

# 2018

# Higher

# **OUR DYNAMIC UNIVERSE part 1**



J. A. Hargreaves
Lockerbie Academy
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### **CHAPTER 1: BACKGROUND INFORMATION**

#### BACKGROUND INFORMATION





SCALAR QUANTITY	VECTOR QUANTITY		
(magnitude + unit)	(magnitude, direction + unit)		
Distance	Displacement		
Speed	Velocity		
Mass	Acceleration		
Energy	Momentum		
Power	Impulse		
Temperature	Field Strength		
Time	Force		
Etc.	(including weight, friction, tension, upthrust etc)		

Distance = "how far we've travelled"

 $\Rightarrow$  symbol d

 $\Rightarrow$  units metres, m

 $\Rightarrow$  scalar quantity



Displacement = "how far we've travelled in a straight line (from A to B)" (include your direction)

 $\Rightarrow$  symbol s

 $\Rightarrow$  units, metres, m

 $\Rightarrow$  Vector quantity

⇒ Must quote the direction



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(metres per second) 
$$speed = \frac{distance}{time} \frac{(metre)}{(second)}$$

(metres per second) 
$$velocity = \frac{displacement}{time}$$
  $\frac{(metre)}{(second)}$ 

What was the average speed for the journey from John O'Groats to Land's End?

d = 1193 miles

 $t = 26\frac{1}{2} \text{ hours}$ 

$$speed = \frac{1193}{26 \cdot 5} = 45 \, mph$$

What was the average velocity for the journey?

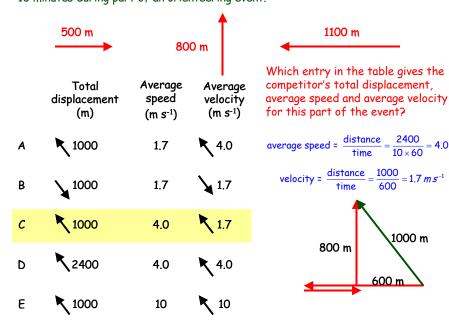
s = 615 miles

 $t = 26\frac{1}{2}$  hours

$$speed = \frac{615}{26 \cdot 5} = 23 \cdot 2 \, mph$$

#### 1992 Higher Paper 1, Q 4

A competitor completes the following sequence of displacements in 10 minutes during part of an orienteering event.



#### Remember

Displacement ≤ distance

Displacement = distance is the whole journey is in a straight line

Velocity ≤ speed

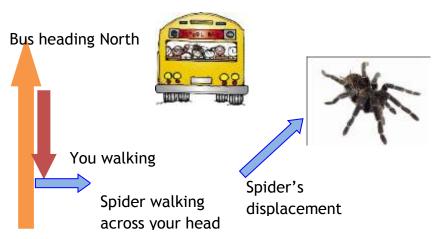
Velocity = speed if whole journey is in a straight line.

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#### RESOLVING VECTORS OR FINDING THE COMPONENT OF A VECTOR;

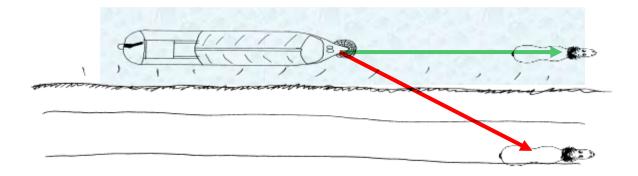
#### WHAT IS THE DIFFERENCE?

When you find the resultant of a number of different vectors you are finding the effect of all the vectors acting on an object. For example if a spider is walking across your head as you are walking down a bus which is travelling north, the spider's displacement is not just the direct path across your head but also depends on your direction down the bus as well as the direction of the bus.

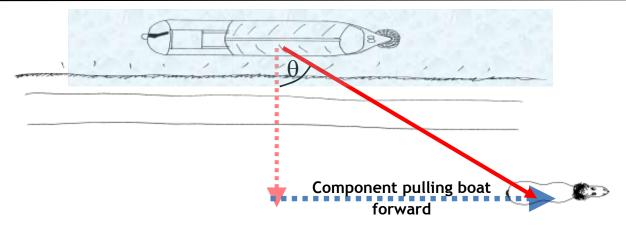


Sometimes however it is useful to break vectors down into how they affect the object say horizontally or vertically, forward or sideways.

For example, consider a horse pulling a barge along a canal. We are only interested in the ability of the horse to pull the boat through the water. For maximum forward motion the horse should be pulling directly in front of the boat. The problem with this is that not many horses walk on water and are consigned to the towpath. When this happens part of the work done by the horse pulls the boat towards the bank. We can resolve the force from the horse, transmitted through the rope into the part that pulls the boat forward and the part that pulls the boat towards the bank.



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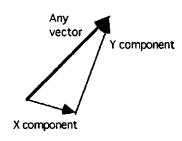


Weetabix Question: Explain why the boat doesn't crash into the bank.

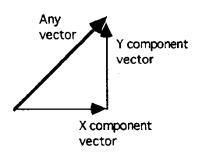
### BREAKING DOWN VECTOR QUANTITIES INTO COMPONENT PARTS.

Any vector can be made by adding two components which for convenience we put at right angles to each other. We call them rectangular components, X along and Y up.

The X and Y vectors are the rectangular components of the main vector.



We have an infinite choice of rectangular components so we can choose the pair which suits us. Usually we take components as horizontal and vertical, but

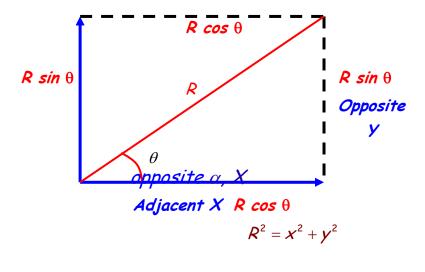


sometimes it is more convenient to take these as parallel to a slope and into the slope.

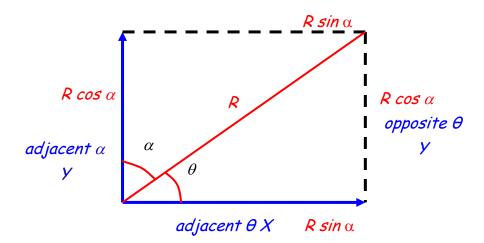
All components are calculated like this:



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$$\tan \theta = \frac{y}{x}$$



$$\alpha = 90 - \theta$$

# PYTHAGORAS

$$c^{2} = a^{2} + b^{2}$$

$$\sin \theta_{1} = \frac{b}{c}$$

$$\cos \theta_{1} = \frac{a}{c}$$

$$\sin \theta_{2} = \frac{a}{c}$$

$$\cos \theta_{2} = \frac{b}{c}$$

$$\sin \theta_{1} = \cos \theta_{2}$$

$$\tan \theta_{1} = \frac{b}{a}$$

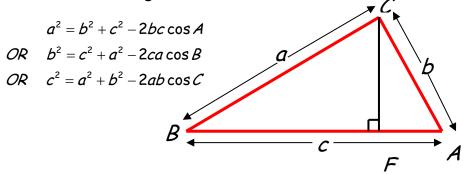
$$\tan \theta_{2} = \frac{a}{b}$$

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# **COSINE RULE**

The cosine rule for a triangle states that:



To prove these formula consider the following triangle, ABC:

Drop a line from C to form a perpendicular with AB at F.

$$CF = b \sin A$$
 and  $AF = b \cos A$   
so  $BF = AB - AF = c - b \cos A$ 

Using Pythagoras' theorem in the triangle BFC:

$$BC^{2} = BF^{2} + CF^{2}$$
or
$$a^{2} = (c - b\cos A)^{2} + b^{2}\sin^{2} A$$

$$= c^{2} - 2bc\cos A + b^{2}(\sin^{2} A + \cos^{2} A)$$

$$= b^{2} + c^{2} - 2bc\cos A$$

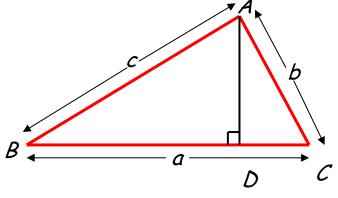
# SINE RULE

The sine rule for a triangle states that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To prove these formula consider the following triangle, ABC:

Drop a line from C to form a perpendicular with BC at D.



$$AD = c \sin B = b \sin C$$

$$\therefore \frac{b}{\sin B} = \frac{c}{\sin C}$$

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### **RESOLVING VECTORS**

#### **RESULTANT FORCES**

The resultant of a number of forces is the single force which has the same effect as the several forces actually acting on the object.

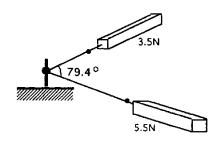
To find the resultant of several vectors we add them. The direction of vector quantities makes their addition more difficult.

e.g. Two forces, each of 2 N, can be added to give any result between zero and 4 N.

FORCES	RESULT
2N 2N	4N
2N 2N	3.7N
2N 2N	2.8N 45°
2N 2N	zero

### ADDING VECTOR QUANTITIES.

A force of 3.5 N pulls at an angle of 79.4° to a force of 5.5 N as shown.



What single force are these equal to?



This is the same as

and is equivalent to



which is 7 N at an angle of  $29^{\circ}$  to the direction of the 5.5 N force.

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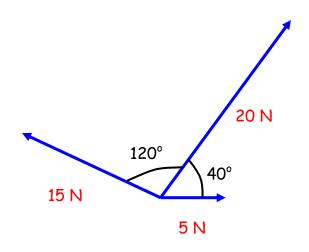
#### RESOLVING VECTORS BY SCALE DIAGRAMS

- 1. Always draw a small plan to show all vectors. Include the direction.
- 2. Choose an adequate scale (as large as possible but which fits on the page). Use an easy conversion.
- 3. Use a sharp pencil and draw the first vector (any one to start).
- 4. Put an arrow on the end and ensure you have clearly indicated the start.
- 5. On the head of the first vector draw the next vector. Ensure the angles are measured correctly and accurately.
- 6. Keep adding vectors until all are drawn.
- 7. CHECK!
- 8. Draw a line from the START of the FIRST to the HEAD of the FINAL vector. This is the RESULTANT.
- 9. Find the angle and measure or calculate it. DON'T MISS THIS OFF!

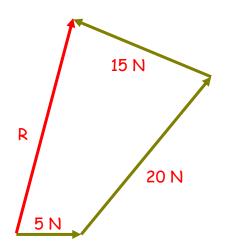
### **EXAMPLE OF RESOLVING VECTORS**

We've met scale diagrams before, especially with forces. Remember: HEAD TO TAIL RULE or COMPONENTS. Don't forget that FORCES DOWN A SLOPE ARE A SPECIAL CASE which we'll deal with in the Forces section.

### RESOLVING BY SCALE DIAGRAM



Measure:19 N at 71° N of E.



#### RESOLVING BY COMPONENTS

# Example 1

Horizontally (L-R +ve) 
$$F_{H} = 5 + 20\cos 40 - 15\cos 20$$
$$= 5 + 15.3 - 14.1$$
$$= 6.2 \text{ N}$$

Vertically (Upwards +ve)  $F_V = 20 \sin 40 + 15 \sin 20$  = 12.9 + 5.1= 18.0 N

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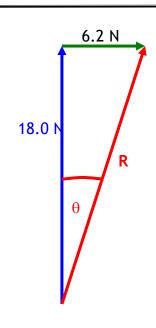
$$F_v^2 = 6.2^2 + 18^2 = 38.4 + 324 = 362.4$$

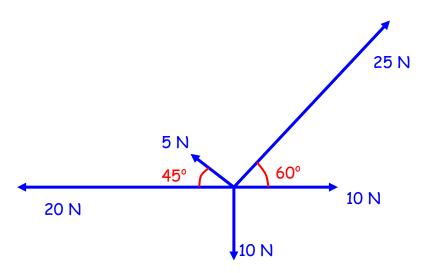
$$F_v^2 = \sqrt{362.4} = 19.0N$$

$$tan\theta = \frac{6.2}{18} = 0.344$$

<u>θ=19°</u>

Example 2





- 1. Resolve vectors horizontally:
- 2. Choose a direction to be positive.
- 3. Find components to each vector.

Horizonal vector  $F_h = 10 + 25\cos 60 - 5\cos 45 - 20 + 0 = -1$ N

4. Resolve vectors vertically:



5. Choose a direction to be positive.

6. Find components to each vector.

Vertical vector 
$$F_v = 0 + 25\sin 60 + 5\sin 45 - 10 + 0 = 15.2$$
N

7. Find the resultant vector by Pythagoras:

Pythagoras states that the square of the long side of a right-angled triangle is the sum of

the squares of the other two sides.

$$R^{2} = a^{2} + b^{2}$$

$$R^{2} = 15.2^{2} + 1^{2}$$

$$R^{2} = 231.04 + 1$$

$$R = \sqrt{232}$$

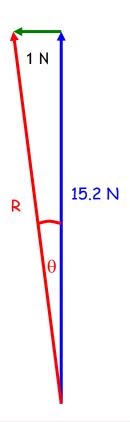
$$R = 15.2 N$$

#### Find $\theta$ :

$$\tan \theta = \frac{b}{a}$$

$$\tan \theta = \frac{1}{15.2} = 0.07$$

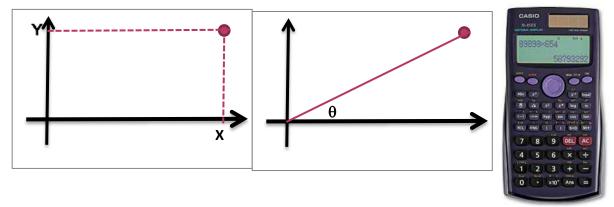
$$\theta = 3.9^{\circ}$$



#### **VECTORS / TUTORIAL 1**

#### POLAR AND RECTANGULAR COORDINATES

INTRODUCTION In maths you've probably used SOHCAHTOA to calculate angles and lengths of sides of right angled triangles. Polar and rectangular coordinates is a quick way of using SOHCAHTOA to find either X and Y if you know hyp and  $\theta$  or to find R and  $\theta$  if you know X and Y.

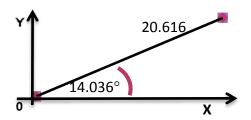


The first few questions are to help you understand the difference between polar and rectangular coordinates and to interchange between the two. Your calculators will do these conversions if you use the "Pol" and "Rec" functions. e.g. to convert a polar coordinate 5, 45 to rectangular coordinate use <SHIFT> Rec(5,45)=. Use the "Pol" in a similar way to get polar coordinates from rectangular ones. Polar coordinates suggest a resultant vector and it's corresponding angle and rectangular coordinates suggest components in the X and Y direction

On the Casio fx-83 Pol is SHIFT+, Rec is SHIFT- and comma is SHIFT)

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1. An object is located at a distance of 20.616 from the origin at an angle of 14.0360 to the X-axis. Determine the rectangular components of its position.



- 2. Calculate the distance and direction to a point whose rectangular coordinates are (49.737, 40.277).
- Find the rectangular coordinates of the points that have the following polar 3. coordinates.
  - (a)  $(100, 36.87^{\circ}),$
- (b)  $(26.93, 68.2^{\circ}),$
- (c) (45.34, 41.42°)
- 4. A room measures 5.64 m by 2.05 m and a plumber has to lay a pipe diagonally across it. Determine the length of the pipe and what angle does it make with the longer wall of the room?
- 5. What shape is plotted out by the following eight points?
  - (a) (8,12)

- (b) (14,12) (c) (20,12) (d) (18,18) (e) (22,15)

- (f) (24,18) (g) (12,18) (h) (10,15)

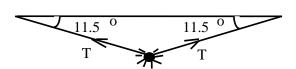
### VECTORS /TUTORIAL 2

1. Look through this list of items and pick out those you think are vector quantities.

Acceleration Area Charge Energy Force Volume Frequency Work Momentum Velocity Radius Mass Temperature Latent heat Power

- 2. A girl walks at 2 ms<sup>-1</sup> across the deck of a ferry while the boat is moving steadily forwards at 8 ms<sup>-1</sup>.
  - (a) Calculate the distance she move across the deck in 2 s.
  - (b) Calculate the distance the ferry move forward during this time.
  - (c) Determine the girl's displacement from her starting point after 2 s.
  - (d) Determine the girl's velocity.

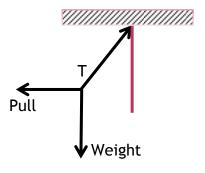
3.



The tension in each of the two strands of spider's web is  $6.3x10^{-5}$  N

- Determine the 'Y' component of this tension. (a)
- Determine the weight of the spider? (b)

- 4. The weight is 60N and the pull to the side is 25N
- (a) Determine the resultant of these two forces?
- (b) Determine the tension in the supporting string.
- 5. A toy boat is held stationary under the influence of three forces. One is 12 N acting due east and one is 6 N acting at 30° from north, Determine the third force.



#### **VECTORS / TUTORIAL 3**

- 1. A building worker lays a line of bricks 9.2 m long and then lays another line at right angles to this so that he ends up 10.5 m from his start. Calculate the length of his second line of bricks.
- 2. A fly with a sore wing runs down a vertical rope with a velocity of 18 cms<sup>-1</sup> while the rope is being raised up with a velocity of 14.5 cms<sup>-1</sup>. The fly starts at a height of 17.5 cm and there are 69.5 cm of rope left coiled on the ground. Calculate whether the fly gets to run onto the ground or falls off the end of the rope.
- 3. A model plane racecourse is set out as shown: -

State which of the following choices gives the displacement of a plane at position B relative to the start, and the distance travelled to reach B.

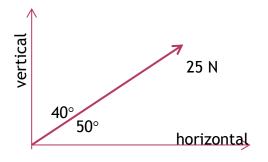
	Displacement	Distance	A
(A)	5 km due east	5 km	3km 4km
(B)	7 km due east	5 km due east	
(C)	5 km due east	7 km due east	В
(D)	7 km due east	7 km	
(E)	5 km due east	7 km	

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4. A force of 25 N acts on a shopping trolley handle as shown: -

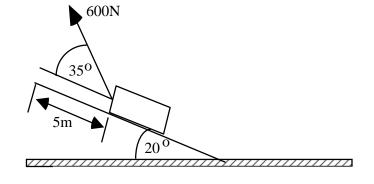
Which entry in the table below correctly shows the horizontal and vertical components of the force?

	ı	I
	Horizontal component	Vertical component
Α	25 sin50°	25 sin40°
В	25 cos50°	25 sin40°
С	25 sin50°	25 cos50°
D	25 cos40°	25 sin40°
Е	25 cos50°	25 sin50°



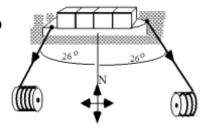
5. A block of weight 800 N is pulled up a  $20^{\circ}$  ramp at a constant speed by a winch cable acting at  $35^{\circ}$  to the ramp: -

When the block has moved 5 m along the ramp, the gravitational potential energy gained by it is calculated from,



6. A barge is about to be pulled nearer the dockside by two horizontal cables attached to winches as shown: -

If the force applied through each winch cable is  $1 \times 10^5$  N, Determine the resultant force on the barge.

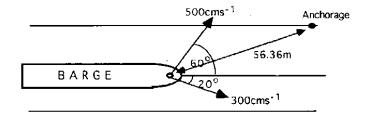


#### VECTOR ADDITION PROBLEMS.

- 1. Determine the resultant of two forces, each of 10 N, acting at  $60^{\circ}$  to each other?
- 2. Two 14 N forces act at  $70^{\circ}$  to each other and are attached to a ring. Determine the resultant force on the ring?

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- 3. A ceiling microphone weighs 6 N and hangs at an angle of 34° to the vertical because it is pulled to the side by a horizontal wire exerting 4 N. Determine the resultant of these two forces.
- 4. A boat travelling at 5 m s<sup>-1</sup> is pushed at an angle of 40° by a wind of 5 m s<sup>-1</sup>. Determine the resultant velocity of the boat.
- 5. Three forces of 9 N, 3 N and 4.5 N act on a balloon as shown. Determine their resultant?
- a) Determine the result of adding a displacement of 2 m east to one 3 m northeast.
  - b) Determine the result of adding a displacement of 8 m east to one 12 m northeast.
  - c) Compare your results to parts (a) and (b), and then state a theory about adding pairs of vectors which are multiples of another pair.
- 7. Determine the least distance between a dog's kennel and its gate if it gets to the gate by walking 6 m 48.7 cm east then 9 m 61 .7 cm north.
- 8. Determine the length of bannister rail needed if the stairs it is for rise a vertical height of 3 m 60 cm in a forward distance of 6 m 23.5 cm?
- 9. A shepherd's trailer moves east at 5 m s<sup>-1</sup> while the sheepdog on it walks north at 2 m s<sup>-1</sup>. A tick on the top of the dog's head rushes along at 50 cm s<sup>-1</sup> at an angle of 50° east of north. Determine the resultant velocity of the tick.
- 10. A plane travels at 90 m s<sup>-1</sup> northwards while a 16.9 m s<sup>-1</sup> wind pushes it at 78° west of north. Determine the distance the plane is from its destination if its resultant velocity carries it straight there in 25 minutes.
- 11. A barge is being pulled towards its anchorage as shown. If the resultant velocity pulls the boat directly to its anchorage, how long does it take to reach there?



- 12. A girl walks at 2 m s<sup>-1</sup> across the deck of a ferry while the boat is moving steadily forwards at 8 m s<sup>-1</sup>.
  - a) How far does she move across the deck in 2 s?
  - b) How far does the ferry move forward during this time?
  - c) Determine the girl's displacement from her starting point after 2 s?
  - d) Determine the girl's velocity?

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#### ACCELERATION- RATE OF CHANGE OF VELOCITY

Acceleration is the rate of change of velocity. Acceleration is a vector quantity and is measured in metres per second squared, (m s<sup>-2</sup>).



Measurements

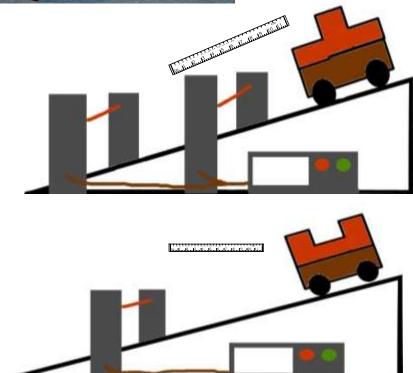
length of mask

$$(ms^{-2})$$
 acceleration=  $\frac{\text{change in velocity}}{\text{time for the change}} \left(\frac{ms^{-1}}{s}\right)$ 

$$a = \frac{dv}{dt} = \frac{v - u}{t}$$

Acceleration can be measured with a double mask and one set of light gates/ light bridges or a single mask and two sets of light gates/bridges.

