors charge can flow up or down the wire if there is a p.d. between A and B
This is the basis of a Wheatstone bridge, which is no longer on the syllabus, but you should know that if there is a p.d. between A \& B then charge will flow.For more details see the notes prior to the CfE Higher.


## MOVEMENT OF CHARGE

Which way does the current flow? Here is the information that you need.
The +ve terminal of a power supply has a high potential.
The -ve terminal of a power supply has a low potential.
Conventional current suggests that +ve charge moves from areas of HIGH potential to LOW potential.

conventional current

electron flow

Electron flow says that electrons (-ve charge) moves from areas of LOW potential to HIGH potential.

TUTORIAL 1: CURRENT, VOLTAGE, POWER AND RESISTANCE

1. There is a current of 40.0 mA in a lamp for 16 s . Calculate the quantity of charge that passes any point in the circuit in this time.
2. A flash of lightning lasts for 1.0 ms . The charge transferred between the cloud and the ground in this time is $5 \cdot 0 \mathrm{C}$. Calculate the value of the average current in this flash of lightning.
3. The current in a circuit is $2.5 \times 10^{-2} \mathrm{~A}$. Calculate the time it takes for $500 \cdot 0 \mathrm{C}$ of charge to pass any given point in the circuit?
4. There is a current of 3.0 mA in a $2 \mathrm{k} \Omega$ resistor. Calculate the p.d. across the resistor.
5. Calculate the values of the readings on the meters in the following circuits.

6. Calculate the unknown values $R$ of the resistors in the following circuits.

(a)

7. Calculate the total resistance between $X$ and $Y$ for the following combinations of resistors.

8. In the following circuit the reading on the ammeter is 2.0 mA . Calculate the reading on the voltmeter

9. Calculate the power in each of the following situations.
(a) A 12 V battery is connected to a motor. There is a current of 5 A in the motor.
(b) A heater of resistance $60 \cdot 0 \Omega$ connected across a 140 V supply.
(c) A current of $5 \cdot 0 \mathrm{~A}$ in a heater coil of resistance $20 \cdot 0 \Omega$.
10. The heating element in an electric kettle has a resistance of $30 \cdot 0 \Omega$.
(a) What is the current in the heating element when it is connected to a 230 V supply?
(b) Calculate the power rating of the element in the kettle.
11. A 15 V supply produces a current of 2.0 A in a lamp for 5.0 minutes.

Calculate the energy supplied in this time.
12. Calculate the readings on the ammeter and the voltmeter in the circuit shown below.

13. Each of the four cells in the circuit shown is identical.

14. A voltage of 12 V is applied across a resistor. The current in the resistor is 50 mA . Calculate the resistance of the resistor.
15. The LED in the circuit below is to emit light.
(a) What is the required polarity of $A$ and $B$ when connected to a 5 V supply so that the LED emits light?
(b) What is the purpose of the resistor R in the circuit?
(c) The LED rating is 20.0 mA at 1.50 V .


Calculate the resistance of resistor $R$.
16. Write down the rules which connect the (a) potential differences and (b) the currents in series and parallel circuits.
17.

(a) State the name given to the circuit shown.
(b)Write down the relationship between $\mathrm{V}_{1}$, $V_{2}, R_{1}$ and $R_{2}$.
18. Calculate the values of $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ of the circuit in question 17 when:
(a) $\mathrm{R}_{1}=1.0 \mathrm{k} \Omega \quad \mathrm{R}_{2}=49 \mathrm{k} \Omega$
(b) $\mathrm{R}_{1}=5.0 \mathrm{k} \Omega \quad \mathrm{R}_{2}=15 \mathrm{k} \Omega$
19.

The light dependent resistor in the circuit is in darkness. Light is now shone on the LDR. Explain what happens to the readings on $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$.

20. Calculate the p.d. across resistor $R_{2}$ in each of the following circuits.
(a)

21. Calculate the p.d. across $A B$ (voltmeter reading) in each of the following circuits.

(a)

(b)

(c)
22. A circuit consisting of two potential dividers is set up as shown.

(a) Calculate the reading on the voltmeter.
(b)(i) Suggest a value of a resistor to replace the $9 \mathrm{k} \Omega$ resistor that would give a reading of 0 V on the voltmeter.
(ii) Suggest a value of resistor to replace the $3 \mathrm{k} \Omega$ resistor that would give a reading of 0 V on the voltmeter.
23. In the circuits shown the reading on the voltmeters is zero. Calculate the value of the unknown resistors $X$ and $Y$ in each of the circuits.