**Calculations** 



$t_1$ time to pass first light gate	$u=\frac{1}{t_1}$
t <sub>2</sub> time to pass second light gate	$V=rac{1}{t_2}$
t₃ time between light gate	$a=\frac{V-U}{t_3}$

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#### PRACTICAL 1: ANGLE OF SLOPE AND ACCELERATION

Using equipment of your choice investigate the effect of the angle of a slope on acceleration. Look back at these notes to be sure that you know what relationship you should be looking for.

Produce a full write up including an estimation of uncertainty in the experiment. A sheet is available to help you with your write up.Do not complete if you've already done this in the introduction booklet.

#### **VECTORS / TUTORIAL ANSWERS**

#### **TUTORIAL 1**

- 1. The rectangular coordinates are (20.000, 5.000)
- 2. The distance from the origin is 64.000 units and it is at an angle of 39.000 to the X-axis.
- 3. (a)  $(100, 36.87^{\circ}) = (80,60)$ 
  - (b)  $(26.93, 68.2^{\circ}) = (10,25)$
  - (c)  $(45.34, 41.42^{\circ}) = (34,30)$
- 4. The pipe is 6m long at an angle of 20° to the longer wall.
- 5. Parallelogram.

#### TUTORIAL 2

- 1. acceleration, momentum, velocity, force.
- 2. (a) 4 m
  - (b) 16 m
  - (c) A scale diagram gives **R** as 16.5m at 14° to the side of the deck.
  - (d) Velocity =  $8.25 \text{ ms}^{-1}$  at  $14^{\circ}$  to the deck's side.
- 3. (a) Y component of tension is  $1.25 \times 10^{-5} \text{ N}$ 
  - (b) The spider weighs  $2.5 \times 10^{-5} \text{ N}$
- 4. (a) Resultant = 65 N at an angle of  $67^{\circ}$ 
  - (b) Tension = 65 N at an angle of 22.6° to the vertical.
- 5. 16 N, 200°

# TUTORIAL 3

- 1. The bricklayer lays a second line 5.1 m long.
- 2. The fly runs out of rope and falls off the end.
- 3. E
- 4. E
- 5. A
- 6. 179 kN due south.

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### VECTOR ADDITION PROBLEM ANSWERS

- 1. 17 N at an angle of 30° to one of the component forces.
- 2. The third force is 22.9N at an angle of 35° to one of the 14N forces.
- 3. 7.2 N @ 34°
- 4. It travels at 9.4 ms -1 at an angle of 20° to the direction of the wind. This does depend on the way you have indicated the wind direction!
- 5. R is 7.5 N,  $\theta$  is 53° north of east.
- 6. (a) A scale drawing gives the resultant R as 4.6m at an angle of  $27.2^{\circ}$ 
  - (b) A similar drawing to the above gives us R as 18.5m at 27.2°
  - (c) If one pair of vectors is a simple multiple of another pair, then their resultants will be the same multiple of each other.
- 7. The least distance is 11.6m.
- 8. The bannister B works out to be 7.2m long.
- 9. The resultant velocity of the tick is 5.6 ms<sup>-1</sup> at an angle of 26.3° north of east.
- 10. It is 142.5 km from its destination.
- 11. It takes 9.1 s go reach the anchorage.
- 12.
  - a. She moves 4 m across the deck
  - b. The ferry moves 16 m in this time
  - c. The girls displacement is 16.5 m at 14° to the deckside
  - d. The girls velocity is 8.25 ms<sup>-1</sup> at 14° to the deckside

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### **CHAPTER 2: EQUATIONS OF MOTION**

### SUMMARY OF CONTENT EQUATIONS OF MOTION

No	CONTENT			
3.	Equations of Motion			
<b>₽</b> eq	$d = \overline{v}t$ , $s = \overline{v}t$ ; $s = \frac{1}{2}(u+v)t$ ; $v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$			
<b>₽</b> b)	I can use the equations of motion to find distance, displacement, speed, velocity, and acceleration for objects with constant acceleration in a straight line.			
<b>₽</b> c)	I can interpret and draw motion-time graphs for motion with constant acceleration in a straight line, including graphs for bouncing objects and objects thrown vertically upwards.			
<b>(</b> d)	I know the interrelationship of displacement-time, velocity-time and acceleration- time graphs.			
<b>P</b> e)	I can calculate distance, displacement, speed, velocity, and acceleration from appropriate graphs (graphs restricted to constant acceleration in one dimension, inclusive of change of direction).			
<b>∰</b> e)	I can give a description of an experiment to measure the acceleration of an object down a slope			

REVISION PROBLEMS- TRY THESE IF YOU'VE NOT DONE PHYSICS OR NEED TO REVISE

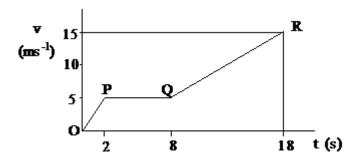
#### **SPEED**

- 1) The world downhill speed skiing trial takes place at Les Arc every year.

  Describe a method that could be used to find the average speed of the skier over the 1 km run. Your description should include:
  - (i) any apparatus required
  - (ii) details of what measurements need to be taken
  - iii) an explanation of how you would use the measurements to carry out the calculations.
- 2) An athlete ran a 1500 metres race in 3 minutes 40 seconds. Calculate his average speed for the race.
- 3) Calculate the distance between the sun and the Earth if it takes light 8 minutes to reach Earth.

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- 4) Concorde travels at an average speed of Mach 1.3 between London and New York.Calculate the time for the journey to the nearest minute. The distance between London and New York is 4800 km. (Mach 1 is the speed of sound. Take the speed of sound to be 340 m s<sup>-1</sup>).
- 5) The speed time graph below represents a girl running for a bus. She starts from a standstill at O and jumps on the bus at Q.



Find:

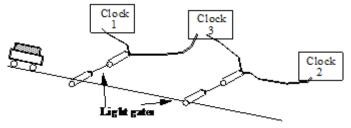
- (i) the steady speed at which she runs
- (ii) the distance she runs
- (iii) the increase in the speed of the bus while the girl is on it
- (iv) how far the bus travels during QR
- (v) how far this girl travels during OR.
- 6) A ground-to-air guided missile accelerates from rest at 150 m s<sup>-2</sup> for 5 seconds. Calculate the speed reached.
- 7) An Aston Martin accelerated from rest at 6 m s $^{-2}$ . Calculate the time it takes to reach a speed of 30 m s $^{-1}$ .
- 8) If a family car applies its brakes when travelling at its top speed of 68 m s<sup>-1</sup>, and decelerates at 17 m s<sup>-2</sup>, calculate the time it takes to reduce its speed 34 m s<sup>-1</sup>.

#### **ACCELERATION**

- 9) An armour-piercing shell, travelling at 2000 m s<sup>-1</sup>, buries itself in the concrete wall of a bunker. If it decelerates at 20 000 m s<sup>-2</sup>, calculate the time it takes to come to rest after striking the wall.
- 10) A skateboard running from rest down a concrete path of uniform slope reaches a speed of 8ms<sup>-1</sup> in 4 s.
  - a) Determine the acceleration of the skateboard.
  - b) Calculate the time after it started when the skateboard take to reach a speed of 12 m s<sup>-1</sup>.
- 11) In the Tour de France a cyclist is travelling at 20 m s<sup>-1</sup>. When he reaches a downhill stretch his speed increases to 40 m s<sup>-1</sup>. It takes 4 s for him to reach this point on the hill.

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- a) Determine the acceleration of the cyclist on the hill.
- b) Assuming he maintains this acceleration, calculate the speed he will be travelling after a further 2 s .
- c) Calculate the time it takes the cyclist to reach a speed of 55 m s<sup>-1</sup>.
- 12) Use the information given below to calculate the acceleration of the trolley.



Length of card = 5 cm

Time on clock 1 = 0.10 s (time taken for card to interrupt top light gate)

Time on clock 2 = 0.05 s (time taken for card to interrupt bottom light gate)

Time on clock 3 = 2.50 s (time taken for trolley to travel between top and bottom light gate)

13) A pupil uses light gates and a suitably interfaced computer to measure the acceleration of a trolley as it moves down an inclined plane.

The following results were obtained:

acceleration (m s <sup>-2</sup> )	5.16,	5.24,	5.21,	5.19,	5.20,	5.20,	5.17,	5.19
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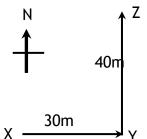
Calculate the mean valve of the acceleration and the corresponding random uncertainty.

# **VECTORS**

14) A car travels 50 km N and then returns 30 km S. The whole journey takes 2 hours.

Calculate

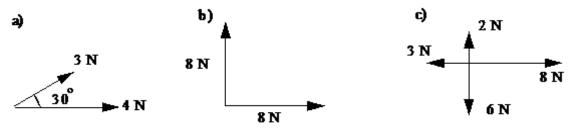
- a) the distance travelled
- b) the average speed
- c) the displacement
- d) the average velocity.



- 15) A girl delivers newspapers to three houses, X, Y and Z, as shown in the diagram. She starts at X and walks directly from X to Y and then to Z.
  - a) Calculate the total distance the girl walks.
  - b) Calculate the girl's final displacement from X.
  - c) The girl walks at a steady speed of 1 m s<sup>-1</sup>.
    - i) Calculate the time she takes to get from X to Z.
    - ii) Calculate her resultant velocity.

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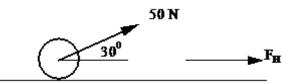
16) Determine the resultant force in the following cases:



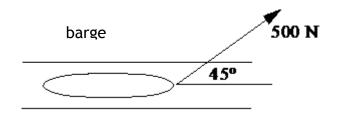
- 17) An aircraft has a maximum speed of 1000 km h<sup>-1</sup>.

  If it is flying north into a headwind speed 100 km h<sup>-1</sup> Determine the maximum velocity of the aircraft.
- 18) A model aircraft is flying north with a velocity of 24 m s<sup>-1</sup>. A wind is blowing from west to east at 10 m s<sup>-1</sup>. Determine the resultant velocity of the plane.
- 19) An aircraft pilot wishes to fly north at 800 km h<sup>-1</sup>. A wind is blowing at 80 km h<sup>-1</sup> from west to east. Determine the speed and course he must select in order to fly the desired course.
- 20) State what is meant by a vector quantity and scalar quantity. Give two examples of each.
- 21) Find the average speed and average velocity of the following.

  An orienteer who runs 5 km due South, 4 km due West and then 2 km North in 1 hour.
- 22) A ship is sailing East at 4 m s<sup>-1</sup>. A passenger walks due North at 2 m s<sup>-1</sup>. Determine the resultant velocity of the passenger relative to the sea? (Use both scale drawing and trigonometry).
- 23) A man pulls a garden roller with a maximum force of 50 N.
  - a) Find his effective horizontal force.
  - b) Without changing the force applied, explain how he could increase this effective force.



24) A barge is dragged along a canal as shown below.



Calculate the component of the force parallel to the canal.

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- 25) A toy train of mass 0.2 kg, is given a push of 10 N at an angle of  $30^{0}$  to the rails. Calculate
  - a) the component of force along the rails
  - b) the acceleration of the train.
- 26) A football is kicked up at an angle of 70<sup>0</sup> at 15 m s<sup>-1</sup>. Calculate
  - a) the horizontal component of the velocity
  - b) the vertical component of the velocity.

### THE EQUATIONS OF MOTION

There are "lots" of equations of motion that you need to know for this course and some that it is helpful to know. For all equations you are expected to know and define the terms.

s = displacement, u = initial velocity, v = final velocity a = acceleration, t = time h = horizontal, v = vertical

Vital equations	Helpful Equations
V = U + at	$S = Vt - \frac{1}{2}at^2$
$V = \frac{S}{t}$	$v = u + at^2$
$v^2 = u^2 + 2as$	$v^2 = u^2 + 2as$
$S = ut + \frac{1}{2}at^2$	$s = ut + \frac{1}{2}at^2$
$\frac{1}{v} = \frac{(v+u)}{2}$ (for a vehicle with constant acceleration)	

#### PROVING THE FIRST EQUATION OF MOTION

$$v = u + at$$

This equation comes from the definition of acceleration

"acceleration is the rate of change of velocity"

$$a = \frac{dv}{dt} = \frac{\text{change in velocity}}{\text{time for the change}}$$

$$a = \frac{v - u}{t_v - t_u}$$

$$a = \frac{v - u}{t} \text{ where t is the time for the change of velocity}$$

$$rearrange$$

$$v = u + at$$

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### PROVING THE SECOND EQUATION OF MOTION

$$S = ut + \frac{1}{2}at^2$$

This equation comes from the fact that displacement is the area under a velocity time graph.

displacement = area under a velocity time graph

$$s = (b \times h) + (\frac{1}{2}b \times h)$$

$$S = ut + \frac{1}{2}t \times (v - u)$$

but

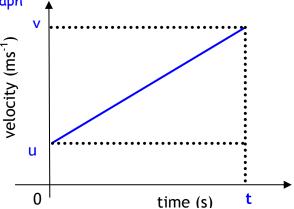
$$a = \frac{v-u}{t}$$
 or  $(v-u) = at$ 

so substitute

$$S = ut + \frac{1}{2}t \times (at)$$

rearrange

 $S = ut + \frac{1}{2}at^2$ 



# PROVING THE THIRD EQUATION OF MOTION

There are many different ways of proving this equation and I have looked through them all. I found this one easiest to tackle but if you like another method then use that but do check it out with me first to make sure that it is viable!

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start with the equation closest to what we want to prove! v = u + atsquare both sides  $v^2 = (u + at)^2$ expand the brackets (here is the first problem!)  $v^2 = (u + at)(u + at)$  $v^2 = u^2 + uat + uat + a^2t^2$ (make sure you've squared both a and t in the final term) collect the terms  $v^2 = u^2 + 2uat + a^2t^2$ it is beginning to look close to what we want! (here is the second problem! substituting x1 changes nothing but makes life a little easier) Substitute  $\frac{2}{2} = 1$  $v^2 = u^2 + 2uat + \frac{2}{2}a^2t^2$ Take out the common factor, 2a  $v^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$ BUT (and here is the good bit!)  $S = ut + \frac{1}{2}at^2$ Replace  $v^2 = u^2 + 2as$ 

#### **EQUATIONS OF MOTION / TUTORIAL 1**

- 1. An object moving with velocity 15 m s $^{-1}$  is accelerated for 8 s so that its new velocity is 27 m s $^{-1}$  in the same direction as before. Calculate its acceleration.
- 2. A feather is hit by a gust of wind, which accelerates it at 20 m s<sup>-2</sup> for 0.5 s. Calculate its new velocity if it was moving in the same direction as the wind at 2 m s<sup>-1</sup> to begin with.
- 3. Determine the time taken for a sports car to reach 30 m s<sup>-1</sup> from a standing start if it can accelerate at 15 ms<sup>-2</sup>.

#### **EQUATIONS OF MOTION / TUTORIAL 2**

1.(a) A child's ball rolls down a pavement and passes a mark at  $1.5 \text{ m s}^{-1}$ . If its velocity is  $6.6 \text{ m s}^{-1}$  three seconds later, Calculate its acceleration.

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- (b) The same ball rolls round a corner at 7 m s<sup>-1</sup> onto a new slope which gives it an acceleration of  $2 \text{ m s}^{-2}$ . Calculate the time it take for its velocity to rise to  $10 \text{ ms}^{-1}$ .
- 2. A water chute has a constant slope which gives the sliders an acceleration of  $4 \text{ m s}^{-2}$ . If a girl jumps on the chute at a velocity of  $3 \text{ m s}^{-1}$ , calculate the distance she slides in 5 s.
- 3. An air-gun pellet has an initial velocity of zero and a muzzle velocity of  $225 \text{ m s}^{-1}$ . If the barrel of the rifle is 0.5 m long, calculate the pellet's acceleration.
- 4. A football player catches up on a ball rolling at 3 m s<sup>-1</sup> and kicks it ahead along the same line with an acceleration of 200 m s<sup>-2</sup> while his foot is in contact with it. If his boot moves with the ball over a distance of 13.75 cm, calculate the speed the ball moves after the kick.
- 5. A ball is thrown upwards out of a window at 15 m  $s^{-1}$  and strikes the ground with a velocity of 30 m  $s^{-1}$ . Determine the height of the window.
- 6. A balloon is released with an initial upwards velocity of 1 m s $^{-1}$  and accelerates at 2 m s $^{2}$  for the first 12 s of its climb.

Calculate its average velocity during these 12 s.

(b) Calculate its displacement during this time.

#### **MOTION GRAPHS**

Type of graph	Gradient	Area under graph
Displacement - time	Velocity	
Velocity - time	Acceleration	Distance travelled (displacement)
Acceleration - time		Change in velocity (v-u)

A displacement - time graph is describing how the displacement of an object is changing with time. We know that:

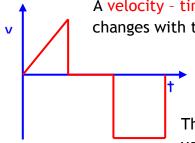
$$v = \frac{ds}{dt}$$
 i.e.  $\frac{\text{change in displacement}}{\text{time taken to change}}$ 

$$v = \frac{s_2 - s_1}{t_2 - t_2}$$

†

Therefore velocity must be equal to the gradient of a displacement - time graph.

gradient = 
$$\frac{\text{rise}}{\text{run}} \left( \frac{\text{displacement}}{\text{time}} \right)$$



A velocity - time graph describes how the velocity of an object changes with time. We know that

$$a = \frac{dv}{dt} = \frac{\text{change in velocity}}{\text{time to change}}$$

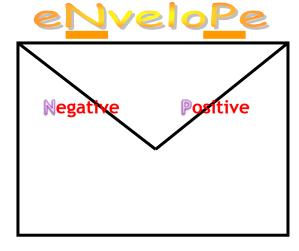
Therefore acceleration must be equal to the gradient of a velocity - time graph

gradient = 
$$\frac{\text{rise}}{\text{run}} \left( \frac{\text{velocity}}{\text{time}} \right)$$

The area under a velocity - time graph = base  $\times$  height

must therefore be equal to displacement,  $s = v \times t$ .

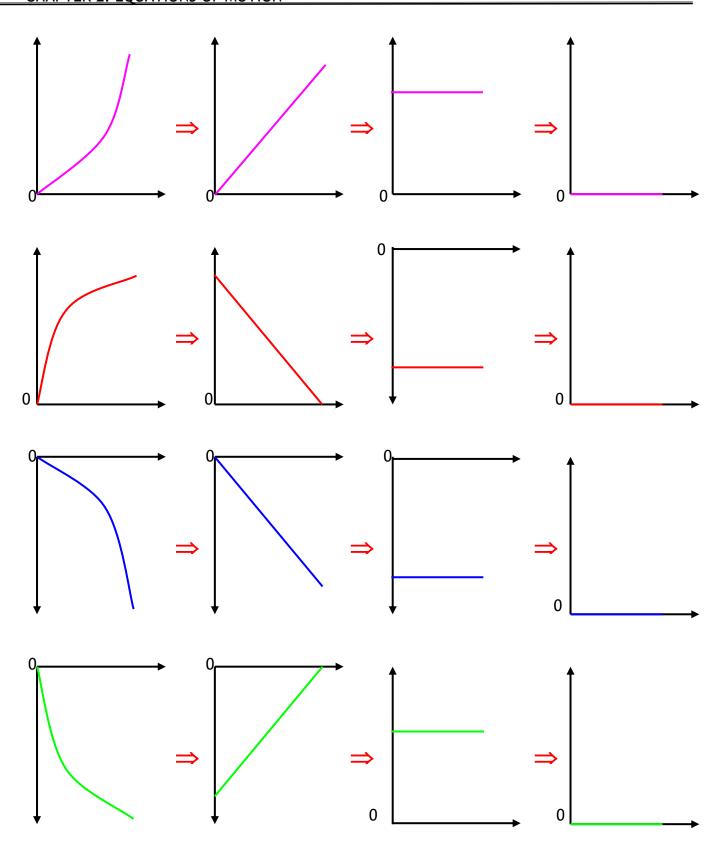
#### **GRADIENTS OF GRAPHS**



On the next page you'll see a set of graphs. Each graph shows a kinematic quantity plotted against time. You can decide on the quantity. The arrows indicate that the graph on the right is a graph of the gradient of the previous graph. For example if the second graph on the second line represents displacement against time the graph to the right of this would be the corresponding velocity-time and the last one on the second row is the corresponding acceleration-time graph. Therefore if you learn

the patterns in these graphs it will help you in your revision and should speed up how you manage to answer some questions.

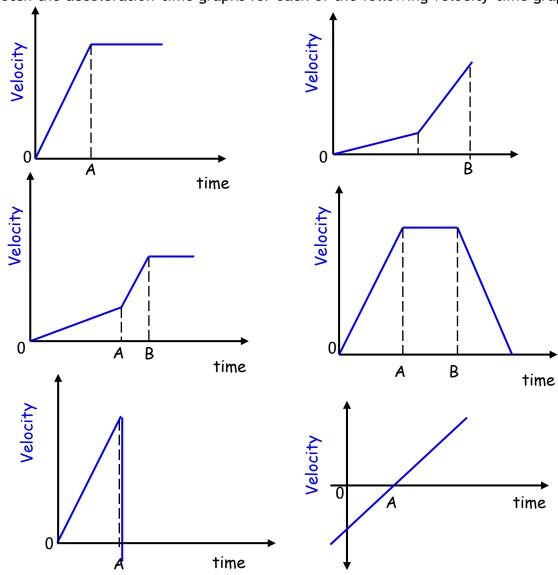
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**TASK** 

Sketch the acceleration-time graphs for each of the following velocity-time graphs.



#### PATTERNS OF RESULTS IN TABLES

Constant velocity results are the most obvious to spot. They are either steadily increasing if they are displacement measurements or they are all about the same if they are velocity measurements.

With constant acceleration, the acceleration figures are constant while the velocity figures increase steadily.

Tables of results for constant acceleration that show displacement figures are a little more difficult because they are not always quoted in the same way. If it is a straightforward set of 'distance from the start' figures, then they show an increase that is itself increasing.

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An object falling under the influence of the force of gravity.

Distance fallen /m	0	5	20	45	80	125	180	245	320	405	500
Time of fall /s	0	1	2	3	4	5	6	7	8	9	10

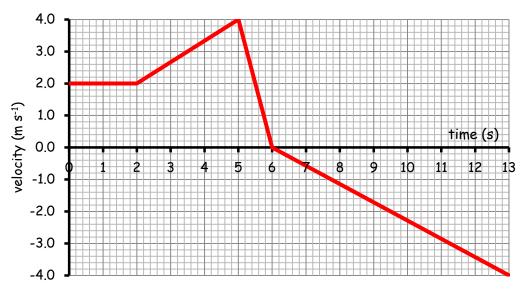
These displacement figures are proportional to  $t^2$ . Sometimes the displacement measurements are given as part of a strobe photograph exercise in which case the figures are displacements that occur during each interval of time. In this case steady acceleration shows up as a steady increase in the displacement measurements.

A stroboscopic photograph using a 1.0 Hz flash of the same object as above gives the following information:

Displacement between Exposures /m			25	35	45	55	65	75	85	95
Time from start /s	1	2	3	4	5	6	7	8	9	10

Find the displacement from the area under the velocity-time graph and the acceleration from the gradient of the velocity-time graph.

The figure below represents the velocity of a particle moving in a straight line.

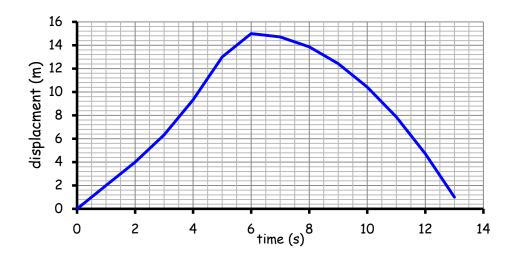


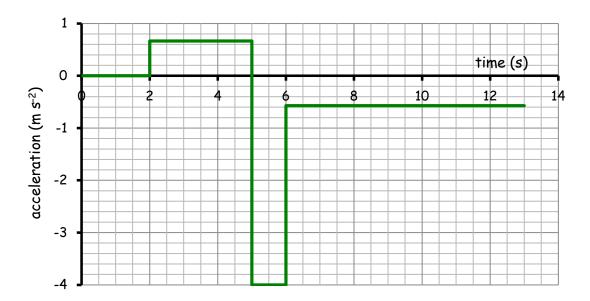
Firstly, describe the graph in words to yourself (roughly).

- Constant velocity / zero acceleration for 2 seconds.
- Constant acceleration from 2 to 5 seconds.
- Deceleration to rest in 1 second.
- Acceleration in opposite direction for 7 seconds.

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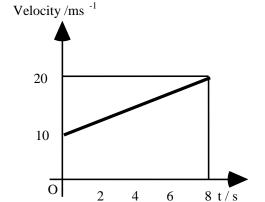
Below are the corresponding displacement-time and acceleration-time graphs.





### **EQUATIONS OF MOTION / TUTORIAL 3**

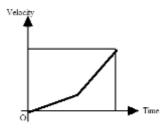
1. Here is a typical velocity-time graph



- (a) Determine the average velocity.
- (b) State the time this occurs.
- (c) Calculate the displacement from the start up to the time recorded in part (b).
- (d) Determine the total displacement.
- (e) Sketch an acceleration-time graph that corresponds to the velocity-time graph above.

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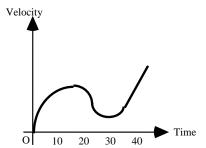
- 2 (a) Describe the motion of the object whose velocity-time graph is shown here.
- (b) Sketch the corresponding acceleration-time graph.



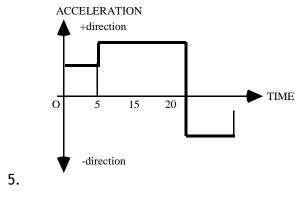
3 Explain what the figures below tell us about the acceleration of the object that produced them.

Time / s	0	10	20	30	40
Velocity / m s <sup>-1</sup>	0	25	55	90	130

- a) If the object described in the graph travels in a straight line;
- (a) State the time at which the speed is zero on this graph.
- (b) State the time at which the acceleration is zero on this graph.



4. Sketch a velocity -time graph that corresponds to the acceleration-time graph shown here:



6. Here is a set of results taken from a strobe photograph whose frequency is 10 Hz, i.e. a photograph is taken every 0.1s.

Displacement between exposures /m	0	0.2	0.6	1.0	1.4	1.8
Time from start /s	0	0.1	0.2	0.3	0.4	0.5

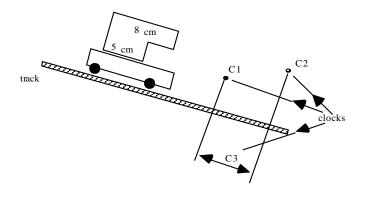
- (a) Calculate the displacement measurements when measured from the start.
- (b) State whether these results show constant acceleration or constant velocity. You must jusify your answer.

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### **EQUATIONS OF MOTION / TUTORIAL 4**

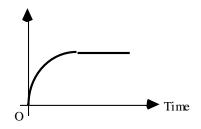
1. The clock times recorded in the following arrangement are as follows: The low level clock,  $C_1$ , reads 0.1 s, the clock which is raised further from the track,  $C_2$ , reads 0.02 s and a handheld clock,  $C_3$ , which reads the time taken for the trolley to travel between  $C_1$  and  $C_2$  reads 0.5 s

Determine the acceleration of the vehicle as it rolls down the track.



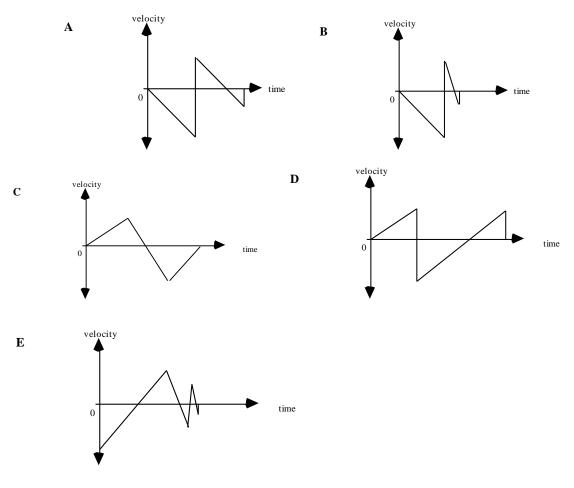
- 2. An object moving at 24 m s<sup>-1</sup> decelerates steadily to a standstill and is displaced 96 m in the process.
  - a) Calculate its average velocity while it slows down.
  - b) Calculate the time it take to stop.
  - c) Determine the acceleration.
- 3. An object starts from rest and accelerates at 3 cm s $^{-2}$  for 6 s. This motion is recorded on a strobe photograph exposed by flashes occurring with a frequency of 1.0 Hz.
  - a) Calculate the lengths between each image on the film.
  - b) State the connection between your answer to part (a) and the following figures.

4. An object is dropped out of a balloon and the graph below made from observations of its motion. Suggest what is plotted on the Y-axis and give your reasons.

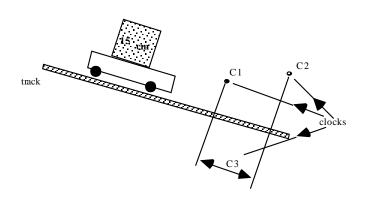


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5. A ball -bearing is dropped onto a metal plate where it bounces a few times before stopping. State which of the following graphs represents its motion from the time of release to its second strike on the plate.



6.



C1 reads  $0.30 \text{ s} \pm 0.01 \text{ s}$ C2 reads  $0.10 \text{ s} \pm 0.01 \text{ s}$ C3 reads  $0.80 \text{ s} \pm 0.20 \text{ s}$ Length of card is  $0.15 \text{ m} \pm 0.01 \text{ m}$ Distance between C1 and C2 is  $0.50 \text{ m} \pm 0.01 \text{ m}$ 

The measurement showing the largest percentage error is: -

- A The reading on C1
- B The reading on C2
- C The reading on C3
- D The length of the card
- E The length of the track between C1 and C2.

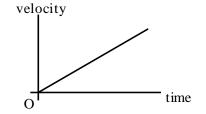
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- 7. State which of the following contains two vectors and one scalar quantity?
  - Work, potential energy, heat. Α
  - В Distance, mass, kinetic energy.
  - C Acceleration, power, velocity.
  - D Displacement, time, speed.
  - Ε Velocity, acceleration, force.

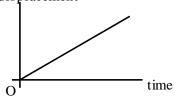
# **EXAM QUESTIONS (PRE 2000)**

If an object moves with constant acceleration, state which of the following 8. shows the corresponding velocity-time and displacement-time graphs?

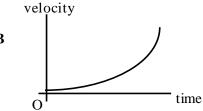




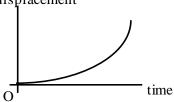




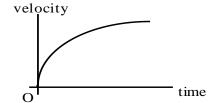
В



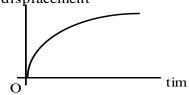




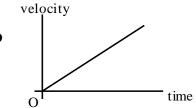
 $\mathbf{C}$ 



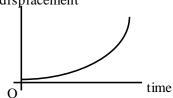
displacement



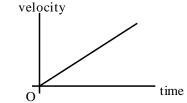
D



displacement



 $\mathbf{E}$ 

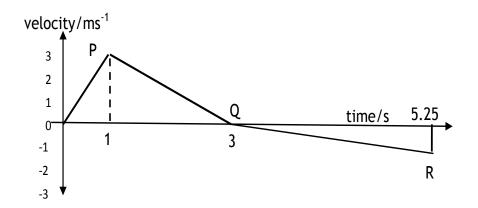


displacement



9. The velocity of a trolley on a slope is investigated using a motion sensor linked to a computer that prints out the velocity-time graph of the trolley's motion.

One such graph for a trolley, which is given an initial push up the slope, is shown here: -



- a) If the position marked P corresponds to the instant the experimenter lets the trolley go, determine the time the trolley at its maximum displacement from P.
- b) Draw the acceleration-time graph of this motion including the initial push.
- 10. A cockroach travels along a straight section of skirting board at a constant speed of 50 cm s<sup>-1</sup>. A cat with tired legs and a reaction time of 0.2 s sees the roach at the instant it passes the end of its nose. The cat accelerates after the roach at a constant rate for 1.5 s until it reaches a velocity of 60 cm s<sup>-1</sup>. It keeps this velocity up for 3.5 s after which it decelerates at 20 cm s<sup>-2</sup> until it reaches 50 cm s<sup>-1</sup> at which point it bumps into the door as the cockroach runs under it.
  - a) Calculate the distance covered by the cat.
  - b) At the instant the cat hits the door, how far is the 'roach on the far side assuming it kept at the same velocity?
  - c) Draw an accurate acceleration-time graph for the cat from the time the cockroach passed its nose until its nose hits the door.

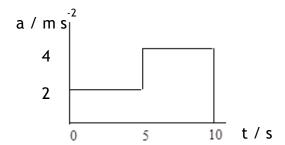
## **EXAM QUESTIONS (POST 2000)**

These questions are in the homework booklet. Do at least the questions suggested by your teacher. Complete other questions as part of your revision for the unit assessment and prelim.

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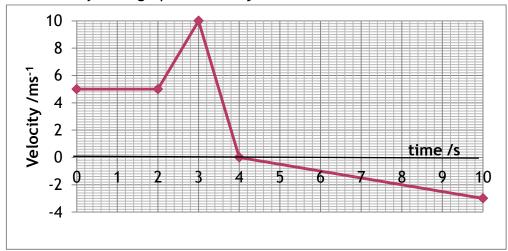
# EXTRA PRACTICE ON THE EQUATIONS OF MOTION FOR THOSE THAT NEED IT

1) The graph below shows how the acceleration of an object varies with time. The object started from rest.



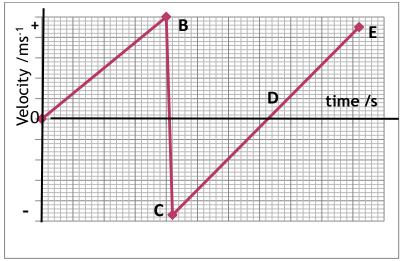
Draw a velocity time graph for the first 10 s of the motion.

2. The velocity time graph for an object is shown below.



Draw the corresponding acceleration-time graph. (Put numerical values on time axis).

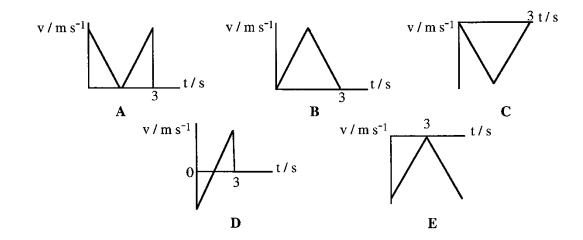
3. The graph shows the velocity of a ball which is dropped and bounces from a floor. A downwards direction is taken as being positive.



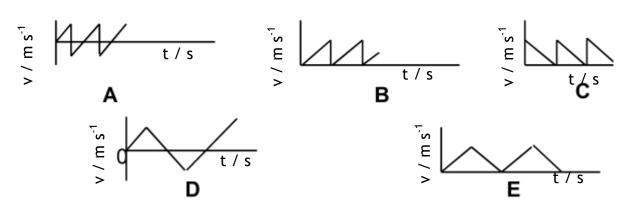
- a) During section OB of the graph
  - i) in which direction is the ball travelling?
  - ii) what can you say about the speed of the ball?

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- b) During section CD of the graph
  - i) in which direction is the ball travelling?
  - ii) what can you say about the speed of the ball?
- c) During section DE of the graph
  - i) in which direction is the ball travelling?
  - ii) State what can you say about the speed of the ball.
- d) State what happened to the ball at point B on the graph.
- e) What happened to the ball at point C on the graph?
- f) What happened to the ball at point D on the graph?
- g) State how the speed of the ball immediately after rebound compare with the speed immediately before.
- 4. State which velocity-time graph below represents the motion of a ball which is thrown vertically upwards and returns to the thrower 3.0 seconds later.

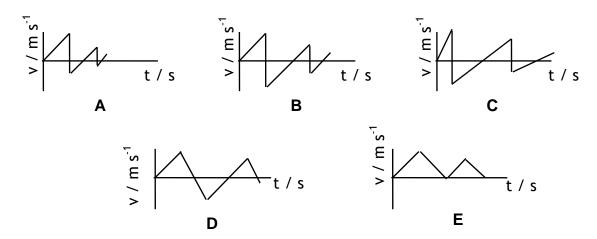


5. A ball is dropped from a height and bounces up and down on a horizontal surface. Assuming that there is no loss of kinetic energy at each bounce, select the velocity-time graph which represents the motion of the ball from the moment it is released.



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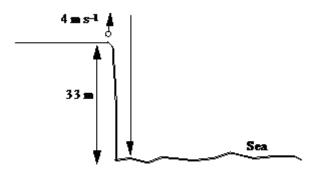
6. A ball is dropped from rest and bounces several times, losing some kinetic energy at each bounce. Selected the correct velocity - time graph for this motion.



- 7. An object accelerates uniformly at 4 m s<sup>-2</sup> from an initial speed of 8 m s<sup>-1</sup>. Calculate the distance travelled in 10s.
- 8. A car accelerates uniformly at 6 m s<sup>-2</sup>, its initial speed is 15 m s<sup>-1</sup> and it covers a distance of 200 m. Calculate its final velocity.
- 9. A ball is thrown to a height of 40 m above its starting point, determine the velocity with which it was thrown.
- 10. A car travelling at 30 m s<sup>-1</sup> slows down at 1.8 m s<sup>-2</sup> over a distance of 250 m. Calculate the time it takes to stop.
- 11. If a stone is thrown vertically down a well at 5 m s<sup>-1</sup>. Calculate the time taken for the stone to reach the water surface 60 m below.
- 12. A tennis ball launcher is 0.6 m long and the velocity of a tennis ball leaving the launcher is 30 m s<sup>-1</sup>. Calculate:
  - a) the average acceleration of a tennis ball
  - b) the time of transit in the launcher.
- 13. In an experiment to find "g" a steel ball falls from rest through 40 cm. The time taken is 0.29 s. Determine the value for "g".
- 14. A trolley accelerates down a slope. Two photo-cells spaced 0.5 m apart measure the velocities to be 20 cm s<sup>-1</sup> and 50 cm s<sup>-1</sup>. Calculate
  - a) the acceleration of the trolley
  - b) the time taken to cover the 0.5 m.

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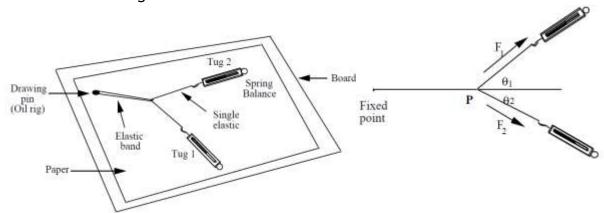
- 15. A helicopter is rising vertically at 10 m s<sup>-1</sup> when a wheel falls off. The wheel hits the ground 8 s later. Calculate at what height the helicopter was flying when the wheel came off.
- 16. A ball is thrown upwards from the side of a cliff as shown below.
  - a) Calculate:
  - i) the height of the ball above sea level after 2 s
  - ii) the ball's velocity after 2 s.
  - b) Determine the total distance travelled by the ball from launch to landing in the sea.



#### **PRACTICALS**

# PRACTICAL 1: RESULTANT OF TWO FORCES

Aim: To compare the resultant of two forces exerted by two tugs on an oil rig with the single force which produces the same effect on the oil rig.



Apparatus: 2 Newton balances, elastic bands, one wooden board, white paper and drawing pins.

# Instructions

- Set up the apparatus as shown.
- With elastic pulled to point P, note the values of F<sub>1</sub>, F<sub>2</sub> and trace their direction on the paper below.
- Mark point P
- Using one spring balance, pull the elastic until point P is reached. Note the value required  $F_T$ .
- Now combine  $F_1$  and  $F_2$  and compare this with the value of  $F_T$ .

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## PRACTICAL 2: ACCELERATION

In this exercise, you are asked to use your understanding of the definition of acceleration to guide your selection and use of the apparatus.

The apparatus consists of a ramp, a trolley with a mask attached for operating photo-diode switches, and two photo-diode switches each connected to an electronic timer. In addition, you can use a hand-held stopwatch to find the time taken by the trolley to pass between the two photo-diode switches.

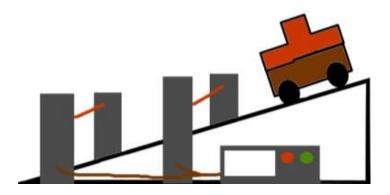
Use the apparatus to find the acceleration of the trolley as it rolls down a moderately sloped ramp.

Provide a written account of your method including how you arrived at your value for the acceleration.

## PRACTICAL 3: ACCELERATION

Aim: To measure the acceleration of a trolley moving down a slope using a computer.

Apparatus: 1 ramp, 1 trolley and single mask, 2 light gate, computer and interface, 1 power supply



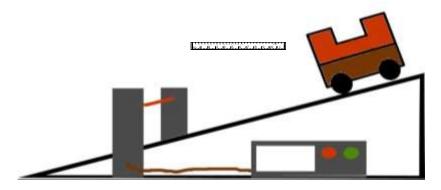
#### Instructions:

- Set up the apparatus as shown in the diagram.
- After selecting the acceleration programme, allow the trolley to run down the track.
- Note the value of the acceleration.
- Repeat 5 times. Calculate the mean acceleration and random uncertainty.
- Explain, in detail, how the mask arrangement allows the computation of the acceleration.

# PRACTICAL 4 ACCELERATION

Aim: To measure the acceleration of a trolley moving down a slope using a computer.

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Apparatus: 1 ramp, 1 trolley and double mask, 1 light gate, computer and interface, 1 power supply

#### Instructions:

- For 5 different angles of slope find the corresponding acceleration.
- Using an appropriate format to find the relationship between the angle of slope and the acceleration.

# PRACTICAL 5 ESTIMATING THE ACCELERATION DUE TO GRAVITY

Aim: To measure the acceleration due to gravity.

Apparatus: 1 light gate, 1 power supply, 1 ALBA interface and light bridge, 1 double mask.

#### Instructions:

- Connect the interface, power supply and light gate.
- Lay the light gate horizontally over the edge of the bench so that the mask can fall through the gate.
- Drop the mask squarely so that it cuts the light beam.
- Record the acceleration from the motion QED.
- Repeat at least five times. Calculate the mean value of the acceleration due to gravity and the random uncertainty.
- Suggest any improvements to the experiment

## PRACTICAL 6: VELOCITY TIME GRAPHS

Title: Velocity-time graphs (acceleration-time graphs)

Aim: To obtain the velocity - time graph for

a) a ball thrown upwards and caught, b) a ball dropped continuing to bounce

Apparatus: computer, interface, motion sensor, football.

Instructions

Α

Set up the interface to plot a velocity-time (and acceleration-time graph).

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- Throw the ball upwards towards the motion sensor and catch it on the way down.
- Print the velocity time graph obtained.

В

- Repeat only this time allow the ball to bounce on the floor beneath the motion-sensor until it comes to rest.
- Print the graph(s) obtained.

On each graph label the following positions:

- a) the highest point of the ball
- b) motion upwards
- c) motion downwards.

# PRACTICAL 7: VELOCITY-TIME GRAPHS (ACCELERATION-TIME GRAPHS)

Aim: To obtain the velocity-time (and acceleration-time) graph for:

- a) a trolley pushed up a slope and allowed to roll back down
- b) a trolley released, bouncing against a buffer at the bottom.

(If an interface is available an acceleration-time graph can also be obtained).