TUTORIAL 2- WHEATSTONE BRIDGES

1. Determine the resistance of the unknown resistor shown in this circuit when the micro-ammeter reads zero?

2. Calculate the resistance of $\mathbf{R}$ in the following circuit when the microammeter reads zero.

3. State whether there is a p.d. between $A$ and $B$ in the following circuit.


## TUTORIAL 3: PAST PAPER QUESTIONS

## SQA Higher Paper 2010

## Q25

The headlights on a truck are switched on automatically when a light sensor detects the light level falling below a certain value.

The light sensor consists of an LDR connected in a Wheatstone bridge as shown.

(a) The variable resistor, Rv , is set at $6000 \Omega$.
(i) Calculate the resistance of the LDR when the bridge is balanced.
(ii) As the light level decreases, the resistance of the LDR increases. Calculate the reading on the voltmeter when the resistance of the LDR is $1600 \Omega$.

SQA Higher 2011 Q9 and 10
9. A Wheatstone bridge circuit is set up as shown.


The reading on the voltmeter is zero. The value of resistor $R$ is

A $3.0 \Omega$
B $4.0 \Omega$
C $18 \Omega$
D $21 \Omega$
E $24 \Omega$.
10. A Wheatstone bridge circuit is set up as shown.


When the variable resistor $R$ is set at $600 \Omega$ the bridge is balanced. When $R$ is set at $601 \Omega$ the reading on the voltmeter is +2.5 mV . R is now set at $598 \Omega$. The reading on the voltmeter is

A -7.5 mV
B -5.0 mV
C -2.5 mV
D +5.0 mV
$\mathrm{E}+7.5 \mathrm{mV}$

## SQA Higher Paper 2012 Q9

The diagram shows part of an electrical circuit.


What is the resistance between X and $Y$ ?

A $0.2 \Omega$
B $5 \Omega$
C $10 \Omega$
D $20 \Omega$
E $50 \Omega$

TUTORIAL ANSWERS

## TUTORIAL 1:

1. $\quad 0.64 \mathrm{C}$
2. $5 \times 10^{3} \mathrm{~A}$
3. $2 \cdot 0 \times 10^{4} \mathrm{~s}$
4. 6 V
5. (a) $I=0.1 \mathrm{~A} \quad$ (b) $\quad \mathrm{I}=0.5 \mathrm{~A}, \mathrm{~V}=4.5 \mathrm{~V}$ (c) $\mathrm{I}=2 \mathrm{~A}, \mathrm{~V}=10 \mathrm{~V}$
6. 

(a) $5 \Omega$
(b) $6 \Omega$
7.
(a) $25 \Omega$
(b) $25 \Omega$
(c) $24 \cdot 2 \Omega$
(d) $13 \cdot 3 \Omega$
(e) $22 \cdot 9 \Omega$
(f) $\quad 14 \cdot 7 \Omega$.
8. $3.75 \times 10^{-3} \mathrm{~V}$
9.
(a) 60 W
(b) 327 W
(c) 500 W
10. (a) $7 \cdot 7 \mathrm{~A}$
(b) 1763 W
11. 9000 J
12. $I=0.67 \mathrm{~A}, \mathrm{~V}=4 \mathrm{~V}$
13. (a) 0.67 A
(b) 0.13 A
(c) 1.34 V
14. $240 \Omega$
15. (c) $175 \Omega$
18. (a) $\mathrm{V}_{1}=0.2 \mathrm{~V}, \mathrm{~V}_{2}=9.8 \mathrm{~V}$
(b) $\quad \mathrm{V}_{1}=2.5 \mathrm{~V}, \mathrm{~V}_{2}=7.5 \mathrm{~V}$
20.
(a) 4 V
(b) 1 V
(c) 3 V
21.
(a) 3 V
(b) -0.8 V
(c) 0 V
22.
(a) 0.6 V
(b)
(i) $12 \mathrm{k} \Omega$
(ii) $4 \mathrm{k} \Omega$
23. $X=9 \Omega, Y=45 \Omega$

TUTORIAL 2

1. $266 \Omega$
2. $R=150 \Omega$
3. Yes; the resistors do not have the same ratio in each branch and so the micro-ammeter registers a current which requires a p.d. across it. If you care to calculate the actual values, you find that $A$ is at 8 V and B at 6 V . This gives a 2 V p.d. between $\mathbf{B}$ and A .

TUTORIAL 3
25. (a) (i) $\frac{R 1}{R 2}=\frac{R 3}{R 4}$
(ii) $V_{P}=4 \cdot 0 \mathrm{~V}$
$R_{1}=6000 \times \frac{800}{4000}$
$\mathrm{VQ}=4.8 \mathrm{~V}$ Voltmeter reading $=0.8 \mathrm{~V}$
$\mathrm{R}_{1}=1200 \Omega$

SQA 2011 Q9:

SQA 2011 Q10: B

SQA 2012 Q9: B

CHAPTER 4: ELECTRICAL SOURCES \& INTERNAL RESISTANCE

## SUMMARY OF CONTENT

## Electrical sources and internal resistance

| T | eq | $E=V+I r \quad V=I R$ |
| :---: | :---: | :--- |
|  | a) | I can correctly use and explain the terms electromotive force <br> (E.M.F), internal resistance and terminal potential difference (t.p.d) <br> ideal supplies, short circuits and open circuits. |
| $\boldsymbol{T}$ | b)I can use $E=V+I r$ and $V=I R$ to solve problems involving EMF, lost <br> volts, t.p.d., current, external resistance, and internal resistance. |  |
| c)I can describe of an experiment to measure the EMF and internal <br> resistance of $a$ cell. |  |  |
| d)I can determine electromotive force, internal resistance and short <br> circuit current using graphical analysis. |  |  |

## BASIC DEFINITIONS

In the circuit below, when $S$ is closed the free charges in the conductor experience a force that causes them to move. In copper wires the free charges are electrons and these will tend to drift away from the negatively charged end towards the positively charged end.


Current is the rate of flow of charge, i.e. the flow of charge (coulombs) per second:

$$
I=\frac{Q}{t}
$$

The energy required to drive the electron current around this circuit is provided by a chemical reaction in the cell. The electrical energy that is supplied by the cell is transformed into other forms of energy in the components that make up the circuit.

Potential difference (voltage) is defined as the energy transferred per unit charge.

$$
V=\frac{W}{Q}
$$

In this section $E_{\mathrm{w}}$ or $W$ will be used for the work done, i.e. energy transferred, therefore 1 volt = 1 joule per coulomb ( $\mathrm{J} \mathrm{C}^{-1}$ ).

When energy is being transferred from an external source to the circuit, the voltage is referred to as an electromotive force (e.m.f.). When energy is
transformed into another form of energy by a component in the circuit, the voltage is referred to as a potential difference (p.d.). Terminal p.d is the total voltage available in the external circuit.


Energy is supplied to the circuit: chemical $\Rightarrow$ electrical An e.m.f. (voltage) can be measured

Energy is provided by the circuit: electrical $\Rightarrow$ light + heat A p.d. (voltage) can be measured

SOURCES OF E.M.F.
E.M.F.s can be generated in a great variety of ways - see the table below.

| Chemical cell | Chemical energy drives the current <br> (e.g. battery) |
| :--- | :--- |
| Thermocouple | Heat energy drives the current <br> (e.g. temperature sensor in an oven) |
| Piezo-electric generator | Mechanical vibrations drive the current <br> (e.g. acoustic guitar pickup) |
| Solar cell | Light energy drives the current <br> (e.g. solar panels on a house) |
| Electromagnetic generator | Changes in magnetic field drive the current <br> (e.g. power stations) |

## EMF-ELECTROMOTIVE FORCE

Definition- the EMF of a source is the electrical potential energy supplied to each coulomb of charge as it passes through the source.

$$
W=Q V
$$

## Where

$V$ is voltage measured in volts
$Q$ is charge, measured in coulombs and
$W$ is electrical potential energy measured in joules

Be careful in this unit and in other texts. Don't confuse $E$ when thinking about EMF with $E$ which is energy or $E$ which is electric field strength. In this case $E$ is EMF and use $W$ as electrical potential energy.

## CELLS

Cells and other sources of electrical energy are not $100 \%$ efficient. There is a resistance inside the source. Energy is transferred to heat as the charge passes through the resistance within the cell (source).

This can be represented as a resistor in series with the source.


Therefore an electrical source is equivalent to a source with an EMF with a resistor in series (internal resistance). We use ' $r$ ' to represent the internal resistance.

So according to Ohm's Law, V = Ir. This V is equal to the lost volts as the charge passes through the source.

$$
\begin{gathered}
\text { voltage }=\frac{\text { energy }}{\text { charge }} \\
V=\frac{W}{Q}
\end{gathered}
$$

W is the "push" shoving the charges around the circuit or the energy supplied to each coulomb of charge. (not a definition to use for your SQA answers!)

For each coulomb of charge
Energy supplied = Energy used in the circuit.

$$
E=V+V_{\text {lost }}
$$

energy supplied $=$ voltage dropped $+\quad$ lost volts by the source in the external circuit

$$
\begin{gathered}
\text { but } V=I R \\
E=I R+I r \\
E=V+V_{\text {lost }} \\
E=I R+V_{\text {lost }} \\
E=V+I r \\
E=I(R+r)
\end{gathered}
$$

EMF-An open circuit has a no complete circuit.
In an open circuit no charge flows

$$
\text { If } \quad \begin{aligned}
& E=V+I r \\
& I=O A \\
& E=V+0 \\
& E=V
\end{aligned}
$$



Therefore for an open circuit the p.d. across the terminals is equal to the EMF.
©Hmm! How come you can measure this with a voltmeter across the terminal?