#### Instructions

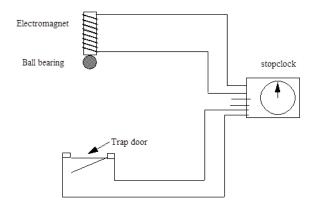
- Set up the apparatus as shown above.
- Practice pushing the trolley up the slope so it reaches no nearer than 40 cm to the motion sensor. (This is the minimum distance the sensor will register).
- Stop the trolley when it returns to its starting position.
- Set up the interface to plot the velocity-time (and acceleration-time) graph.
- Now use the computer to record the data.
- Print the graph(s) obtained.
- Mark on the graphs the relative position of the trolley on the track.

# PRACTICAL 8: EQUATIONS OF MOTION

Aim: To calculate the acceleration due to gravity using

Apparatus: electromagnet, trap door, stop clock, metre stick, ball bearing.

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## Instructions:

- Measure the distance between the bottom of the ball bearing and the trap door.
- Allow the ball to fall several times and find the average time it takes to reach the trap door.
- Show that  $s = ut + \frac{1}{2}at^2$  the equation in this case reduces to  $h = \frac{1}{2}at^2$ ,

where h is the height dropped and g is the acceleration due to gravity.

- Use the equation to estimate the acceleration due to gravity.
- Estimate the uncertainty in your answer.
- How could the method be improved.

# PRACTICAL 9 USING $v^2 = u^2 + 2as$

- Hold the card above the light gate and next to the ruler so that its height above the gate may be measured carefully.
- Release the card so that it cuts through the light beam; a velocity measurement should appear in the table on the screen.
- Repeat this measurement from the same height several times; enter the height value in the height column of the table in the computer program.
- Repeat this procedure for a new starting height 2 cm above the first.
- Collect a series of measurements, each time increasing the height by 2 cm.

## **Analysis**

- Depending upon the software, the results may be displayed on a bar chart as the experiment proceeds. Note the relative increase in values of velocity as greater heights are chosen.
- The relationship between velocity and height fallen is more precisely investigated by plotting a XY graph of these two quantities. (Y axis: velocity; X axis: height fallen.)
- Use a curve matching tool to identify the algebraic form of the relationship. This is usually of the form 'velocity is proportional to the square root of height'.
- Use the program to calculate a new column of data representing the square of the velocity. Plot this against height on a new graph. A straight line is the

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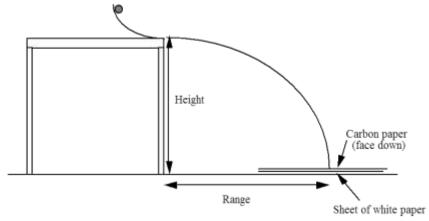
usual result, showing that the velocity squared is proportional to the height fallen.

NB Some of these practicals could form part of an Assignment for Higher.

# PRACTICAL 10: PROJECTILES

Aim: To find the horizontal speed of a projectile.

Apparatus: projectile launcher, metre stick, carbon paper, marble or ball bearing.



#### Instructions:

- Measure the height the ball travels after it leaves the launcher and hits the floor.
- Use  $s = ut + \frac{1}{2}at^2$  to calculate the time of flight, (remember  $u_v = 0$  at the top)
- Launch the ball and find where it strikes the floor. Place white paper and carbon paper as shown in this area.
- Measure the horizontal distance travelled (range).
- Use your results to calculate horizontal speed.
- Repeat this five times and find the mean horizontal speed. (Ensure that the ball starts from the same point each time).
- Estimate the random uncertainty.

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# TUTORIAL ANSWERS

# **REVISION**

Speed	Vectors
1	14. a) 80 km
2. 6.8 m s <sup>-1</sup>	b) 40 km h <sup>-1</sup>
3. 1.44 × 10 <sup>11</sup> m	c) 20 km h <sup>-1</sup> (000)
4. 181 minutes	d) 10 km h <sup>-1</sup> (000).
5. a) 5 m s <sup>-1</sup>	15. a) 70 m
b) 35 m	b) 50 m (037)
c) 10 m s <sup>-1</sup>	c) 70 s
d) 100 m	d) 0.71 m s <sup>-1</sup> (037)
e) 135 m	16. a) 6.8 N (077)
6. 750 m s <sup>-1</sup>	b) 11.3 N (045)
7. 5 s	c) 6.4 N (129)
8. 2 s	17. 900 km h <sup>-1</sup> (000)
Acceleration	18. 26 m s <sup>-1</sup> (023)
9. 0.1 s	19. 804 km h <sup>-1</sup> (354)
10. a) 2 m s <sup>-2</sup>	20
b) 6 s	21. a) 11 km h <sub>-1</sub>
11. a) 5 m s <sup>-2</sup>	b) 5 km h <sup>-1</sup> (233)
b) 50 m s <sup>-1</sup>	22. 4.5 m s <sup>-1</sup> (063)
c) 7 s	23. 43N b) reduce the angle-
12. 0.2 m s <sup>-2</sup>	24. 353.6 N
13. (5.20 + 0.01) m s <sup>-2</sup>	25. 43.5 m s <sup>-2</sup>
	26. $v_x = 5.1 \text{ m s}^{-1} v_y = 14.1 \text{ m s}^{-1}$

# **EQUATION OF MOTION TUTORIAL ANSWERS**

# TUTORIAL 1

- 1. Acceleration= 1.5ms<sup>-2</sup>
- 2. Its new velocity is 12ms<sup>-1</sup>.
- 3. The sportscar takes 2s to reach 30ms<sup>-1</sup>.

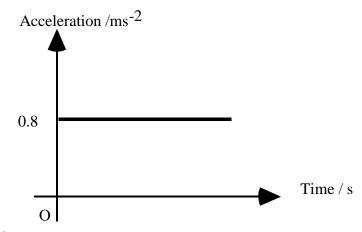
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# TUTORIAL 2

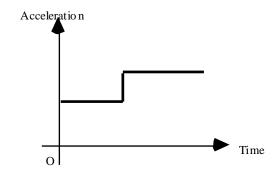
- a. The acceleration is  $1.7 \text{ms}^{-2}$
- b. It takes 1.5 seconds
- 2. She slides 65m
- 3. The pellet accelerates at 50625ms<sup>-2</sup>
- 4. The ball is moving at 8ms<sup>-1</sup> after the kick.
- 5. The window is 33.75m above the ground.
- 6.
- a. The average velocity is  $13\,\mathrm{ms}^{-1}$
- b. The balloon rises 156m

# TUTORIAL 3

- 1. (a)  $15 \text{ms}^{-1}$ .
  - (b) 4s
  - (c) 50m
  - (d) 120m
  - (e)



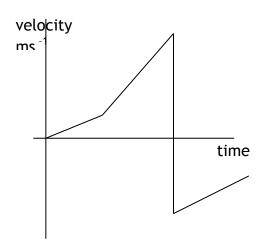
- 2.
- a. constant acceleration followed by a greater constant acceleration.
- b.



- 3. It is a steadily increasing acceleration
- 4. (a) At the origin.
  - (b) At 20s and again near 30s.

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5.



# 6.a)

displacement (m)	0	0.2	0.8	1.8	3.2	5.0
time (s)	0	0.1	0.2	0.3	0.4	0.5

b) These results show constant acceleration. You can tell this as the displacement results measured from the start are increasing by a different amount with each time period

# TUTORIAL 4

- 1. The acceleration is  $7.0 \text{ ms}^{-2}$
- 2. (a) It is  $12 \text{ms}^{-1}$ .
  - (b) It takes 8s.
  - (c) The deceleration is 3ms<sup>-2</sup>
- 3. (a)

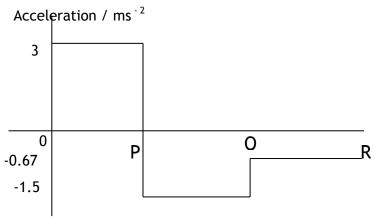
Interval	Initial vel. /cms <sup>-1</sup>	Final vel. /cms <sup>-1</sup>	Average vel. /cms <sup>-1</sup>	Length / cm
1	0	3	1.5	1.5
2	3	6	4.5	4.5
3	6	9	7.5	7.5
4	9	12	10.5	10.5
5	12	15	13.5	13.5
6	15	18	16.5	16.5

- (b) The figures quoted are displacements from the start.
- 4. The Y-axis shows the velocity of the falling object which will increase steeply to begin with but level off as the air drag increases.
- 5. D.

- 6. B.
- 7. C.

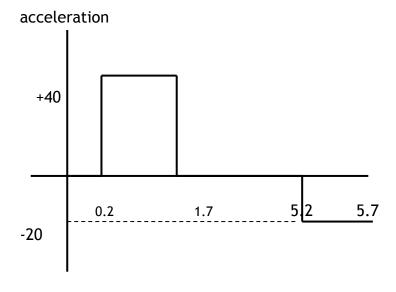
# **EXAM QUESTION ANSWERS**

- 8. D.
- 9. 2 s
- b)



- 10a) The cat travels 282.5cm.
- b) The cockroach is 2.5cm in front when the cat hits the door.

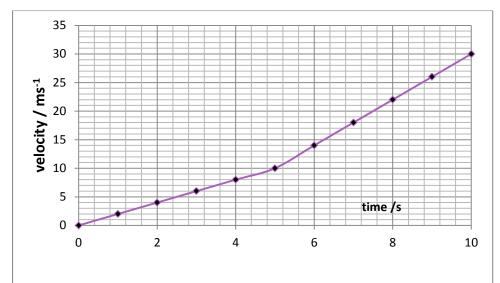
10c).



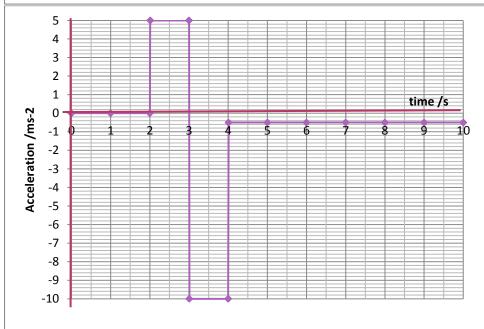
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# EXTRA PRACTICE ON THE EQUATION OF MOTION TUTORIAL ANSWERS

1.



2.



- 3. a)i) down ii) increasing constantly /uniformly
  - b) i) up ii) decreasing constantly/uniformly
  - c) i) down ii) increasing constantly /uniformly
  - d) At point B the ball hits the floor and rebounds / receives an additional force
  - e) At point C the ball has rebounded and loses contact with the floor/ receives an additional force
- 4.
- 5. A
- 6. A
- 7. 280 m
- 8. 51.2 m s<sup>-1</sup>
- 9. 28 m s<sup>-1</sup>
- 10. 16.7 s
- 11. 3.1 s
- 12. 750 m s<sup>-2</sup>
- 13. 9.5 N kg<sup>-1</sup>
- 14. 0.21 m s<sup>-2</sup>
- 15. 234 m
- 16. a) i) 21.4 m ii) 15.6 m s<sup>-1</sup> b) 34.6 m

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## CHAPTER 3: FORCES, ENERGY AND POWER

# SUMMARY OF FORCES, ENERGY & POWER

# 4. Forces, energy and power



eq W = mg

F = ma

 $E_W$  or W = Fd

Ep = mgh

 $Ek = \frac{1}{2} mv^2$ 

E = Pt

1

a) I can use vector addition and appropriate relationships to solve problems involving balanced and unbalanced forces, mass, acceleration and gravitational field strength.

P

b) I can identify and explain the effects of friction on moving objects. I do not need to use reference to static and dynamic friction.



c) I can identify and explain terminal velocity, in terms of forces



d) I can interpret and produce velocity-time graphs for a falling object when air resistance is taken into account.



e) I can analyse motion using Newton's first and second laws.



f) I can use free body diagrams and appropriate relationships to solve problems involving friction and tension.



g) I can resolve a vector into two perpendicular components.



h) I can resolve the weight of an object on a slope into a component acting parallel (down the slope) and a component acting normal to the slope.



i) I can use the principle of conservation of energy and appropriate relationships to solve problems involving work done, potential energy, kinetic energy and power.

## **NEWTONS THREE LAWS OF MOTION**

From previous courses it is assumed that you learned Newton's Three Laws of Motion, a corner stone of Physics.

## **NEWTON'S FIRST LAW:**

A body will remain at rest or move at steady speed in a straight line unless acted upon by an unbalanced force.



Or

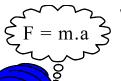
Unless an unbalanced force acts on an object the object will move at constant velocity (which means constant speed in a straight line without quoted direction)

Or

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A body will remain at rest or travel at constant velocity, unless acted upon by an unbalanced force.

## **NEWTON'S SECOND LAW**



We normally write as a formula:

$$F_R = ma$$

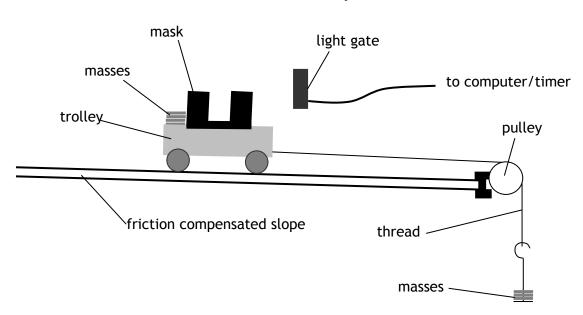
Resultant Force = mass × gravitational field strength (Newtons) = (Kilogram) × (Newtons per kilogram)

Originally written as The rate of change of momentum equals force

$$\frac{\Delta mv}{t} = F$$

$$F = \frac{m(v-u)}{t}$$

Where 
$$a = \frac{(v-u)}{t}$$



The apparatus shown above can be used to investigate the relationship between unbalanced force, mass and acceleration.

To find a relationship between three variables then two experiments must be carried out keeping one of the variables constant in each.

It is found that:

 $a \propto F$ 

when m is constant

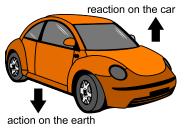
 $a \propto \frac{1}{m}$ 

when F is constant

Hence:

 $a \propto \frac{F}{m}$ 

## **NEWTON'S THIRD LAW:**



To every action there is an equal and opposite reaction.

(the action and reaction act on different bodies)

For every action there is an equal but opposite reaction.

or

If A exerts a force on B, B exerts an equal but opposite force on A

### BALANCED FORCES VERSUS NEWTON PAIRS

If the ship is travelling at constant velocity then:-

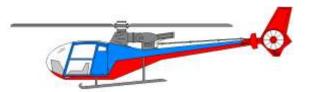
Weight is equal to the buoyancy force or upthrust, but in the opposite direction. Engine force is equal to the



drag forces but in the opposite direction. These are the balanced forces and are directly caused by each other. Balanced forces are two separate forces acting on one object. A Newton Pair is a pair of forces acting on the two interacting objects. The size of the force on the first object equals the size of the force on the second object.

TASK: For each of these 4 balanced forces state the Newton Pairs.

If the helicopter is hovering at constant height the engine force is equal in size but opposite in direction to the weight. These are balanced forces. **TASK: State the** 



Newton Pairs for each of these Forces.



What forces are acting on the bike?

 $F_w$  (weight) =  $F_r$  (reaction force from ground)

 $F_F$  (frictional) =  $F_E$  (engine)

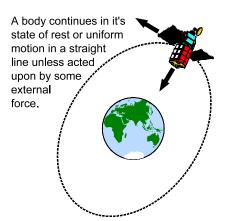
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#### TASK:

# Which of these equal forces are Newton Pairs and which are balanced forces?

So how do we know that there is a resultant force on the satellite?

Name the balanced forces on the paraglider.



#### THE NEWTON

One Newton is equal to the force which causes an acceleration of one metre per second squared when applied to a mass of one kilogram.

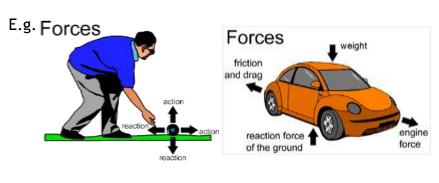
#### **RESULTANT FORCES**

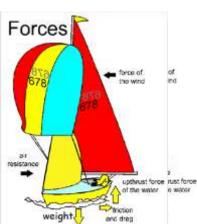
The resultant of a number of forces is the single force which has the same effect as the several forces actually acting on the object.

#### FREE BODY DIAGRAMS

Free body diagrams are diagrams that show all the forces (or sometimes components of forces) acting on a mass. If the size of the forces are known these can be added, if not they can be written in under the names of the forces.







TASK: Draw a freebody diagram of:

- a) you standing on the floor or sitting on a chair
- b) Mostly Harmless (Mrs H's canal boat) going at 4 mph along the Forth and Clyde Canal.

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## **VECTORS AND TENSION**

## WHAT IS A VECTOR?

A vector is a quantity where magnitude and direction are both important.

## WHY ARE VECTORS IMPORTANT?

Calculations can be positive or negative e.g. equations of motion/momentum.

Sometimes we want to use components of vectors e.g. in projectile motion we only use the vertical or horizontal components in our calculations.

## WHICH QUANTITIES ARE VECTOR QUANTITIES?

Force including Tension, Weight, Friction, etc.

Acceleration Displacement Field strength

Velocity Impulse Momentum

## WHEN DO WE QUOTE DIRECTION FOR VECTOR QUANTITIES?

In all equations of motion questions direction should be noted, also in momentum calculations. If in doubt, add a direction, even if it is an arrow or just a left or right, up or down.

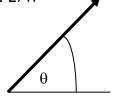
## CASES TO BE AWARE OF:

In equation of motion questions you always use components if they are at an angle, but it may ask for the final velocity which is a vector quantity. Therefore you must recombine horizontal and vertical components.

## A SPECIAL CASE OF RESOLVING VECTORS: FORCES DOWN A SLOPE.

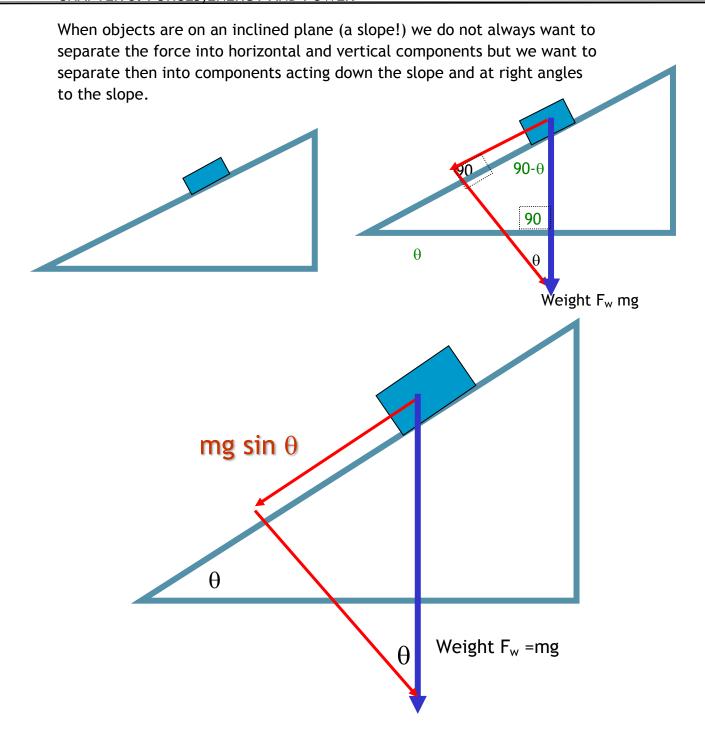
We can take COMPONENTS of any vector quantity to help us with our calculations

Usually these are horizontal and vertical or N/S and E/W



 $F_V = F \sin \theta$   $\theta$   $F_h = F \cos \theta$ 

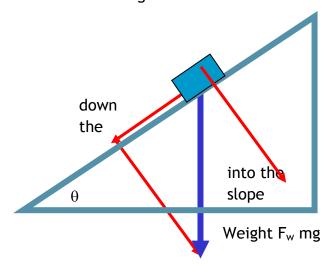
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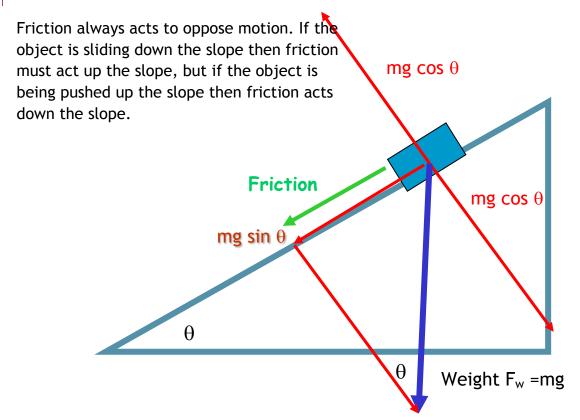
So the important component that gives the block the motion down the slope is  $\mathop{\text{mg sin}} \theta$ 

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The component  $\mbox{mg}\cos\theta$  pushes into the slope. This force is balanced by an equal but opposite force from the slope and is called the reaction forceThis leavethe component parallel to the slope as the part responsible for causing the acceleration down the slope, but friction also plays a part in the accelerationaccording to Newton's second law.

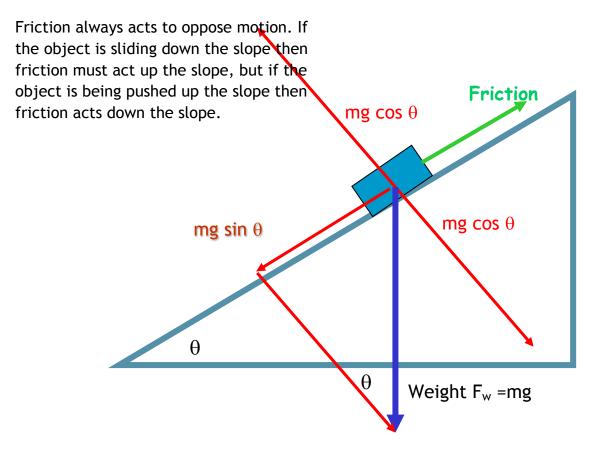


# FORCES WHEN AN OBJECT MOVES UP THE SLOPE

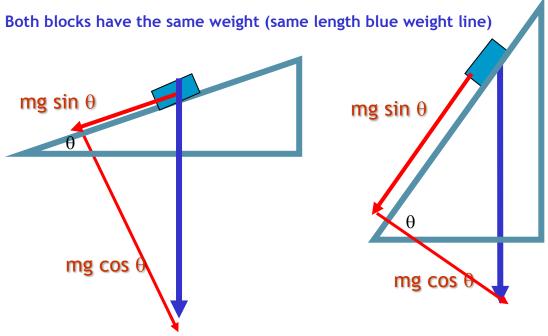


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# FORCES WHEN AN OBJECT MOVES DOWN THE SLOPE



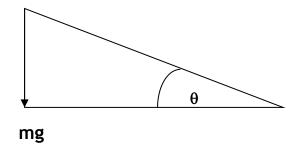
Notice as the slope gets steeper the component of weight down the slope increases, so there is more chance of the object moving!