## Doesn't this lead to a flow of charge?

Answer: The resistance inside a voltmeter is very large. If $R$ is very large then I is extremely small (from $V=I R$ ), in fact it is just about negligible. ©AHH!

## PRACTICAL 1 - A DRY CELL

Use a dry cell and measure: -
(i) Its EMF using a voltmeter (i.e. place a voltmeter directly across the terminals of the 1.5 V cell without a load.)
(ii) Now add a bulb or resistor across the cell and measure the new voltage across the cells. This is the cells terminal p.d. and the corresponding current when an external resistor is connected to it.
(iii) Add an Ammeter in the circuit and find the internal resistance of the cell with this load.

Calculate the internal resistance of the cell using $E=V+I r$.

## PRESCRIBED PRACTICAL

## PRESCRIBED PRACTICAL

You can work in 3 groups, everyone must be actively involved or you can fail this assessment

## AIM

To find the EMF and internal resistance of a 1.5 V cell

## INSTRUCTIONS

1. Set up the experiment shown in the diagram using the Alba interface and EMF board

2. Alter the variable resistor or the resistance of the external circuit so that I changes
3. Take readings of I and V from the ammeter and voltmeter.
4. Repeat for other values of I and V
5. Plot a graph of $V$ against I
$E=I R+I r$
$E=V+I r$
$V=E-I r$
using
$y=m x+c$
$V=-r I+E$

## UNCERTAINTIES

Think about where and how you are getting uncertainties in your measurements.

## RISK ASSESSMENT

I want you to think about

## HAZARDS

What are your hazards?
What could go wrong and how?

## RISK

How likely is it that each thing goes wrong?
How serious would it be if the above did go wrong (these two are called the risk)

## CONTROL MEASURES

How can you reduce the risk (seriousness and likelihood) of something going wrong?

## RESULTS

Remember it is best if you can plot your results and graph them as you go along and then you can tell if you have got a dodgy point.
$\checkmark$ How many repeats?
$\checkmark$ How many different points?
$\checkmark$ How close should they be? Evenly spaced or more at a certain point?
$\checkmark$ What do you need to measure?
$\checkmark$ What are the best measuring instruments?

## HOMEWORK

Hand in from everyone an individual piece
$\checkmark$ An excel table and graph of your results!
$\checkmark$ Uncertainties quantified
$\checkmark$ Results and Conclusion
$\checkmark$ Evaluation, did you plan well enough or launch in and make mistakes (hint don't!)

References: For more info go into Assignment and look at the Intro to Risk Assessment,


From the graph you can also find the short circuit current. This is the highest current that can be provided by a source. It is found from the intercept on the Xaxis. The examples below indicate why you should always be extremely careful when using car batteries.

Note!
In cars the negative electrode is connected to the body. If you want to take out a car battery, disconnect the minus connector of the battery first. If your spanner gets contact to any metal part of the car while unscrewing, nothing will happen. If you would start with the plus terminal and you get contact with the body, you would short-circuit the battery with dangerous consequences (no fuse, extremely high current, overheating and danger of explosion). After disconnecting the minus terminal you are allowed to disconnect the plus connector. A contact with the body now has also no consequences.

Installing the battery has to be:
For removing a battery out of a car
Always disconnect minus first and Always connect minus at last.

A 12 V car battery has an internal resistance of $0.001 \Omega$. What is its "short" circuit current?

A short circuit means $\mathrm{R}=0 \Omega$
$E=12 \mathrm{~V}$
$r=0.001 \Omega$
$E=I r$
$12=I \times 0.001$
$\frac{12}{0.001}=I=12000 \mathrm{~A}$

What is the short circuit current of a 1.5 V dry cell of internal resistance $1.25 \Omega$ ?