A common mistake that students often make is that they think mg or weight is the vertical component and draw the diagram as below. They then try to calculate the force down the slope and find the hypotenuse. This gives them a value greater than the weight. THIS IS NOT POSSIBLE. Remember the weight is the hypotenuses of your diagram and you are taking components. The problem with this is that the component of the weight down the

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slope would be greater than the weight. To avoid this always draw the weight line longer than the slope (see previous diagrams)



wrong method so don't you do it

$$\sin = \frac{opp}{hyp}$$

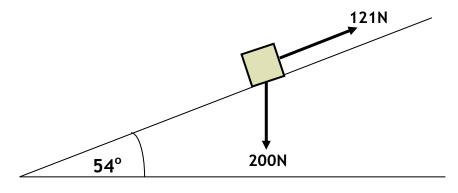
$$\sin = \frac{mg}{hyp}$$

$$hyp = \frac{mg}{\sin \theta} \text{ wrong answer!}$$

The object will accelerate if there is an unbalanced force on an object. This comes from the **resultant** of all the forces. In most of the cases the acceleration is equal to the component of weight down the slope less the friction which slows the object down.

# Example 1

- a) Determine the mass of the block shown?
- b) Determine the resultant force on the block?
- c) Determine the acceleration of the block shown?



$$F_{w} = mg$$
$$m = 20.4kg$$

 $F_{w} \sin \theta = \text{component down the slope}$ 

$$F_{un} = F_{w} \sin \theta - F$$

$$F_{un} = 200\sin 54 - 121$$

$$F_{un} = 40.8N$$

Notice  $F_{w} \cos \theta$  is balanced from a reaction force from the slope

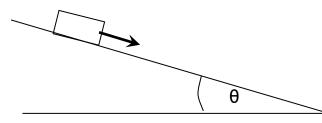
 $200 = m \times 9.8$ 

$$F_{un} = ma$$

$$40.8 = 20.4 \times a$$

$$a = 2ms^{-2}$$

# MOTION ON A SLOPE REVISION TEST



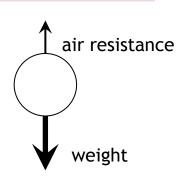
If an object is on a slope then it may move down the slope. Depending on the forces acting on the object it may be travelling at a constant velocity or it may be accelerating.

- 1. Describe the forces acting on the object when:
- (a) it is moving with constant velocity.
- (b) it is accelerating down the slope.
- 2. Explain what you think will happen to the acceleration if the angle is increased

For further notes on motion down a slope refer to the Powerpoint presentation on the network.

### **FREEFALL**

When an object is allowed to fall towards the Earth it will accelerate because of the force due to gravity acting on it. This will not be the only force acting on it though. There will be an upwards force due to air resistance.



## Air resistance increases with speed.

If an object is allowed to fall through a large enough distance then the force due to air resistance may increase to become the same magnitude as the force due to gravity.

When this situation occurs the forces acting on the object will be balanced. This means the object will fall with constant velocity or terminal velocity.

## PARACHUTES- AN EXAMPLE OF TERMINAL VELOCITY



The instant the parachutist leaves the plane

- ✓ Her vertical speed is zero
- ✓ Air resistance is zero
- ✓ Weight is the unbalanced force on the parachutist
- ✓ The parachutist accelerates downwards according

to F=ma

✓ The parachutist accelerates at 9.8 m s $^{-2}$ 

# As the parachutist falls her speed increases

- ✓ AIR RESISTANCE/ DRAG increases
- ✓ WEIGHT remains constant

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- ✓ There is still an unbalanced force on the parachutists but this is less than before.
- √ The parachutist accelerates downwards but the acceleration is less than 9.8ms<sup>-2</sup>
- ✓ The parachutist does not slow down but speeds up slower!

# Finally AIR RESISTANCE equals WEIGHT

- ✓ The forces on the parachutist are now BALANCED (overall effect ZERO acceleration)
- ✓ The parachutist travels at CONSTANT SPEED (acceleration is zero)
- ✓ The parachutist travels at TERMINAL VELOCITY

# The parachute opens

- ✓ AIR RESISTANCE/ DRAG INCREASES
- ✓ WEIGHT remains constant
- ✓ AIR RESISTANCE>>WEIGHT
- ✓ There is an unbalanced force on the parachutist upwards
- ✓ The parachutist decelerates (slows down very quickly)

Is it true that you go upwards when the parachute is open? That is what you see on the TV. The Camera operator hasn't opened her parachute so begins to fall faster than the parachutist she is filming. To keep filming the parachutist, the camera operator, has to point her camera upwards, giving the illusion that the parachutist has shot upwards.

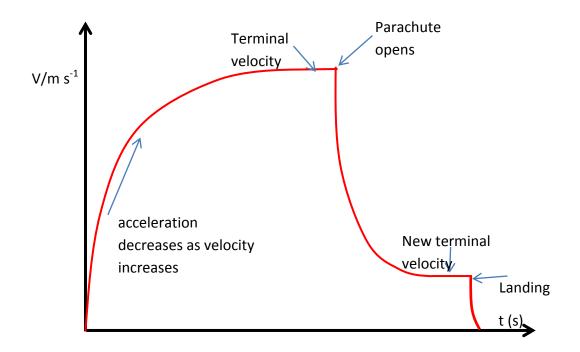
## As the parachutist decelerates

- ✓ AIR RESISTANCE DECREASES
- ✓ Air resistance decreases until it equals the weight.
- ✓ The forces on the parachutist are now balanced.
- ✓ The parachutist travels at CONSTANT SPEED (acceleration is zero)
- ✓ The parachutist travels at a NEW TERMINAL VELOCITY
- ✓ Obviously less than before but still enough to break a leg on impact.



1:http://www.bbc.co.uk/ schools/gcsebitesize/scie nce/add\_gateway\_pre\_20 11/forces/fallingrev1.sht

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## **FRICTION**

In winter sports, we need friction to be as low as possible so that we can achieve high speeds.

Ice skaters actually move on a layer of water, and don't skate on ice at all. When ice is subjected to high pressure it melts.

The narrow blades of the skates create a very high pressure and thus the skaters glide along on a layer of water they've just melted. The water refreezes as soon as they've moved on.

This is called "regelation" (sounds like something that happens to a football team, but it's spelt differently!)



#### **ROCKETS**

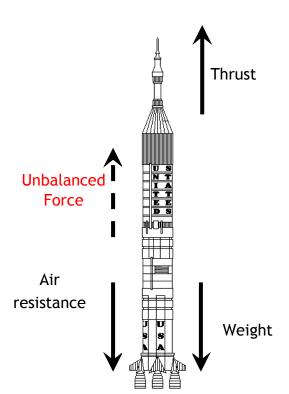
Whenever you consider rockets, always start by drawing a freebody diagram.

The motion of a rocket once it starts to lift off is complex. We need to consider all the forces acting on the rocket.

At the instant of lift off: in order for the rocket to lift off, the thrust must be greater than the weight. This produces an unbalanced force which makes the rocket accelerate upwards.

unbalanced Force = Thrust- weight

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$$a = \frac{F_u}{m}$$

where  $F_U$  = Thrust - weight, easy to calculate if we know the thrust and the mass of the rocket.

So far so good, but we've only moved a couple of centimetres at this point.

As the lift off continues: when the rocket begins moving, the air resistance will increase as the speed increases. This acts against motion so the unbalanced force will be reduced. But wait a minute, as the rocket uses up fuel its mass will decrease, reducing the weight increasing the unbalanced force.

Not only that as the rocket gets further from the Earth the gravitational field strength will reduce making the weight smaller again.

As altitude increases, the air will get thinner, thus reducing the air resistance.

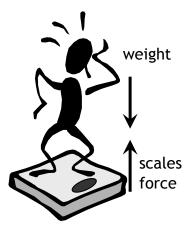
All of this is based on the assumption that the thrust remains constant. If thrust changes then this becomes an even bigger headache.

How do all of these different factors act together to affect the acceleration of the rocket?

Essentially the net effect is that the acceleration increases as the rocket rises.

https://www.youtube.com/watch?v=wbSwFU6tY1c

# THE PHYSICS OF LIFTS



Have you noticed that when you are in a lift you experience a strange feeling when the lift starts to move and as it begins to slow to a stop. However when the lift is in the middle of its journey you cannot tell if you are moving at all.

This is because at the start and end of the journey you will experience an acceleration and consequently an unbalanced force. This unbalanced force is what you 'feel'

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Direction	Lift	Notes	Free body diagram
Stationary		waiting together - stationary The forces are balanced	T=mg F=mg
Moving at constant speed up or down.		Moving at constant velocity up or down the forces are balanced	T=mg F=mg
Accelerating downwards.		unbalanced forces downwards, less tension in the lift cable as some of weight used for acceleration	T=mg-ma T=m(g-a) F=mg
decelerating downwards.		unbalanced force upwards. The cable is having to slow the lift down and support its weight	T=mg+ma T=m(g+a) F=mg
accelerating upwards		The cable must support the weight & provide an accelerating/unbalanced Force	T=mg+ma T=m(g+a) F=mg
decelerating upwards.		Part of the weight can be used as a decelerating force	T=mg-ma F=mg

When you stand on a set of scales the scales is actually measuring the **upwards** (reaction) force. This is the force the scales exert on you. Now this is fine when you are in your bathroom trying to find your weight.

Normally, you and your bathroom scales will be stationary and so your weight will be equal to the upwards force [Newton Pair].

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When you weigh yourself when you are accelerating the reading on the scales will <u>not</u> be equal to your weight. The reading will give you an indication of the unbalanced force acting on you, which could then be used to calculate an acceleration. This unbalanced force could be acting up or down depending on the motion of the lift.

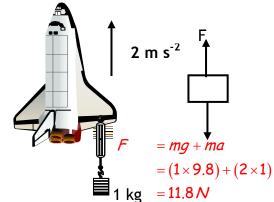
Accelerometers which can use this principle are found in more and more electronic devices. Game consol handsets, phones where you can move between functions by shaking the handset, laptops that know when they're falling and protect themselves before they hit the ground, the list goes on and on.

The forces are similar for an object accelerating upwards and decelerating downwards.

The forces are similar for an object accelerating downwards or decelerating upwards.

A mass of 1 kg hangs from a spring balance which is suspended from a rocket.

State the reading on the balance if the rocket accelerates upwards at 2 m s<sup>-2</sup>.



#### WHAT IS TENSION?



Tension is a force, measured in Newtons.

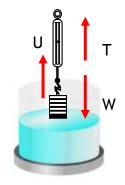
Think of it as the "balancing force". It acts in the direction to balance out forces to:

prevent movement



Examples of the way tension acts.

As soon as other forces start to act on the object the size and/or the direction of the tension alters.



At the start, W=T.

As you submerge the mass the reading on the spring balance decreases as the upthrust, U, increases.

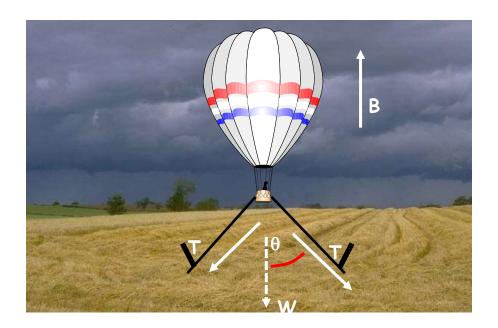
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Once the mass is totally immersed the upthrust remains constant so the reading on the spring balance will remain constant.

$$W = T + U$$

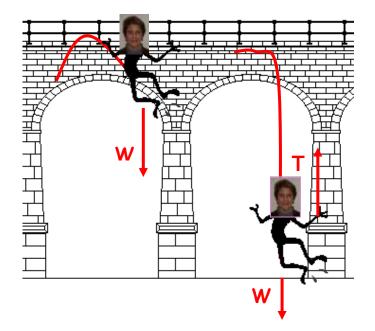
$$T = W - U$$

#### TENSION AS A BALANCING FORCE



The tension, T, is acting downwards. If it wasn't then the balloon would rise.

$$B = W + 2T \cos \theta$$



When the jumper leaps off she is in freefall.

W acts downwards. The tension in the bungee cord is zero Newtons.

At the end of the jump the elastic provides a stopping force as well as providing the reaction force to weight.

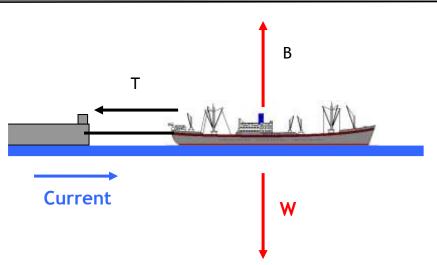
T acts upwards (a large force) and overcomes W as well as providing the decelerating force.

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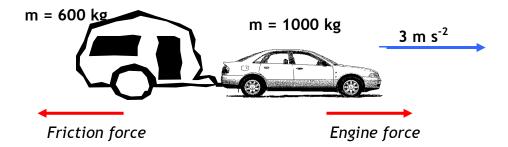
In the boat example opposite.

The tension, T, in the rope will depend on the strength of the current.

You will often have to use components when the current



The tension in the towbar, T, is the size of the force required to accelerate the caravan with the same value as the car. However, you must remember to add the frictional force. Assuming the frictional force,  $F_v$ , of the caravan is 400 N then the tension is given by:



Assuming the frictional force, F<sub>c</sub>, of the car is also 400 N, the total force provided

$$T = ma + F_{v}$$

$$T = (600 \times 3) + 400$$

$$T = 2200 N$$

$$Force = (m_{1} + m_{2}) a + F_{v} + F_{c}$$

$$= (1000 + 600) 3 + (400 + 400)$$

$$= 4800 + 800$$

$$= 5600 N$$

by the car is that needed to accelerate the car + caravan is:

If the vehicle is travelling at constant speed then since the forces are balanced:

$$T = F_{\nu} = 400 \, \text{N}$$

## NEWTON'S SECOND LAW: ENERGY AND POWER

#### Newton's Second Law F=m a

Gravitational Potential Energy= mass  $\times$  gravitational field strength  $\times$  height

$$E_p = mgh$$

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Kinetic Energy=  $\frac{1}{2}$  × mass × velocity squared

$$E_{\nu} = \frac{1}{2} m v^2$$

 $work\ done = Force \times distance$ 

$$E_w = F \times d$$

#### LAW OF CONSERVATION OF ENERGY

Energy cannot be created or destroyed, it can only be changed from one form to another (or from one object to another).

# Example

(a) Thomas drops a football of mass 0.2 kg from a height of 2.25 m. Calculate the velocity of the ball at the instant before it hits the ground. Ignore air resistance

Before the ball is dropped, it possesses only gravitational potential energy.

At the instant before the ball hits the ground, all the gravitational potential energy has been converted to kinetic energy. So, provided that there are no energy losses,

$$\triangle$$
 Ep lost =  $\triangle$  Ek gained
$$\triangle$$
 mgh =  $\triangle$  ½ mv²
$$gh = \frac{1}{2}v^2$$

('m' appears on both sides of equation, so can be cancelled out).

9.8 x 2.25 = 0.5 
$$v^2$$
  
22.1 = 0.5  $v^2$   
 $v^2$  = 22.1/0.5 = 44.2  
 $v = \sqrt{44.2} = 6.6 \text{ ms}^{-1}$ 

Another important concept it power. Power is the equivalent to the energy transformed or dissipated per second.

You have probably not met that Power can equal force  $\times$  velocity but you should be

$$P = \frac{E}{t} = \frac{E_w}{t} = \frac{Fs}{t} = Fv$$

The equation above is found by combining some of the equations below

$$E_p = mgh$$
,  $E_k = \frac{1}{2}mv^2$ ,  $E_w = Fs$ ,

able to see that this is equivalent. Other used of power and energy are given

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below. We have yet to meet I as equal to irradiance, but it will turn up in the next unit.

$$Power = \frac{Energy}{Time}$$

$$P = IV, = I^2 R, = \frac{V^2}{R}$$

Where the *I* in the last equation is **irradiance** and **not current**, **so don't get confused!** 

$$P = IA$$

 $Power = Irradiance \times area$ 

#### NEWTON'S SECOND LAW / TUTORIAL 1

The first few questions here are to give you an idea of what you ought to be able to answer from Nat 5. You ought to be familiar with this type of problem. If you are not confident then it is worth spending time revising this section until you are happy to go on.

- 1. What force is needed to give a 100000kg tank an acceleration of 5ms<sup>-2</sup>?
- 2. An airgun pellet is accelerated at  $2x10^{4}$ ms<sup>-2</sup> by the air in the barrel exerting an average force of 100N on it. Determine the mass of the pellet?
- 3. Determine the acceleration of a bike and rider with a combined mass of 80kg when coasting downhill with a gravitational force of 140N acting on them?

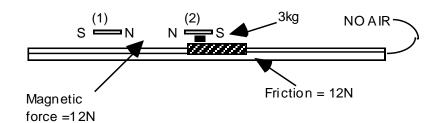
#### NEWTON'S SECOND LAW / TUTORIAL 2

You will find the following tutorial easier if you ensure that you include a free body diagram. Students think that free body diagrams take too much time to complete but they usually save time and are more likely to lead to a correct answer. It makes you stop and think about all the forces acting on a body as these are often missed out at first glance, so don't get caught in the questions below.

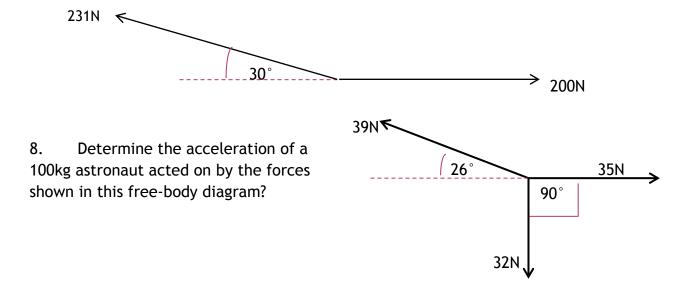
- 1. A 150g bird drops straight from a tree towards the garden with a steady frictional force of 0.97N acting on it. Determine its acceleration?
- 2. A snowball is given an acceleration of 1000 ms<sup>-2</sup> by the thrower's arm which exerts a 100N force. If the jersey of her coat causes a constant retarding force of 20N when it is being moved, Determine the mass of the snowball?
- 3. A roller skate starts to roll down a hill and takes 5s to reach a velocity of 7.5ms<sup>-1</sup>. If the roller skate has a weight of 7.84N, Determine the force causing the acceleration?

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- 4. A falling feather of mass 0.01g feels a frictional force of  $95.5\mu N$  as it falls. Determine the acceleration during this time?
- 5. Two forces of 6N and 8N are at right angles to each other and both act on a 2.5kg mass at the same time. How much acceleration do they cause?
- 6. (a) Draw a free-body diagram for the vehicle shown below, then calculate its acceleration.
- (b) Air is pumped in so that the friction becomes zero. Draw the new free-body diagram and use it to find the acceleration of the vehicle.

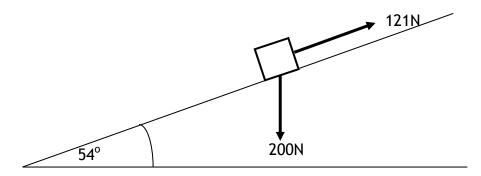


7. Determine the acceleration of a 1.5kg ball kicked by two players simultaneously as shown on this free-body diagram?

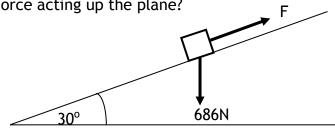


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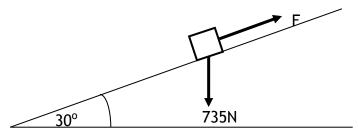
9.



- a) Determine the mass of the block shown?
- b) Determine the resultant force on the block?
- c) Determine the acceleration of the block shown?
- 10. If the block shown in the diagram accelerates down the plane at 2.5ms<sup>-2</sup>, Determine the frictional force acting up the plane?



11. If the block starts from rest and reaches a velocity of 6ms<sup>-1</sup> in 2s, Determine the frictional force acting up the plane?



### NEWTON'S SECOND LAW / TUTORIAL 3

The next few tutorial questions revise the work that you should have done in Nat 5 on energy and power and then incorporates work that you have done as part of this higher course.

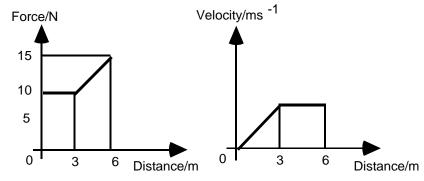
- 1. In a low orbit the space shuttle has a mass of 75000 kg and a payload mass of 25000 kg. It travels at a speed of 8 kms<sup>-1</sup>.
- (a) Find the shuttles kinetic energy in orbit.
- (b) What work is done bringing the shuttle to rest on the ground?

If the shuttle takes 53 minutes and twenty seconds to come to rest after the reentry engines are fired

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- (c) Find the total distance travelled,
- (d) Find the frictional force of the air.
- 2. A golf ball of mass 0.1kg rises vertically to 40m above the ground. Calculate the gravitational potential energy the ball gains at that height.
- 3. A man mows a lawn in 10 minutes by exerting an average force of 100 N over a distance of 100 m. Calculate his average power.
- 4. Two objects of unequal mass but identical kinetic energies are moving in the same direction. If the same frictional force is applied to each, how does their stopping distances compare?
- 5. A large packing case of mass 500 kg rests on the ground. A fork-lift truck raises it 1.5 m, transports it at a steady speed of 2.0 ms<sup>-1</sup> and deposits it on a loading platform of a lorry.
- (a) Determine the minimum upward force exerted by the fork-lift truck?
- (b) How much potential energy is gained by the packing case?
- (c) Calculate the kinetic energy of the packing case while it is being transported at steady speed.
- (d) What happens to the kinetic energy when the fork-lift truck stops?
- (e) If the fork-lift truck uses energy at the rate of 25 kW and the lifting operation takes 3.0 seconds, calculate the apparent efficiency of the operation.