A short circuit means $\mathrm{R}=0 \Omega$
$E=1.5 \mathrm{~V}$
$r=1.25 \Omega$
$E=I r$
$1.5=I \times 1.25$
$\frac{1.5}{1.25}=I=1.2 \mathrm{~A}$

EMF (GRAPHS)

GRAPH 1-V AGAINST I

## For these Questions

$E=$ intercept of $y$ axis measured in VOLTS
-gradient $=r$ (internal resistance of the cell)
Or $\quad m=-r$
Short circuit current occurs when the external resistance is
zero $(R=0)$ which occurs where the line passes the $x$-intercept. Short circuit current

$$
\begin{aligned}
& R=0 \Omega \\
& \Rightarrow I_{\max } \\
& \Rightarrow I R=0 \\
& \Rightarrow V=0 \\
& \Rightarrow E=0+\text { lostvolts }
\end{aligned}
$$



GRAPH 2 - R AGAINST 1/I

$$
E=I(R+r)
$$

For these $Q$
$R=\frac{E}{I}-r$
$R=\frac{E(\times) 1}{I}-r$
fits into the equation for a straight line or
$y=m x+c$
$m=E$
$+c=-r$ so the y intercept is $-r$
Short circuit current occurs when the external resistance is zero $(R=0)$ which occurs where the line passes the $x$-intercept. However remember that the $x$-axis is $1 / I$ so to calculate I take the inverse of the intercept.

## A graph of $R$ against 1/I



GRAPH 3-GRAPH OF LOST VOLTS AGAINST R
For these $Q$
Lost volts =Ir
when the external resistance is zero $(R=0)$ which occurs where the line passes the $y$-intercept then the $E=I r$ or $E=$ lost volts So the $y$-intercept is equal to the $E M F$.

The voltage available to the circuit or terminal p.d will equal the $y$ intercept subtract the lost volts for a particular external resistance. The resistance in this graph is the external resistance so
$E(y$-intercept $)=I \times R$ (value from $x$ axis)-Lost volts ( $y$ value for $x$-axis value)
From this I can be found
Then LOST VOLTS=Ir

e.g. $\mathrm{EMF}=6 \mathrm{~V}$ When the external resistance of the circuit is 2 ohms
then the terminal p. $\mathrm{d}=6 \mathrm{~V}-1.2 \mathrm{~V}$ (read from the y axis) $=\underline{3.8 \mathrm{~V}}$

1. A cell has an internal resistance of $3.0 \Omega$ and an EMF of 2.0 V and another cell has an internal resistance of $1 \Omega$ and EMF of 1.50 V .

Calculate the resistance of the wire which, when connected to either cell, will produce the same current.

In this question there are 2 different cells, however it is one piece of wire which tells us the external resistance is constant and the question tells us that for these conditions of $r$ and $E$ the current remains constant so

$$
\begin{aligned}
& E=I(R+r) \\
& I=\frac{E}{(R+r)} \\
& I=\frac{E_{1}}{\left(R+r_{1}\right)}=\frac{E_{2}}{\left(R+r_{2}\right)} \\
& I=\frac{2.0}{(R+3.0)}=\frac{1.5}{(R+1)} \\
& \text { cross multiply } \\
& 2.0(R+1)=1.5(R+3.0) \\
& \exp \text { and } \\
& 2 R+2=1.5 R+4.5 \\
& 2 R-1.5 R=4.5-2 \\
& 0.5 R=2.5 \\
& R=\frac{2.5}{0.5} \\
& R=5 \Omega
\end{aligned}
$$

POWER MATCHING
[APPLICATION OF INTERNAL RESISTANCE]
This appears to have been removed from the Higher course from 2018, although could form a problem solving question.

In order to have the most efficient transfer of energy between a source and an output the resistances of both must be considered. It is possible to calculate the optimum ratio of resistance to give maximum output. This is shown below, fortunately you do not need to be able to reproduce this explanation

Maximum Power in a simple d.c. circuit.
We know that

$$
\begin{equation*}
P=I^{2} R \tag{1}
\end{equation*}
$$

If we have a simple circuit consisting of a power supply of EMF, $E$, internal resistance, $r$, passing current through a resistor, $R$, then:

$$
\begin{aligned}
& E=I(R+r) \\
& \therefore I=\frac{E}{(R+r)}
\end{aligned}
$$

Substituting for $I$ in equation (1) gives:

$$
P=\left(\frac{E}{(R+r)}\right)^{2} R=\frac{E^{2} R}{R^{2}+2 R r+r^{2}}
$$

Dividing top and bottom of the r.h.s. by $R$ gives:

$$
P=\frac{E^{2}}{R+2 r+\left(\frac{r^{2}}{R}\right)}
$$

Thus the power dissipated in the resistor, $R$, will be at a maximum when the denominator is at a minimum. Remember that $r$ is effectively a constant. Thus if we differentiate the denominator with respect to $R$ we can find the turning points. i.e.
$\frac{d}{d R}\left\{R+2 r+\left(\frac{r^{2}}{R}\right)\right\}=\frac{d}{d R}\left\{R+2 r+r^{2} R^{-1}\right\}$
$=1-r^{2} R^{-2}=1-\frac{r^{2}}{R^{2}}=0 \quad$ (to find turning points)
$\frac{r^{2}}{R^{2}}=1$
$r^{2}=R^{2}$
$r=R \quad$ (Resistors cannot have negative values)
If $\mathrm{R}<\mathrm{r}$ then gradient is negative, if $\mathrm{R}>\mathrm{r}$ then gradient is positive, thus this IS a minimum

Since resistances cannot be negative we conclude that for the maximum power to be produced we must set $R=r$. You will test this in an experimental activity.