6.



Answer the following questions from these two graphs.

- (a) How much work is done on the object in the first 3 metres?
- (b) Into what energy form is this work transferred?
- (c) Assuming that the object moves horizontally, how do we know that *heat energy* is being produced as it moves from 3 to 6 metres?

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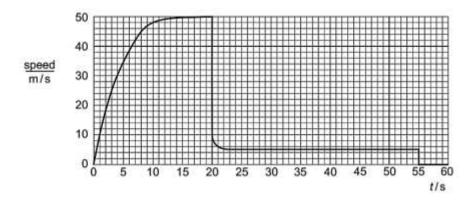
### NEWTON'S 2ND LAW, ENERGY AND POWER- / EXTRA HELP

- 1. State Newton's 1st Law of Motion.
- 2. A lift of mass 500 kg travels upwards at a constant speed. Calculate the tension in the lifting table.
- 3. A fully loaded oil tanker has a mass of  $2.0 \times 10^3$  kg. As the speed of the tanker increases from zero to a steady maximum speed of  $8.0 \text{ m s}^{-1}$  the force from the propellers remains constant at  $3.0 \times 10^6 \text{ N}$



- (a) Calculate the acceleration of the tanker just as it starts from rest
- (b) Determine the size of the force of friction acting on the tanker when it is travelling at the steady speed of 8.0 m s<sup>-1</sup>?
- (c) When its engines are stopped, the tanker takes 50 minutes to come to rest from a speed of 8.0 m s<sup>-1</sup>. Calculate its average deceleration.

4.



The graph shows how the speed of a parachutist varies with time after having jumped from the aeroplane. With reference to the letters, explain each stage of the journey.

- 5. Two girls push a car of mass 2000 kg. Each applies a force of 50 N and the force of friction is 60 N. Calculate the acceleration of the car.
- 6. A boy on a skateboard rides up a slope. The total mass of the boy and the skateboard is 90 kg. He decelerates uniformly from 12 m s<sup>-1</sup> to 2 m s<sup>-1</sup> in 6 seconds. Calculate the resultant force acting on him.
- 7. A box is pulled along a rough surface with a constant force of 140 N. If the mass of the box is 30 kg and it accelerates at  $4 \text{ m s}^{-2}$  calculate:
  - a. the unbalanced force causing the acceleration.
  - b. the force of friction between the box and the surface.

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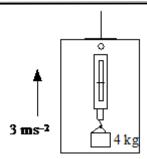
- 8. An 800 kg Metro is accelerated from 0 to 18 m s<sup>-1</sup> in 12 seconds.
  - a. Determine the resultant force acting on the Metro?
  - b. How far does the car travel in these 12 seconds?
  - c. At the end of the 12 s period the brakes are operated and the car comes to rest in a distance of 50 m. Determine the average frictional force acting on the car.

9.

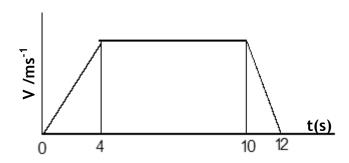
- (a) A rocket of mass 40000 kg is launched vertically upwards. Its engines produce a constant thrust of 700000 N.
- (i) Draw a diagram showing all the forces acting on the rocket.
- (ii) Calculate the initial acceleration of the rocket.
- (b) As the rocket rises its acceleration is found to increase. Give three reasons for this.
- (c) Calculate the acceleration of the same rocket from the surface of the Moon if the Moon's gravitational field strength is 1.6 N kg<sup>-1</sup>.
- (d) Explain in terms of Newton's laws of motion why a rocket can travel from the Earth to the Moon and for most of the journey not burn up any fuel.
- 10. A rocket takes off and accelerates to 90 m s<sup>-1</sup> in 4 s. The resultant force acting on it is 40 kN upwards.
  - a. Calculate the mass of the rocket.
  - b. Calculate the force produced by the rocket's engines if the average force of friction is 5000 N.
  - 11. Determine the minimum force required to lift a helicopter of mass 2000 kg upwards with an initial acceleration of 4 m s<sup>-2</sup>. Air resistance is 1000 N.
- 12. A crate of mass 200 kg is placed on a balance in a lift.
  - a. Determine the reading on the balance, in newtons, when the lift was stationary.
  - b. The lift now accelerates upwards at  $1.5~{\rm m~s^{-2}}$ . Determine the new reading on the balance.
  - c. The lift then travels up at a constant speed of 5 m s<sup>-1</sup>. Determine the reading on the balance.
  - d. For the last stage of the journey calculate the reading on the balance when the lift decelerates at 1.5 m s<sup>-2</sup> while moving up.
- 13. A small lift in a hotel is fully loaded and has a mass of 250 kg. For safety reasons the tension in the pulling cable must never be greater than 3500 N.
  - a. Determine the tension in the cable when the lift is:
    - i. at rest
    - ii. moving up at a constant 1 m s<sup>-1</sup>
    - iii. accelerating upwards at 2 m s<sup>-2</sup>
    - iv. accelerating downwards at 2 m s<sup>-2</sup>?
  - b. Calculate the maximum permitted upward acceleration of the fully loaded lift.
  - c. Describe a situation where the lift could have an upward acceleration greater than the value in (b) without breaching safety regulations.

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14. A package of mass 4 kg is hung from a spring balance attached to the ceiling of a lift which is accelerating upwards at 3 m s<sup>-2</sup>. Determine the reading on the spring balance?

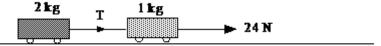


15. The graph shows how the downward speed of a lift varies with time.



- a. Draw the corresponding acceleration/time graph.
- b. A 4 kg mass is suspended from a spring balance inside the lift.

  Determine the reading on the balance at each stage of the motion.
- 16. Two masses are pulled along a flat surface as shown below.

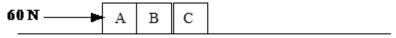


Find the

- a. acceleration of the masses
- b. tension, T between the masses.
- 17. A car of mass 1200 kg tows a caravan of mass 1000 kg. The frictional force on the car and caravan is 200 N and 500 N respectively. The car accelerates at  $2 \text{ m s}^{-2}$ .
  - a. Calculate the force exerted by the engine of the car.
  - b. Calculate the force the tow bar exerts on the caravan.
  - c. The car then travels at a constant speed of 10 m s<sup>-1</sup>. Assuming the frictional forces to be unchanged calculate the new engine force and the force exerted by the tow bar on the caravan.
  - d. The car brakes and decelerates at 5 m s<sup>-2</sup>. Calculate the force exerted by the brakes (assume the other frictional forces remain constant).
- 18. A tractor of mass 1200 kg pulls a log of mass 400 kg. The tension in the tow rope is 2000 N and the frictional force on the log is 800 N. How far will the log move in 4 s assuming it was stationary to begin with?

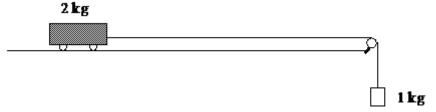
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19. A force of 60 N pushes three blocks as shown. If each block has a mass of 8 kg and the force of friction on each block is 4 N

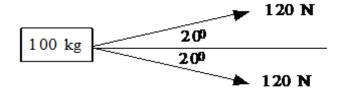


Calculate:

- a.
- i. the acceleration of the blocks
- ii. the force block A exerts on block B
- iii. the force block B exerts on block C.
- b. The pushing force is then reduced until the blocks move at constant speed. Calculate the value of this pushing force.
- c. State whether the force block A exerts on block B now equals the force block B exerts on block C. You must justify your answer.
- 20. A 2 kg trolley is connected by string to a 1 kg mass as shown. The bench and pulley are frictionless.



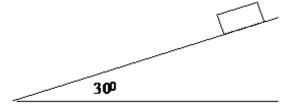
- a. Calculate the acceleration of the trolley.
- b. Calculate the tension in the string.
- 21. A force of 800 N is applied to a canal barge by means of a rope angled at  $40^{\circ}$  to the direction of the canal. If the mass of the barge is 1000 kg and the force of friction between the barge and the water is 100 N find the acceleration of the barge.
- 22. A crate of mass 100 kg is pulled along a rough surface by two ropes at the angles shown.



- a. If the crate is moving at a constant speed of 1 m s<sup>-1</sup> Determine the force of friction.
- b. If the forces were increased to 140 N at the same angle calculate the acceleration of the crate.

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23. A 2 kg block of wood is placed on the slope shown. It remains stationary. Determine the size of the frictional force acting up the slope.



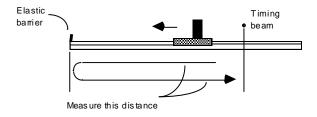
- 24. . A 500 g trolley runs down a runway which is 2 m long and raised 30 cm at one end. If its speed remains constant throughout calculate the force of friction acting up the slope.
- 25. The brakes on a car fail while it is parked at the top of a hill. It runs down the hill for a distance of 50m until it crashes into a hedge. The mass of the car is 900 kg and the hill makes an angle of 15° to the horizontal. If the average force of friction is 300 N. Calculate:
  - a. the component of weight acting down the slope
  - b. the acceleration of the car
  - c. the speed of the car as it hits the hedge
  - d. the force acting perpendicular to the car (the reaction) when it is on the hill.

#### NEWTON'S SECOND LAW ENERGY AND POWER/ PRACTICALS

## PRACTICAL 1: FRICTION

Set up the linear air track with an arrangement that allows you to measure the speed of the vehicle travelling on it using photo-switches interfaced with a computer.

Choose one place along the track for measuring the speed. Set the vehicle off and measure its speed on each pass as well as the distance it travels between each speed measurement.



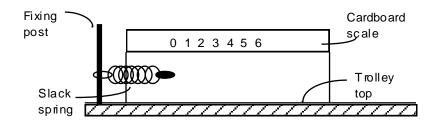
We can find the average value of friction on the air track by using its effect on the vehicles motion. In this case the loss of kinetic energy  $(1/2mv^2)$  equals the work done against friction (force x distance). The equation we need is:

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \text{friction } \times \text{distance}$$

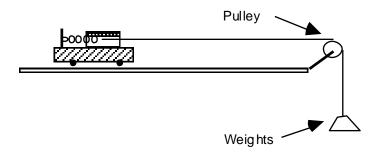
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## PRACTICAL 2 CHECKING NEWTON'S SECOND LAW.

(1) (a) Mark a 10cm length of cardboard with equidistant marks to act as a force scale for a light spring.



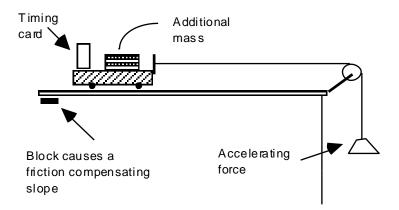
The apparatus above is fixed to one end of a trolley then string and weights are attached to it as shown below:



Use this arrangement to get a graph of acceleration against force.

## PRACTICAL 2B: CHECKING NEWTON'S SECOND LAW.

(2): Set up a trolley as shown in the diagram below:



The purpose of this arrangement is to give us an actual measurement of the effect of inertia by showing that the acceleration of the trolley is inversely proportional to its mass.

i.e. we must show that the product  $a \times m$  is a constant.

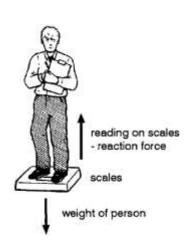
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DISCUSSION OPPORTUNITY: The force causing the acceleration is the weight on the end of the string. This weight also has a *mass*. Explain how this affects the results.

# PRACTICAL 3 TO CALCULATE THE ACCELERATION OF A LIFT.

Apparatus: 1 set of bathroom scales.

#### Instructions:





- Use the scales to determine your weight and your mass.
- Stand on the scales inside of the lift and note the reading on the scales while the lift is stationary.
- Press the button to send the lift downwards.

Note a) The minimum reading on the scales as the lift accelerates downwards.

- b) The reading on the scales as the lift travels at constant speed.
- c) The maximum reading on the scales as the lift decelerates upwards.
- Repeat the experiment for the upward journey of the lift.
- Use your results to calculate the acceleration of the lift at the three stages of each journey.

### **EXTENSION:**

Draw free body diagrams of the forces with the relative sizes for the three stages of each journey.

### PRACTICAL 4WORK DONE BY FRICTION

Aim: To calculate the work done by the force of friction acting on a trolley moving down a slope.

Apparatus: 1 trolley, 1 light gates, 1 ramp, 1 stopcock, 1 power supply. (Or use the motion QED), metre stick, scales.

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#### Instructions:

- a) Measure the mass of the trolley, m.
- b) Set up the apparatus as shown in the diagram, marking the initial position of the centre of the trolley on the slope.
- c) Measure the distance, d, travelled by the trolley to the light gate.
- d) Release the trolley, allowing it to cut through the light beam.
- e) Note the time on the timer at the light gate and calculate the speed of the trolley, v, through the light gate.
- f) Measure the difference in height between the starting position and the position of the light gate, h.
- g) Calculate the loss in  $E_p$  of the trolley.
- h) Calculate the gain in  $E_k$  of the trolley.
- i) Hence calculate the difference due to friction.
- j) By considering the work done by friction on the trolley, calculate the average force of friction acting on the trolley over the distance, *d*, down the runway.

#### **EXTENSION:**

By considering the uncertainties in the measured quantities in the above experiment, estimate a value for the absolute uncertainty in the average force of friction for the journey.

## SQA EXAM QUESTION: FORCE, ENERGY AND POWER

For exam questions turn to the homework book and complete the homework questions from there.

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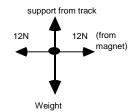
### NEWTON'S SECOND LAW/ TUTORIAL ANSWERS:

### **TUTORIAL 1**

- 1. The force needed is  $5x \cdot 10^5 N$
- 2. The pellet has a mass of 5g
- 3. The acceleration is  $1.75 \text{ms}^{-2}$

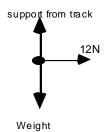
#### TUTORIAL 2

- 1.  $a = 3.3 \text{ ms}^{-2} \text{ downwards}$
- 2. m = 0.08kg or 80g
- 3. F = 1.2N
- 4.  $a = 0.25 \text{ ms}^{-2}$
- 5. The acceleration caused is 4ms<sup>-2</sup> at an angle of 53° to the 6N force as shown on the diagram.
- 6. (a)



There is no unbalanced force so a=0

(b)



The 12N unbalanced force causes an acceleration of 4ms<sup>-2</sup> to the right in the 3kg vehicle.

- 7. The acceleration is  $77 \text{ ms}^{-2}$
- 8. The acceleration is 0.149 ms<sup>-2</sup> downwards.
- 9. a) 20.4 kg
  - b) 40.8N
  - c) 2 ms<sup>-2</sup>
- 10. 168N
- 11. 142.5N

### **TUTORIAL 3**

- 1. (a)  $320 \times 10^{10} \text{ J}$ 
  - (b)  $320 \times 10^{10} \, \text{J}$
- 2. 40 J
- 3. 17 W
- 4. The kinetic energy is converted into heat as the objects do work against the force of friction. Since their original energies are the same, they each do the same amount of work as they stop so their stopping distances must be equal.

- 5. (a) 5000N
  - (b) 7500 J
  - (c) 1000 J
  - (d) It is converted to heat
  - (e) 10%
- 6. (a) Work done = 30J
- (b) Kinetic energy since we can see from the second graph that the speed is increasing.
- (c) The velocity is steady during this part of the motion and so the forces on the object are balanced. This means that the work done is against the force of friction which only produces heat.

# NEWTON'S SECOND LAW EXTRA HELP ANSWERS

1.	A body will remain at rest or move at constant velocity unless acted upon by an unbalanced force -
2.	4900 N
3.	a) i) $0.015 \text{m s}^{-2}$ ii) $3.0 \times 10^6 \text{ N}$ b) - $0.0027 \text{ m s}^{-2}$
4.	
5.	0.02 m s <sup>-2</sup>
6.	150 N
<i>7</i> .	a) 120 N b) 20 N
8.	a) 1200 N b) 108 m c) 2592 N
9.	a) i) ii) 7.7 m s <sup>-2</sup> b) - c) 15.9 m s <sup>-2</sup> d) -
10.	a) 1778 kg b) 62424 N
11.	28600 N
12.	a) 1960 N b) 2260 N c) 1960 N d) 1660 N
13.	a) i) 2450 N ii) 2450 N iii) 2950 N iv) 1950 N b) 4.2 m s-2 c) -
14.	51.2 N
15.	a) b) 37.2 N, 39.2 N, 43.2 N
16.	a) 8 m s-2 b) 16 N
17.	a) 5100 N b) 2500 N c) 700 N, 500 N d) 10,300 N
18.	24 m
19.	a) 2 m s-2 b) 40 N c) 20 N d) 12 N e)
20.	a) 3.27 m s-2 b) 6.54 N
21.	0.513 ms-2
22.	a) 225.5 N b) 0.376 m s-2
23.	9.8 N
24.	0.735 N
25.	a) 2283 N b) 2.2 m s-2 c) 14.8 m s-1 d) 8520 N

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### **CHAPTER 4: COLLISIONS AND EXPLOSIONS**

#### SUMMARY OF COLLISIONS AND EXPLOSIONS

# 5. Collisions and explosions



e q

p = mv

Ft = mv - mu

 $E_k = \frac{1}{2}mv^2$ 

P

a) I can use the principle of conservation of momentum and an appropriate relationship to solve problems involving the momentum, mass and velocity of objects interacting in one dimension.

P

b) I can explain the role in kinetic energy in determining whether a collision is described as elastic and inelastic collisions or in explosions.



c) I can use appropriate relationships to solve problems involving the total kinetic energy of systems of interacting objects.



d) I can use Newton's third law to explain the motion of objects involved in interactions.



e) I can draw and interpret force-time graphs involving interacting objects.



f) I know that the impulse of a force is equal to the area under a force-time graph and is equal to the change in momentum of an object involved in the interaction.



g) I can use data from a force-time graph to solve problems involving the impulse of a force, the average force and its duration.



h) I can use appropriate relationships to solve problems involving mass, change in velocity, average force and duration of the force for an object involved in an interaction.

#### **COLLISIONS AND EXPLOSIONS**

## **INERTIA**

Inertia is the tendency of a body to remain at rest, or if moving, to continue its motion in a straight line.

(NB the inertia is given by its mass)

Click on this link and watch the example

https://www.youtube.com/watch?v=zWeKRMh3kT8

http://www.stevespanglerscience.com/experiment/trick-with-tablecloth

### MOMENTUM

The momentum of a body is defined as the product of mass and its velocity.

It has the symbol p and units kgms<sup>-1</sup>.

Momentum is a vector quantity.

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### REVISION OF NEWTON'S LAWS OF MOTION

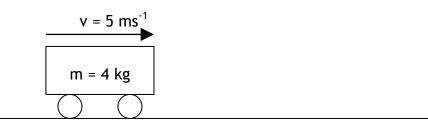
First Law: unless a resultant force acts on a body its velocity will not change.

Second Law: The rate of change of momentum of a body is proportional to the resultant force that acts.

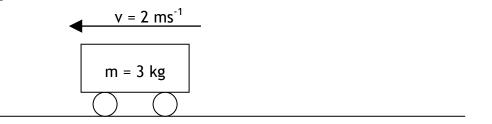
Third Law: In any interaction between two objects the force exerted by object A on object B is equal in size but opposite in direction to the force exerted by object B on A.

For each of the example find the total momentum of the system. Remember that momentum is a vector quantity.

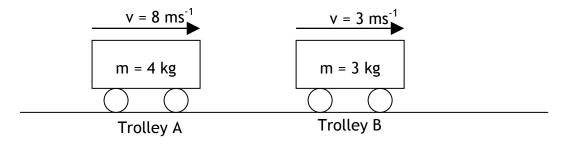
## Example 1



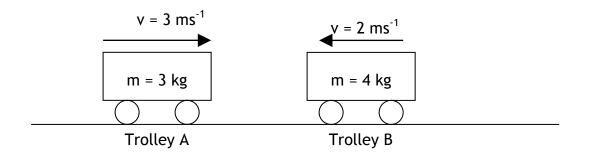
## Example 2



## xample 3



## Example 4



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#### **Answers**

Example 1 = 20kgms<sup>-1</sup> to the right,

Example 2 = 6kgms<sup>-1</sup> to the left,

Example 3 = 41kgms<sup>-1</sup> to the right

Example 4 = 1kgms<sup>-1</sup> to the right

### MOMENTUM AND COLLISIONS

Any object that has mass and moves has momentum. When objects collide momentum can be transferred to the objects with which they collide.

## **TASK**

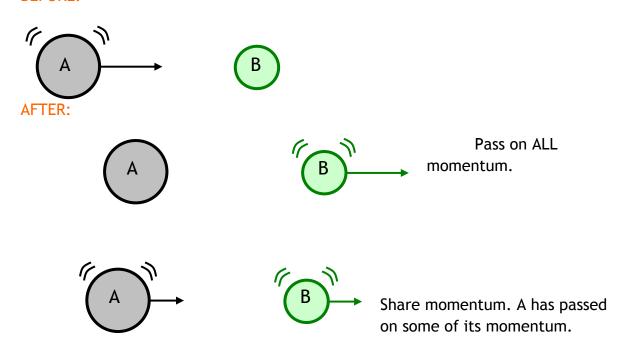
List 10 situations in which momentum is transferred during collisions.

## **Experiments show that**

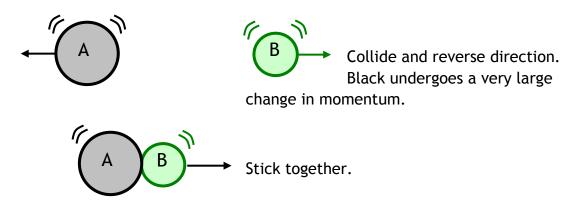
During any collision, in the absence of external forces the total momentum before the collision is equal to the total momentum after the collision.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

### **BEFORE:**



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View the video and think about the conservation of momentum

## http://www.youtube.com/watch?v=fRY7vSarkio

If you think of situations where objects collide you should notice that you have 2 types of collision.

when objects stick together

CLIMAMADV

when the objects bounce off each other after the collision.

Loosely we call the first type of collision *INELASTIC*, and the second type of collision *ELASTIC*. In both types of collision MOMENTUM IS CONSERVED as is TOTAL ENERGY.

For Higher we need a tighter definition for the two types of collision.

An inelastic collision is defined where the total kinetic energy after the collision is reduced, ie kinetic energy is lost in the collision. NB the total energy remains the same but it is converted to other forms mainly as heat and sound.

An elastic collision is one where no kinetic energy is lost. (In reality there are very few examples of perfectly elastic collisions)- Although it should be recognised that particles collide elastically.

SUMMARY	MARY				
Type of Collision	Momentum	Kinetic Energy	Total Energy		
Elastic	Conserved Conserved		Conserved		
Inelastic	Conserved	Reduced	Conserved		
Explosions	Conserved zero at start and finish	here Ep is converted to E <sub>k</sub> so E <sub>k</sub> increases	Conserved		

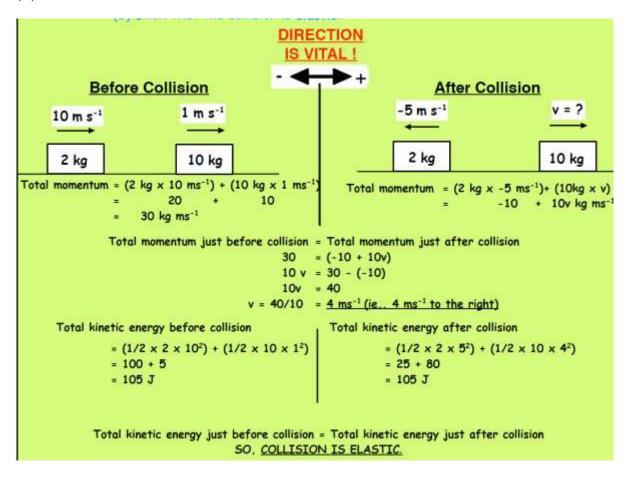
During ALL collisions, remember that TOTAL energy is CONSERVED (as well as momentum). We are just looking at  $\Delta Ek$  for defining inelastic and elastic collisions.

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#### EXAMPLE MOMENTUM PROBLEM

A 2 kg trolley moving to the **right** at **10 ms**<sup>-1</sup> collides with a **10 kg** trolley which is also moving to the **right** at **1 ms**<sup>-1</sup>. Immediately after the collision, the **2 kg** trolley rebounds to the **left** at **5 ms**<sup>-1</sup>.

- (a) Calculate the **velocity** of the **10 kg** trolley immediately **after** the collision.
- (b) Show that the collision is elastic.



#### Note

You should set out all your momentum problems like this - This makes it easier for you (and anybody marking your work) to see exactly what you are doing.

- Always include a sketch to show the masses of the colliding objects and their velocities just before and just after the collision.
- Take plenty of space on your page Some people take a new page for every problem.
- Take care with your calculations and be careful with directions. Remember:

#### **DIRECTION IS VITAL!**

Task: You try this....

Two objects collide. Object 1 has a mass of 0.2kg and object 2 has a mass of 1kg. The smaller object has a starting velocity of 10 ms<sup>-1</sup> and the larger object has an

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initial velocity of 1 ms<sup>-1</sup>. After the collision object 1 rebounds with a velocity of 5 ms<sup>-1</sup> Determine the velocity of object 2?

Is the collision elastic or inelastic? YOU MUST JUSTIFY YOUR ANSWER. To justify your answer you must find out if Ek is lost. If Ek is lost the collision is inelastic.

(Answer 4 ms<sup>-1</sup>, elastic)

## I. ELASTIC COLLISIONS

#### In an elastic collision:

- ✓ the 2 colliding objects bounce apart after the collision.
- ✓ momentum is conserved. (The total momentum just before the collision = the total momentum just after the collision.)
- √ total energy is conserved
- ✓ kinetic energy is conserved. (The total kinetic energy just before the collision = the total kinetic energy just after the collision.)

## II. INELASTIC COLLISIONS

#### In an inelastic collision:

- ✓ the 2 colliding objects stick together after the collision (but only one sign of an inelastic collision, not definitive!)
- ✓ momentum is conserved. (The total momentum just before the collision = the total momentum just after the collision.)
- √ total energy is conserved
- ✓ kinetic energy decreases. (The total kinetic energy just after the collision is less than the total kinetic energy just before the collision.) Some kinetic energy is changed into sound, heat and energy of deformation (which changes the shape of the objects) during the collision.

When objects collide they exert a force on each other according to Newton's Third Law.

$$F_{a} = -F_{b}$$

$$F_{a} = m_{a}a_{a}$$

$$F_{a} = \frac{m(v - u)}{t}$$

$$F_{a}t_{a} = m_{a} (v_{a} - u_{a})$$

This quantity Ft is called the IMPULSE and it is equal to the CHANGE IN MOMENTUM of the object.

Impulse = change in momentum

$$(N s) = (kg m s^{-1})$$

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### REARRANGING MOMENTUM EQUATIONS

In ALL linear collisions and explosions MOMENTUM is CONSERVED.

Momentum Before = Momentum after

$$\Delta p_1 = \Delta p_2$$

Momentum before is equal momentum of object 1 and momentum of object 2 prior to the collision. Momentum after is equal momentum of object 1 and momentum of object 2 after the collision.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Also  $\Delta p$  of body 1 is the same as  $\Delta p$  of body 2

(ie the impulse received by each body in a collision is equal)

Impulse on 1 = impulse on 2

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

Let's see how these equations come about:-

Let's start with Newton's Third Law of Motion. This says that the force acting on object B is equal in size but opposite in direction to that on object A. Newton's Third Law is that same as saying that Momentum is conserved. (NB This only applies to different bodies and not to explosions, which are simply balanced forces).

$$-F_1 = F_2$$

(for ease I have used 1 & 2 instead of A & B for my bodies!)

but Newton's Second Law says:

$$F = ma$$

$$.:-m_1a_1 = m_2a_2$$

but acceleration is equal to  $a = \frac{v - u}{t}$  so substitute

$$-\frac{m_1(v_1 - u_1)}{t_1} = \frac{m_2(v_2 - u_2)}{t_2}$$

During the collision the time body 1 is in contact with body 2 must be the same so that t cancels out,  $-m_1(v_1-u_1)=m_2(v_2-u_2)$ 

This tells us that the change in momentum of body 1 is equal to the change in momentum of body 2.

Expand the brackets.  $-m_1v_1 + m_1u_1 = m_2v_2 - m_2u_2$ 

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Change in  $p_1$  = change in  $p_2$ 

but 
$$-m_1v_1 + m_1u_1 = m_1v_1 - m_1u_1$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

rearrange again

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

momentum before = momentum after

QED!

#### SUMMARY OF MOMENTUM & COLLISIONS

Force is measured in Newtons.

Momentum is measured in kilograms metres per second.

Impulse is measured in Newton seconds and is given by the area under a Force -time graph.

Change in momentum is measured in kilograms metres per second. It is equal to the impulse.

During any collision, in the absence of external forces total momentum before a collision is equal to the total momentum after the collision. HOWEVER, remember that the momentum of each individual object is likely to change, one will lose momentum, and one will gain it!

### MOMENTUM & IMPULSE EQUATION SUMMARY

F=ma

$$-F_1 = F_2$$

p=mv

$$a = \frac{v - u}{t}$$

momentum before = momentum after

$$m_1u_1 + m_2u_2 = m_2v_2 + m_1v_1$$

 $m_1u_1 + m_2u_2 = (m_2 + m_1)v$  if they stick together

Change in  $p_1$  = change in  $p_2$ 

$$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2$$

$$\Delta p_1 = \Delta p_2$$

$$\Delta p = mv - mu$$

$$\Delta p = m(v - u)$$

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Change in momentum = impulse

$$Ft = mv - mu$$

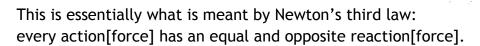
$$\Delta p = Ft$$

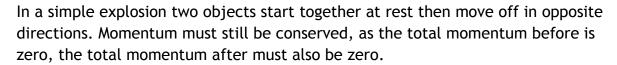
#### **EXPLOSIONS AND NEWTON'S LAWS**

Far from being some abstract physics law from a text book the law of conservation

of momentum has direct consequences in our lives. If you have ever been in a plane or on a jet ski then the motion of these vehicles depends on this law.

The propulsion system works by the engine expelling some form of exhaust at high speed in one direction. The conservation of momentum means that something else must move in the opposite direction to conserve momentum. In the cases above that thing is the engine, which is attached to the vehicle causing it to move.





**Example:** An early Stark Jericho missile is launched vertically and when it reaches its maximum height it explodes into two individual warheads.







www.lron-man-armor.blogspot.com www.marvalcinematicuniverse.wikia .com

force of

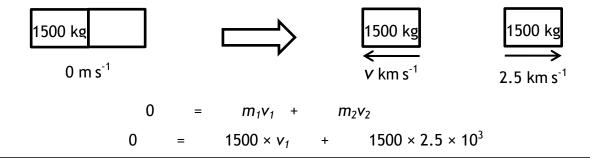
air

force of balloon

Both warheads have a mass of 1500 kg and one moves off horizontally, with a velocity of 2.5 km s $^{-1}$  (Mach 9) at a bearing of 090°.

Calculate the velocity of the other warhead.

Solution:



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$$1500 \times v_1 = -1500 \times 2.5 \times 10^3$$

$$v_1 = -\frac{3.75 \times 10^6}{1500}$$

$$v_1 = -2.5 \times 10^3 \,\mathrm{m s}^{-1}$$

The negative sign in the answer indicates the direction of  $v_1$  is opposite to that of  $v_2$ , i.e.  $270^{\circ}$  rather than  $090^{\circ}$ .

Second warhead is travelling at 2.5 km s<sup>-1</sup> on a bearing of 270°.

## COMPARING EXPLOSIONS AND COLLISION

Explosion	Collision
$p_{total}$ before = $p_{total}$ after	$p_{total}$ before = $p_{total}$ after
$0 = m_1 v_1 + m_2 v_2$	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
$m_1 v_1 = -m_2 v_2$	$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$ - $m_1(v_1 - u_1) = m_2(v_2 - u_2)$
$\frac{m_1v_1}{t} = \frac{-m_2v_2}{t}$	$\frac{-m_1(v_1-u_1)}{t} = \frac{m_2(v_2-u_2)}{t}$
$m_1a_1 = -m_2a_2$	$-m_1a_1 = m_2a_2$
$F_1 = -F_2$	$-F_1 = F_2$

## **FORCE-TIME GRAPHS**

For force-time graphs:

area under graph

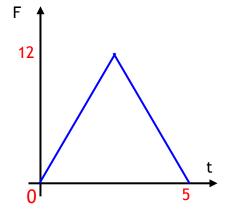
= Ft

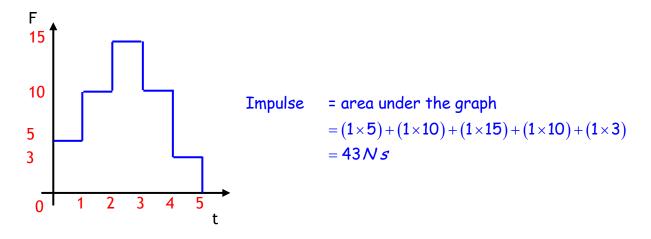
=  $\Delta p$  (change in momentum)

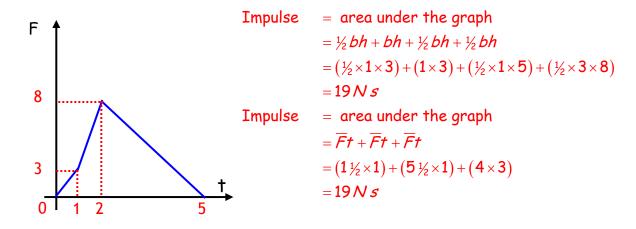
= impulse

Impulse = area under the graph

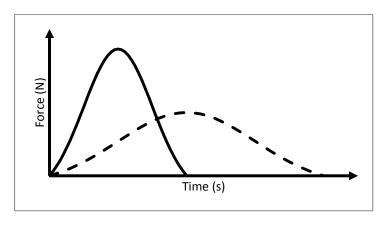
 $= \frac{1}{2} \times 12 \times 5 = 30 \text{ N s.}$ 







A more realistic example of a Ft graph is the one shown below (represented by the solid line), which could be a graph of a collision between two cars that bounce off each other. At Higher you would not be expected to calculate the impulse from this type of graph as it is considered too difficult. However, you could be asked what this graph might look like if safety features were in place. Obviously this would have the same area under the graph as the same momentum change must occur, but safety features are designed to decrease the force on the occupants, resulting in an increased time of contact.



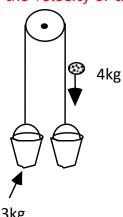
## WORKED EXAMPLES

Two difficult worked examples of momentum and collisions and explosions. Try to work out how you would start each question before you look at the solutions.

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### Worked Example 1

Calculate the velocity of the bucket and the putty after the collision.



A 4kg lump of putty is dropped from 2m into a bucket as shown.

Find the velocity the putty hits the bucket:

$$v = ?$$
 $u = 0 \, m \, s^{-1}$ 
 $s = 2 \, m$ 
 $m = 4 \, kg$ 
 $v^2 = u^2 + 2 \, as$ 
 $v^2 = 0^2 + (2 \times 9.8 \times 2)$ 
 $v = 6.3 \, m \, s^{-1}$ 

This is the INITIAL velocity of the putty during the collision. Notice too that as the buckets are joined it is the combined mass of the buckets that is being accelerated.

momentum before = momentum after

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
  
 $(4 \times 6.3) + (6 \times 0) = (10)v$   
 $25.2 + 0 = 10v$   
 $v = \frac{25.2}{10} = 2.5 \, \text{m s}^{-1}$ 

This is the initial velocity during the collision so this can be used to find the final velocity of the bucket.

#### Worked Example 2

An astronaut of mass 100 kg (including his equipment) is at rest with respect to his spaceship which is 100 m away. He has 1600 g of oxygen left in his tank which he uses to breathe at 0.8 g per second and which he can release in bursts of 50g at a time with a velocity of 100 m s<sup>-1</sup> in order to propel himself towards his ship.



- (a) What velocity does the astronaut have towards his ship if he releases 50 g of oxygen?
- (b) How long does it take him to reach his ship at this speed?

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Momentum before = momentum after = 0 kg m s<sup>-1</sup>

Find the momentum of the gas = mv =  $0.05 \times 100$  =  $5.0 \text{ kg m s}^{-1}$   $\therefore \text{ Momentum of astronaut} = -5.0 \text{ kg m s}^{-1} = mv$  -5.0 = (100 - 0.05)v -5.0 = 99.95v  $v = \frac{-5.0}{99.95} = -0.05 \text{ m s}^{-1}$ 

Assuming he travels at a constant speed  $\bar{v} = \frac{s}{t}$  and directly towards the spacecraft:

$$0.05 = \frac{100}{t}$$

$$t = \frac{100}{0.05} = 2000 \,s$$

#### MOMENTUM & IMPULSE / TUTORIAL 1

- 1) A 20g marble moving at 5ms<sup>-1</sup> strikes a 12g stationary marble. If the 20g marble continues along the same line as before but at the reduced velocity of 2ms<sup>-1</sup>, Calculate the speed at which the other marble move off.
- 2) A 3kg linear air track vehicle moving at 0.5ms<sup>-1</sup> is struck by a 2kg vehicle travelling in the same direction. If the velocity of the 2kg vehicle is reduced to 0.75ms<sup>-1</sup> by the collision, while the other moves off at 2ms<sup>-1</sup>, Determine the original velocity of the 2kg vehicle.
- 3) An exploding pen which goes off accidentally on a bench sends it 20g cap in one direction at 10ms<sup>-1</sup>. State what happens to the remaining 50g body of the pen.

### MOMENTUM & IMPULSE / TUTORIAL 2

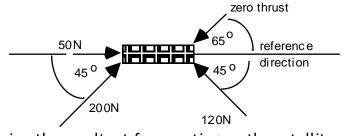
- 1) A 0.2kg ball travelling at 15ms<sup>-1</sup> is stopped by a hockey stick. Calculate the impulse the stick receive from this action.
- 2) An 80kg rugby player moving at 8 ms<sup>-1</sup> receives an impulse of 520 kgms<sup>-1</sup> from another player he accidentally runs into.
  - a) Determine the change in momentum involved in this collision.
  - b) Determine the new momentum of the runner.
- 3) A 2.0 kg bowling ball is moving at 3.5 ms $^{-1}$  before it strikes the skittles, and at 0.5 ms $^{-1}$  afterwards.
  - a) Determine its momentum change.
  - b) If the collisions with the skittles lasted for 1.5 s, Determine the average force acting on the ball during the strike.

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- 4) A 90 kg football player slides into a tackle at 5 ms<sup>-1</sup>. If the grass slows him with an average force of 180 N, calculate the time taken to stop.
- 5) Calculate the impulse needed to change the velocity of a 2000 kg car from 5 m s<sup>-1</sup> to 8 ms<sup>-1</sup>.
- 6) A space probe of mass  $5 \times 10^6$  kg and velocity 9000 ms<sup>-1</sup> receives an impulse of  $5 \times 10^9$  Ns by operating its thrusters. Determine its resulting momentum change.
- 7) A 5.0 kg object changes its velocity from 8 ms<sup>-1</sup> to 11 ms<sup>-1</sup>.
  - a) Determine the momentum change involved.
  - b) If the change takes 1.5 s to complete, determine the average force acting on the object.
  - c) Calculate the impulse the force gives to the object.

### MOMENTUM & IMPULSE / TUTORIAL 3

- 1) A climber is equipped with a shock-stop tape, which takes a constant force of 492 N to unravel it in the event of a fall. If the climber has a mass of 82 kg and drops a distance of 2 m, Determine his speed on reaching the end of the tape.
- 2) An 80 kg shuttle jockey rises vertically from the Moon's surface at 5 ms<sup>-2</sup>.
  - a) Draw her free-body diagram.
  - b) State how you know the resultant force is acting upwards.
  - c) Determine the size of the resultant upward force on her.
  - d) If the upward vertical force exerted by the pilot's seat is 528 N, determine the Moon's gravitational field strength in newtons per kilogram.
- 3) A 2000 kg satellite has three of its adjusting thrusters operating simultaneously in the same plane. There is a fourth thruster on standby which is set to zero thrust.

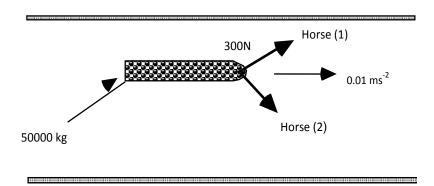


- a) Determine the resultant force acting on the satellite.
- b) If the acceleration of the satellite is  $0.1~{\rm ms}^{-2}$  at an angle of  $65^{\circ}$  to the reference direction, explain what can you deduce about the thruster which is shown as providing zero thrust.
- 4) When a car hits a test wall at 10 ms<sup>-1</sup>, a 10 kg carry cot with a dummy baby inside stretches its safety harness by 0.25 m while it comes to a halt. Calculate the force acting on the *dummy* if its mass is 6.0 kg.

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- 5) An electron is accelerated from rest on the cathode of an electron gun to one tenth the speed of light in a distance of 15 cm. If the mass of an electron is  $9.11 \times 10^{-31}$  kg, Determine the size of the electric force acting on it.
- 6) A 50000 kg canal barge is accelerated parallel to the canal side at 0.01 ms<sup>-2</sup> by two horses towing it as shown:

If the two horses are pulling at right angles to each other, calculate the magnitude of force exerted by the horse. Assume frictional forces are negligible.

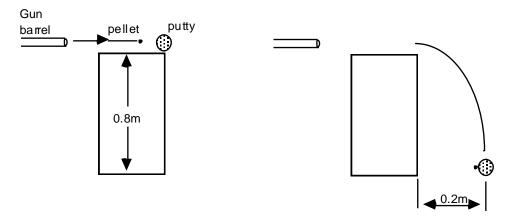


- 7) At a car research centre a solid fuel rocket system is used to accelerate the cars to their impact with a concrete wall. Each car and its variety of occupants must strike the wall at 15 ms<sup>-1</sup> after travelling 22.5 m. To do this, the accelerating rocket is fuelled with bags of propellant, each of which gives 1000 N of thrust when ignited. Calculate the number of bags must be used on an occasion when the car under test has a 'kerb weight' of 1600 kg.
- 8) A hockey player stops a 100 g puck moving at 20 ms<sup>-1</sup> with a 0.5 kg hockey stick.
  - a) Determine the resulting velocity of the stick if it does not have the player holding it.
  - b) If the player hit the approaching puck so that it exactly reverses its velocity, determine the momentum change of the puck.
- 9) A 10.0 N force accelerates a 4.0 kg body from 3.0 ms<sup>-1</sup> to 8.0 ms<sup>-1</sup>.
  - a) Calculate the time the force acts.
  - b) Calculate the impulse received by the body.
- 10) An open, empty coal wagon of mass 1000 kg is moving at 5 ms<sup>-1</sup> when it passes under a coal chute which dumps 4000 kg of coal into it.
  - a) Determine the velocity of the full wagon as it emerges from under the chute.
  - b) Explain how the vertically moving coal is able to transmit a horizontal impulse to the wagon.
- 11) An astronaut of mass 120 kg including her equipment is at rest with respect to his spaceship which is 75.0 m away. She has 1200 g of oxygen left in her tank which she uses to breathe at 0.8 g per second and which she can release in

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bursts of 50 g at a time with a velocity of 100 ms<sup>-1</sup> in order to propel herself towards her ship.