Here are some results for you to try out. Find the total power in the external circuit (load) using $I^{2} R$ and then add this to the power lost in the internal circuit using $I^{2} r$

| EMF E $(\mathrm{V})$ | 12 |
| :--- | :--- |
| Internal res r $(\Omega)$ | 10 |


| Load res R ( $\Omega)$ | 0.00 | 2.00 | 4.00 | 6.00 | 8.00 | 10.00 | 12.00 | 14.00 | 16.00 | 18.00 | 20.00 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Current I (A) | 1.20 | 1.00 | 0.86 | 0.75 | 0.67 | 0.60 | 0.55 | 0.50 | 0.46 | 0.43 | 0.40 |
| Power PL in load (W) |  |  |  |  |  |  |  |  |  |  |  |
| Power lost in internal <br> circuit (W) |  |  |  |  |  |  |  |  |  |  |  |
| Total Power (W) |  |  |  |  |  |  |  |  |  |  |  |



The maximum power transfer theorem states that "THE MAXIMUM AMOUNT OF POWER WILL BE DISSIPATED IN THE LOAD RESISTANCE IF IT IS EQUAL IN VALUE TO THE SOURCE RESISTANCE OF THE NETWORK SUPPLYING THE POWER".

This is seen in the graph below
IN OTHER WORDS, THE LOAD RESISTANCE RESULTING IN GREATEST POWER DISSIPATION MUST BE EQUAL IN VALUE TO THE EQUIVALENT SOURCE RESISTANCE, THEN $R_{\mathrm{L}}=$ R $_{\mathrm{S}}$ BUT IF THE LOAD RESISTANCE IS LOWER OR HIGHER IN VALUE THAN

THE SOURCE RESISTANCE OF THE NETWORK, ITS DISSIPATED POWER WILL BE LESS THAN MAXIMUM.


## TUTORIALS

## TUTORIAL 1-EMF

1. a) If 15C of charge moves between two electrodes with 1200 V between them, calculate the work done by the circuit.
b) If the current in this device lasted for 600 s , calculate its power consumption.
c) Calculate the current in this device.
d) Using 'volts = joules per coulomb' and 'amps = coulombs per second', find what quantity we get by multiplying volts by amps.
2. A 12 V 24 W bulb is used in series with a $4 \Omega$ resistor so that the bulb can run normally form a 20 V supply. Calculate the resistance of the bulb.
3. A battery of EMF. 12 V has a terminal p.d. of 9 V when connected to an external circuit drawing 3A. Calculate the internal resistance of the battery.
4. a) A 12 V car battery has an internal resistance of $0.001 \Omega$. Calculate its 'short circuit' current.
b) Calculate the short circuit current of a 1.5 V dry cell of internal resistance $1.25 \Omega$.
5. A power source has a terminal p.d. of 5.7 V when its external circuit is receiving 1.5 A from it. When the external circuit is changed so that the current drawn from the source is 2 A , the terminal p.d. measurers 4.6 V determine the source's EMF and internal resistance.

## TUTORIAL 2 MORE EMF

1. Calculate the EMF of a cell of resistance $3.0 \Omega$ which can produce a current of 0.20 A in a wire of resistance $6 \cdot 0 \Omega$.
2. A cell of EMF 1.2 V and internal resistance $0.40 \Omega$ maintains a current in an external resistance of $2 \cdot 0 \Omega$. Calculate the p.d. between the terminals of the cell.
3. A voltmeter gives a reading of $2 \cdot 0 \mathrm{~V}$ when connected open circuit to a battery. When it lights a lamp of resistance $3.50 \Omega$ the reading on the meter falls to 1.40 V . Determine the internal resistance of the battery.
4. A cell has a resistance of $3.0 \Omega$ and an EMF of 2.0 V and another cell has a resistance of $1.0 \Omega$ and EMF of 1.5 V .

Calculate the resistance of the wire which, when connected to either cell, will produce the same current.
5. If the EMF of the cell in the diagram is 4.0 V , calculate
a. The t.p.d.
b. The lost volts.
c. What would happen to these values if another $10 \cdot 0 \Omega$ resistance was added in parallel?
6. If the EMF is 16.0 V and the current going through the circuit is 10.0 mA calculate the internal resistance.

7. Calculate the EMF of the cell if the current is 0.40 A .

[^0]8. In the circuit below, $r$ represents the internal resistance of the cell and R represents the external resistance of the circuit. When S is open, the voltmeter reads 2.0 V . When S is closed, it reads 1.6 V and the ammeter reads 0.8 A .
(a) Calculate the e.m.f. of the cell.
(b) Determine the terminal potential difference when S is closed.
(c) Calculate the values of $r$ and $R$.
(d) If $R$ was halved in value, calculate the new
 readings on the ammeter and voltmeter.
9. The cell in the diagram has an e.m.f. of $5 \cdot 0 \mathrm{~V}$. The current through the lamp is 0.20 A and the voltmeter reads 3.0 V . Calculate the internal resistance of the cell.
10. A cell of e.m.f. 4.0 V is connected to a load resistor of $15 \cdot 0 \Omega$. If there is a current of 0.2 A in the circuit calculate the internal resistance of the circuit?

11. A signal generator has an e.m.f. of 8.0 V and internal resistance of $4.0 \Omega$. A load resistor is connected to its terminals and draws a current of 0.5 A , calculate the load resistance.
12. The diagram below shows a circuit comprising a cell with internal resistance connected to a variable resistor.
(a) Calculate the terminal p.d. across the cell in the circuit below.

(b) State whether the current increases or decreases as R is increased, explain your answer.
(c) State whether the terminal p.d. then increases or decreases, explain your answer.
13. A cell with e.m.f. 1.5 V and internal resistance $2.0 \Omega$ is connected to a $3.0 \Omega$ resistor, calculate the current in the circuit.
14. A pupil is given a voltmeter and a torch battery. When he connects the voltmeter across the terminals of the battery it registers 4.5 V , but when he
connects the battery across a $6.0 \Omega$ resistor, the voltmeter reading decreases to 3.0 V .
(a) Calculate the internal resistance of the battery.
(b) What value of resistor would have to be connected across the battery to reduce the voltage reading to 2.50 V .
15. In order to find the internal resistance of a cell, the following sets of results were taken.

| Voltage (V) | 1.02 | 0.94 | 0.85 | 0.78 | 0.69 | 0.60 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Current (A) | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 | 0.12 |

(a) Draw the circuit diagram used.
(b) Plot a graph of these results and from it determine
(i) the e.m.f.
(ii) the internal resistance of the cell.
(c) Use the e.m.f. from part (b) to calculate the lost volts for each set of readings and hence calculate 6 values for the internal resistance.
(d) Calculate the mean value of internal resistance.
16. The voltage across a cell is varied and the corresponding current noted.

The results are shown in the table below.

| Voltage (V) | 5.5 | 5.6 | 5.7 | 5.8 | 5.9 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Current (A) | 5.0 | 4.0 | 3.0 | 2.0 | 1.0 |

Plot a graph of V against I .
(a) What is the open circuit p.d?
(b) Calculate the internal resistance.
(c) Calculate the short circuit current.
(d) A lamp of resistance $1.5 \Omega$ is connected across the terminals of this supply.

Calculate (i) the terminal p.d. and (ii) the power delivered to the lamp.

TUTORIAL 3- PAST PAPERS
1.
a) A rechargeable cell is rated at $0.5 \mathrm{~A} h$ (ampere hour). This means that, for example, it can supply a constant current of 0.50 A for a period of 1 hour. The cell then requires to be recharged.
i. What charge, in coulombs, is available from a fully charged cell?
ii. A fully charged cell is connected to a load resistor and left until the cell requires recharging. During this time the p.d. across the terminals of the cell remains constant at 1.2 V . Calculate the electrical energy supplied to the load resistor in this case
b)
i. State what is meant by the EMF. of a cell
ii. The circuit shown below is used in an experiment to find the EMF. and internal resistance of the rechargeable cell.


The voltmeter and ammeter readings for a range of settings of the variable resistor are used to produce the graph below. Use the graph (below) to find the values for the EMF. and internal resistance of the cell.

2. (a) A cell of EMF. 1.5 V and internal resistance $0.75 \Omega$ is connected as shown in the following circuit.

i. Calculate the value of the reading on the voltmeter.
ii. Calculate the "lost volts" in this circuit?
3. A battery of EMF 6.0 V and an internal resistance, r , is connected to a variable

resistor R as shown in the following circuit diagram.

The graph below shows how the "lost volts" of this battery changes as the resistance of $R$ increases.


Use information from the graph to calculate the p.d. across the terminals of the battery (t.p.d.) when the resistance of R is $1.0 \Omega$.

Calculate the internal resistance, $r$, of the battery.
4. The Circuit below is used to determine the internal resistance $r$ of a battery of EMF. $E$.


The variable resistor provides known values of resistance $R$.
For each value of resistance $R$, the switch $S$ is closed and the current I is noted.

For each current, the value of $\frac{1}{I}$ is calculated.
In one such experiment, the following graph of R against $\frac{1}{I}$ is obtained.

a) Conservation of energy applied to the complete circuit gives the following relationship.

$$
\begin{equation*}
E=I(R+r) \tag{1}
\end{equation*}
$$

Show that this relationship can be written in the form

$$
R=\frac{E}{I}-r
$$

b) Use information from the graph to find:
i) the internal resistance of the battery;
ii) the EMF. of the battery.
c) The battery is accidentally short-circuited. Calculate the current in the battery when this happens.

## TUTORIAL ANSWERS

## TUTORIAL 1

1. a) There are 18000 J of work done.
b) It is a 30 W device.
c) The current in the device is 25 mA
d) The product of volts and amps is watts.
2. The bulb has a resistance of $6 \Omega$
3. The battery's internal resistance is $1 \Omega$
4. a) The short circuit current is 12000 A
b) The short circuit current of a dry cell is 1.2 A
5. The EMF. of the power supply is 9 V and its internal resistance is $\mathbf{2 . 2 \Omega}$.

## TUTORIAL 2 ANSWERS

1. $E=1.8 \mathrm{~V}$
2. $V=1.0 \mathrm{~V}$
3. $r=1.5 \Omega$
4. $R=5 \cdot 0 \Omega, I=0.25 \mathrm{~A}$
5. a) t.p.d $=3.1 \mathrm{~V}$, b) lost volts $=0.9 \mathrm{~V}$ c) As the load resistance decreases current increases meaning the lost volts increases and the tpd decreases.
Calculating this t.p.d $=2.5 \mathrm{~V}$, lost volts $=1.5 \mathrm{~V}$
6. $r=550 \Omega$
7. $E=20 V$
8. 

a) 2.0 V
b) 1.6 V
c) $r=0.5 \Omega$
$R=2.0 \Omega$
d) $I=1 \cdot 3 \mathrm{~A}, \mathrm{~V}=1 \cdot 3 \mathrm{~V}$
9. $r=10 \Omega$
10. $r=5.0 \Omega$
11. $\mathrm{R}=12 \Omega$
12. a) $V=1.3 \mathrm{~V} \quad$ b) The current decreases as $R$ increases $c$ ) $V$ increases as when the current decreases, lost volts decreases so the t.p.d. increases.
13. $\mathrm{I}=0.3 \mathrm{~A}$
14.
a) $r=3.0 \Omega$
b) $3 \cdot 7 \Omega$
15.
a)
b) i)
$1 \cdot 1 \mathrm{~V}$ ii)
$4 \cdot 2 \Omega$
c )

| Voltage (V) | 1.02 | 0.94 | 0.85 | 0.78 | 0.69 | 0.6 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Current (A) | 0.02 | 0.04 | 0.06 | 0.08 | 0.1 | 0.12 |
| Lost Volts (V) | 0.08 | 0.16 | 0.25 | 0.32 | 0.41 | 0.5 |
| $r(\Omega)$ | 4.0 | 4.0 | 4.2 | 4.0 | 4.1 | 4.2 |

Average $r=4.1 \Omega$
16. a) open circuit p.d $=e m f=6 \mathrm{~V}$ b) $r=0.1 \Omega \quad$ c) $I_{\max }=60 \mathrm{~A}$
d) i) $V=5 \cdot 6 \mathrm{~V}$
ii) $\mathrm{P}=21 \mathrm{~W}$

## TUTORIAL 3 PAST PAPERS ANSWERS

1. (a) (i) $\mathrm{Q}=1800 \mathrm{C}$
(ii) $E=2160 \mathrm{~J}$
(b) (i) The EMF is the energy the cell supplies to each coulomb of charge passing through it.
(ii) The EMF $=1.4 \mathrm{~V}$ ( y -intercept)

The internal resistance $r=4 \Omega$
2. (a) i)

$$
V=1 \cdot 2 \mathrm{~V}
$$

ii) $\quad r=0.3 \Omega$
3. When $\mathrm{R}=1.0 \Omega$, lost Volts $=2.0 \mathrm{~V}$ so the t.p.d $=4.0 \mathrm{~V}, \underline{\underline{r=0.5 \Omega}}$
4.
(a) $E=I(R+r)$
$E / I=R+r$
$R=E / I-r \quad$ As required
(b) (i) $r=2.5 \Omega$
(ii) $E=17.1 V$
(c) $\quad \mathrm{I}=\mathrm{E} / \mathrm{r}=17.1 / 2.5=6.8 \mathrm{~A}$


[^0]:    3: Question 7