- a) Calculate the velocity the astronaut has towards her ship if she releases 50 g of oxygen.
- b) Calculate the time taken for her to reach her ship at this speed.
- c) Determine the breathing time she has left after releasing this oxygen.
- d) State if it is better for her survival if she releases 100 g of oxygen instead of 50 g. You must justify your answer.
- 12) The radar track of an incoming missile shows it has a kinetic energy of  $E_k$  and a momentum of p. If it explodes at this point, state the effect this has on the size of (a)  $E_k$  and (b) p
- 13) A 2.0 kg ball moving at 5.0 ms<sup>-1</sup> strikes an 8.0 kg ball moving at 2.0 ms<sup>-1</sup>. Assuming that they stick to each other on impact, calculate the velocity of the balls and the loss in kinetic energy when they are moving.
  - a) in the same direction before impact and
  - b) moving in opposite directions.
- 14) A 20.0 g pellet strikes a 380.0 g block horizontally at 50.0 ms<sup>-1</sup>. The pellet sticks in the block which now slides across the table top before coming to rest after travelling 0.125 m. Determine the force of friction between the block and the table's surface.
- 15)A 200 kg bobsleigh hits the incline of the slow-down area at a speed of 20.0ms-<sup>1</sup>. If the sledge stops after rising a vertical height of 15.0 m, determine the energy turned to heat by the friction between the runners and the snow.
- 16) The diagrams illustrate an experimental method for measuring the speed of an air-gun pellet:



A lump of putty of mass 0.1 kg is resting on the edge of a bench of height 0.80 m. The pellet, of mass  $5.0 \times 10^{-4}$  kg, is fired at the lump of putty.

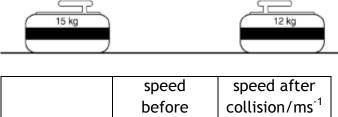
The putty with the pellet embedded in it lands 0.20 m from the foot of the bench as shown.

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- a) Show that the horizontal velocity of the putty after the impact of the pellet is  $0.5~\text{ms}^{-1}$ .
- b) (i) State the principle of the conservation of momentum.
  - (ii) Using this principle, calculate the velocity of the pellet just before it strikes the putty.
- c) Using only the apparatus above, suggest one way of improving the accuracy of this experiment.

### **COURSE QUESTIONS**

1. Two students Angus and Ryan are investigating collisions between 'curling stones' at an ice rink. Ryan releases his stone, of mass 15 kg, so that it collides head on with Angus's stone, of mass 12 kg, which is travelling in the opposite direction. The speeds of the stones before and after collision are given in the table below.



	speed	speed arter
	before	collision/ms <sup>-1</sup>
	collision/ms <sup>-</sup>	
Angus' stone	0.6	V
Ryan's stone	0.8	0

- (a) Calculate the speed, v, of Angus's stone after the collision.
- (b) Show by calculation whether the collision is elastic or inelastic.
- (c) (i) During the collision the stones are in contact for 0.05 s. Calculate the average force exerted by Ryan's stone on Angus's stone during the collision.
- (ii) Determine the average force exerted by Angus's stone on Ryan's stone during the collision?
- 2. Two identical cars X and Y are crashed at 14 m s<sup>-1</sup> into a wall as part of a car safety test. Car X carries a dummy of mass 75 kg. The dummy is restrained by a seatbelt. The dummy is brought to rest by the seatbelt in a time of 35 ms.

Car Y carries an identical dummy that is not restrained by a seatbelt.

- (a) Calculate the average force exerted by the seatbelt on the dummy in car X.
- (b) The dummy in car Y is brought to rest in a time of 15 ms due to collision with the steering wheel.

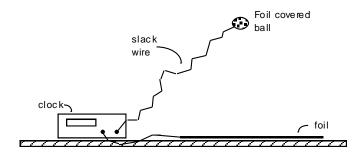
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- (i) Determine the impulse on the dummy in car Y during this time?
- (ii) Which dummy is more likely to be damaged as a result of the crash? You must explain your answer.

### **PRACTICALS**

## PRACTICAL 1: FORCE DURING AN IMPACT

Aim: To find the average force of an impact.



Find the mass of the ball then use your ingenuity to measure the height **d** the ball is dropped from and the height **b** to which it bounces

Use your results and the equation below to calculate the average force on the ball during its bounce.

It can be shown that the average force is given by

$$\bar{F} = \frac{4.47 \ m \times (\sqrt{d} + \sqrt{b})}{t}$$

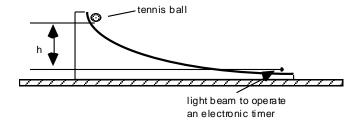
where m is the mass of the ball and t is its contact time with the surface.

## PRACTICAL 2: ENERGY EXCHANGE

Aim: Exchange of potential and kinetic energies.

Instructions:

Part (i): Set up the tennis ball and ramp as shown



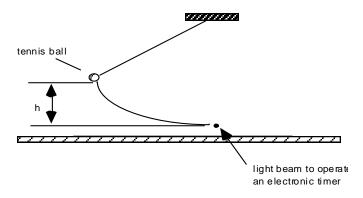
- Measure the mass of the tennis ball.
- Measure h and use E<sub>p</sub> = mgh to calculate the potential energy of the tennis ball.

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• Measure the speed of the ball as its diameter passes through the light beam and use  $E_k = \frac{1}{2}mv^2$  to calculate it kinetic energy.

How much potential energy does not turn up as kinetic energy?

Part (ii): Set up the same tennis ball as a simple pendulum.



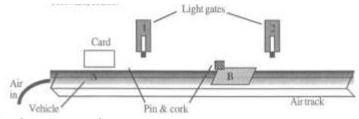
Make the same measurements and calculations as in Part (i) above.

DISCUSSION OPPORTUNITY: Where do you think the 'lost' potential energy goes and why do you think there is such a difference between the results of these two experiments?

### PRACTICAL 3: MOMENTUM

Aim: To compare the total momentum before and after a collision.

Apparatus: 1 linear air track, 2 vehicles (1 with card), 2 light gates, 1 computer &



velocity software, scales.

### Instructions:

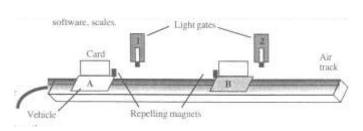
- Note the masses of vehicle A and vehicle B.
- Set the computer to measure TWO velocities.
- Keeping, vehicle B stationary, push A towards B.
- Note: a) the velocity of A through light gate 1
  - o b) the velocity of A and B together through light gate 2.
- Calculate the total momentum before and after the collision.
- Repeat the experiment for different masses of vehicle A and vehicle B.
- Use an appropriate format to compare the total momentum before and after the collision.

PRACTICAL 4: CONSERVATION OF MOMENTUM

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Aim: To compare the total momentum before and after a collision.

Apparatus: 1 linear air track, 2 vehicles with cards, 2 light gates, 1 computer & velocity software, scales.



#### Instructions:

- Note the masses of vehicle A and vehicle B.
- Set the computer to measure 3 velocities.
- Keeping vehicle B stationary, push A towards B.
- Note: a) the velocity of A through light gate 1
- b) the velocity of B through light gate 2
- the velocity of A through light gate 1 or 2 (if at all!).
- Compare the total momentum before and after the collision.
- Repeat for different masses.

### PRACTICAL 5: EXPLOSIONS

Aim: To compare the total momentum before and after a collision using explosive trolleys.

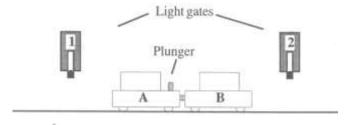
Apparatus: 2 trolleys with cards, 2 light gates, 1 computer & velocity software, scales.

#### Instruction:

- Note the masses of trolleys A and B.
- ii. Set the computer to measure two velocities.
- iii. With both trolleys stationary, strike the plunger.
- iv. Note:
- v. the velocity of A through light gate 1
- vi. the velocity of B through light gate 2.
- vii. Compare the total momentum before and after the explosion. (Remember momentum is a vector quantity).
- viii. Repeat the experiment for different masses of trolleys.

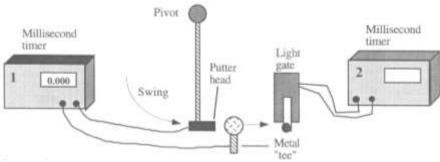
## PRACTICAL 6: IMPULSE

Aim: To calculate average force exerted by a putter on a golf ball.



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Apparatus: 1 "putter" and mountings, 1 metal painted golf ball with metal "tee," 1 light gate, 2 millisecond timers, scales.

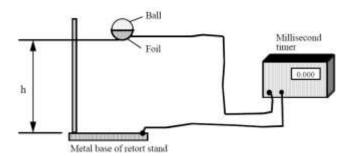


#### Instructions:

- i. Measure the mass of the (golf ball, in, and the diameter of the golf ball. Reset both millisecond timers to zero.
- ii. Place the light gate in front of the ball so that it will pass through the centre of the beam after having been struck by the "putter" head.
- iii. Pull the "putter" head back and allow it to strike the ball.
- iv. Note the time of contact on timer 1.
- v. Using the time recorded on millisecond timer 2, calculate the velocity, v, of the golf ball after being struck by the "putter" head.
- vi. Use the equation Ft = mv ~ mu to calculate the force acting on the ball.

# PRACTICAL 7: IMPULSE 2

Aim: To compare different times of contact for different balls bouncing on a hard surface. Apparatus: 3 balls of different types, covered in metal foil, 1 retort stand, 1 millisecond timer.

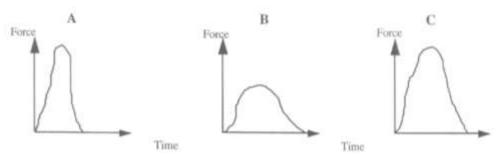


#### Instructions:

- i. Reset the millisecond timer to zero.
- ii. Raise the first ball to a set height h, eg 50 cm, and allow it to bounce once only onto the retort stand base.
- iii. Note down the time, t, recorded on the millisecond timer. This is the time of contact between the ball and the retort stand base during the bounce.
- iv. Repeat 3 or 4 times from the same height and obtain an average value for the time. Repeat the experiment for the other two balls, dropping them from the same height.

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Below are three examples of force-time graphs, showing force varying with time. From the results of your experiment, match the ball used to the graph that could

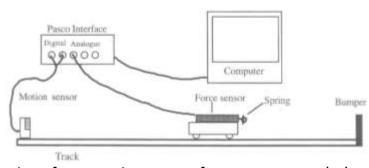


show the variation of force with time during the bounce.

By considering the materials the balls are made from, explain the results.

## PRACTICAL 8: IMPULSE

Aim: To compare force-time graphs for different collisions.



Apparatus: Pasco interface, motion sensor, force sensor, track, bumper, computer.

### Instructions:

- i. Set up the computer to plot graphs of
- ii. force against time
- iii. velocity against time.
- iv. Give the trolley a gentle push so it collides with the bumper.
- v. Use the statistics function to calculate the area under the force time graph for the first collision.
- vi. Read off the velocity before and after the collision.
- vii. Measure the mass of the trolley and hence calculate the change of momentum.
- viii. Compare this with the impulse calculated (area under the graph).
  - ix. Account for any difference.
  - x. Repeat the experiment with a slightly stronger push.
  - xi. Explain any differences in the graphs produced.
- xii. Now replace the spring with clay and repeat the process.
- xiii. Explain the change in the shape of the force-time graph.

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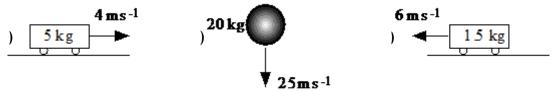
### PRACTICAL 8: IMPULSE 3

Repeat Practical 8 using repelling magnets between the trolley and bumper.

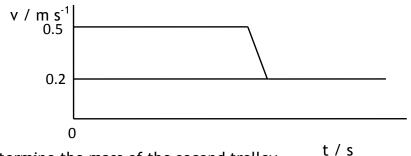
### MOMENTUM AND IMPULSE ADDITIONAL PRACTICE QUESTIONS

If you need additional practice try the following questions.

1) Determine the momentum of the object in each of the following situations:



- 2) A trolley of mass 2 kg and travelling at 1.5 m s<sup>-1</sup> collides and sticks to another stationary trolley of mass 2 kg. Calculate the velocity after the collision. Show that the collision is inelastic.
- 3) A target of mass 4 kg hangs from a tree by a long string. An arrow of mass 100 g is fired with a velocity of 100 m s<sup>-1</sup> and embeds itself in the target. At what velocity does the target begin to move after the impact?
- 4) A trolley of mass 2 kg is moving at constant speed when it collides and sticks to a second trolley which was originally stationary. The graph shows how the speed of the 2 kg trolley varies with time.

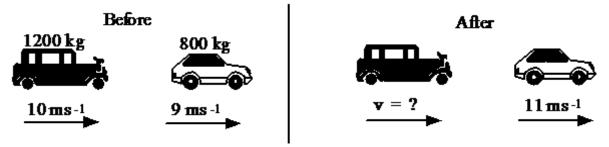


Determine the mass of the second trolley

- 5) In a game of bowls one particular bowl hits the jack 'straight on' causing it to move forward. The jack has a mass of 300 g and was originally stationary, the bowl has a mass of 1 kg and was moving at a speed of 2 m s<sup>-1</sup>.
  - a. Determine the speed of the jack after the collision if the bowl continued to move forward at 1.2 m s<sup>-1</sup>?
  - b. How much kinetic energy is lost during the collision?
- 6) In space two spaceships make a docking manoeuvre (joining together). One spaceship has a mass of 1500kg and is moving at 8 m s<sup>-1</sup>. The second spaceship has a mass of 2000 kg and is approaching from behind at 9 m s<sup>-1</sup>. Determine their common velocity after docking.
- 7) Two cars are travelling along a racing track. The car in front has a mass of 1400 kg and is moving at 20 m s<sup>-1</sup> while the car behind has a mass of 1000 kg and is moving at 30 m s<sup>-1</sup>. They collide and the car in front moves off with a speed of 25 m s<sup>-1</sup>.

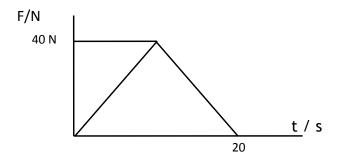
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- a. Determine the speed of the rear car after the collision.
- b. Show clearly whether this collision was elastic or inelastic.
- 8) One vehicle approaches another from behind as shown. The vehicle at the rear is moving faster than the one in front and they collide which causes the vehicle in front to be 'nudged' forward with an increased speed. Determine the speed of the rear vehicle after the collision.

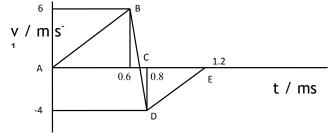


- 9) A trolley of mass 0.8 kg, travelling at 1.5 m s<sup>-1</sup> collides head on with another vehicle of mass 1.2 kg, travelling at 2.0 m s<sup>-1</sup> in the opposite direction. They lock together on impact. Determine the speed and direction after the collision.
- 10) A firework is launched vertically and when it reaches its maximum height it explodes into 2 pieces. One piece has a mass of 200 g and moves off with a speed of 10 m s<sup>-1</sup>. If the other piece has a mass of 120 g what speed does it have?
- 11) Two trolleys in contact, initially at rest, fly apart when a plunger is released. One trolley with a mass of 2 kg moves off with a speed of 4 m s<sup>-1</sup> and the other with a speed, in the opposite direction, of 2 m s<sup>-1</sup>. Determine the mass of this trolley?
- 12) A man of mass 80 kg and woman of mass 50 kg are skating on ice. At one point they stand next to each other and the woman pushes the man who then moves away at 0.5 m s<sup>-1</sup>. With what speed and at what direction does the woman move off?
- 13) Two trolleys in contact, initially at rest, fly apart when a plunger is released. If one has a mass of 2 kg and moves off at speed of 2 m s<sup>-1</sup>, calculate the velocity of the other trolley given its mass is 3 kg.
- 14) A cue exerts an average force of 7 N on a stationary snooker ball of mass 200 g. If the impact lasts for 45ms, with what speed does the ball leave the cue?
- 15)A girl kicks a football of mass 500 g which was originally stationary. Her foot is in contact with the ball for a time of 50 ms and the ball moves off with a speed of 10 m s<sup>-1</sup>. Calculate the average force exerted on the ball by her foot.
- 16) A stationary golf ball is struck by a club. The ball which has a mass of 100 g moves off with a speed of 30ms<sup>-1</sup>. If the average force of contact is 100 N calculate the time of contact.
- 17) The graph shows how the force exerted on a hockey ball by a hockey stick varies with time. If the mass of the ball is 150 g determine the speed of the ball as it leaves the stick (assume that it was stationary to begin with).

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- 18) A ball of mass 100 g falls from a height of 20 cm onto a surface and rebounds to a height of 18 cm. The duration of impact is 25 ms. Calculate:
  - a. the change in momentum of the ball caused by the 'bounce
  - b. the average force exerted on the ball by the surface.
- 19) A rubber ball of mass 40 g is dropped from a height of 0.8 m onto the pavement. It rebounds to a maximum height of 0.45 m. The average force of contact between the pavement and the ball is 2.8 N.
  - a. Calculate the velocity of the ball just before it hits the ground and the velocity just after hitting the ground.
  - b. Calculate the time of contact between the ball and pavement.
- 20) A ball of mass 400 g travels horizontally along the ground and collides with a wall. The velocity / time graph below represents the motion of the ball for the first 1.2 seconds.



- a. Describe the motion of the ball during sections AB, BC, CD and DE.
- b. Determine the time of contact with the wall?
- c. Calculate the average force between the ball and the wall.
- d. How much energy is lost due to contact with the wall?
- 21) Water is ejected from a fire hose at a rate of 25 kg s<sup>-1</sup> and a speed of 50 m s<sup>-1</sup>. If the water hits a wall calculate the average force exerted on the wall. Assume that the water does not rebound from the wall.
- 22) A rocket burns fuel at a rate of 50 kg per second, ejecting it with a constant speed of 1800 m s<sup>-1</sup>. Calculate the force exerted on the rocket.
- 23) Describe in detail an experiment which you would do to determine the average force between a football boot and a football as it is being kicked. Draw a diagram of the apparatus and include all measurements taken and calculations carried out.
- 24) A 2 kg trolley travelling at 6 m s $^{\text{-}1}$  collides with a stationary 1 kg trolley.
  - a. If they remain connected, calculate:
    - i. their combined velocity
    - ii. the momentum gained by the 1 kg trolley
    - iii. the momentum lost by the 2 kg trolley.

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b. If the collision time is 0.5 s, find the force acting on each trolley.

### MOMENTUM & IMPULSE / TUTORIAL ANSWERS

### TUTORIAL 1

- 1. The other marble moves off at 5ms<sup>-1</sup>.
- 2. The original velocity of the 2kg vehicle is 3ms<sup>-1</sup>.
- 3. The remaining part of the pen moves off at 4ms<sup>-1</sup> in the other direction.

## TUTORIAL 2

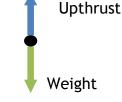
- 1. The stick receives an impulse of 3Ns.
- 2. a) The change in momentum is 520 kgms <sup>-1</sup>.
  - b) The new momentum of the runner is 120 kgms<sup>-1</sup>.
- 3. a) The momentum change is 6 kgms <sup>-1</sup>.
  - b) The average force on the ball is 4 N.
- 4. The player takes 2.5 s to stop.
- 5. The car receives an impulse of 6000 Ns.
- 6. The resulting change in momentum is  $5 \times 10^9$  kgms<sup>-1</sup>.
- 7. a) The momentum change involved is 15 kgms<sup>-1</sup>.
  - b) The average force acting on the object is 10 N.
  - c) The force gives an impulse of 15 Ns to the object.

## TUTORIAL 3

1. The climber reaches a speed of 4ms<sup>-1</sup>.

2.

a)



- b) Because she is *accelerating* and a change in velocity only occurs if there is a resultant force greater than zero.
- c) The resultant force is 400N
- d) The gravitational field strength is therefore 1.6Nkg<sup>-1</sup>.

3.

- a. 250 N @ 65° to the reference line (in the direction of zero thrust)
- b. The thrusters are applying a force of 50N
- 4. The force acting on the dummy is 1200N.
- 5. The electric force acting on it is  $2.7 \times 10^{-15}$  N.
- 6. The force exerted by horse 2 is 400N.

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7. 8 bags would be needed.

8.

- a. The velocity of the stick is 4ms<sup>-1</sup>.
- b. The momentum change of the puck is 4 kgms<sup>-1</sup>.

9.

- a. The force acts for 2s.
- b. The body receives an impulse of 20 kgms<sup>-1</sup>.

10.

- a. The velocity of the full wagon is 1ms<sup>-1</sup>.
- b. As the coal slides over the bed of the wagon, the horizontal friction between them acts to slow the wagon and speed up the coal.
- 11. a) The astronaut moves off with a velocity of 0.05 ms<sup>-1</sup> towards the ship.
  - b) It takes the astronaut 1800s to reach the ship.
  - c) He has 1437.5 seconds of breathing time remaining.
  - d) It is better for him to release 100g at a time.

12.

- a. Since energy is conserved, the kinetic energy of the pieces is equal to their previous kinetic energy plus the potential energy stored in the explosive.
- b. Since momentum is conserved, the total momentum of the pieces is the same as the total momentum before the explosion.
- 13. a) The velocity of the balls after impact is 2.6 ms <sup>-1</sup>. 7.2J of kinetic energy are lost.
  - b) The velocity of the balls after impact is 0.6 ms<sup>-1</sup>. 39.2J of kinetic energy are lost.
- 14 The force of friction is 10N opposing the direction of motion.
- 15. 10 600J of energy is turned to heat.
- 16. b)ii) The pellet strikes the putty at 100.5 ms<sup>-1</sup> in the direction shown.

## **COURSE QUESTION ANSWER**

- (a) 0.4 m s-1
- (b) inelastic since Ek changed
- (c) (i) F = 240 N
  - (ii) Same in magnitude but in opposite direction [must state both] {1}
  - 2. (a) F = 30 kN
  - (b) (i) 1050 Ns

(ii) Y force larger since same change in momentum occurs in smaller time

## MOMENTUM AND IMPULSE

- 1. a) 20 kg m s<sup>-1</sup> to the right b) 500 kg m s<sup>-1</sup> downwards c) 9 kg m s<sup>-1</sup> to the left
- 2. 0.75 m s<sup>-1</sup>
- 3. 2.4 m s<sup>-1</sup>
- 4. 3 kg
- 5. a) 2.7 m s<sup>-1</sup> b) 0.19 J
- 6. 8.6 m s<sup>-1</sup>
- 7. a) 23 m s<sup>-1</sup>
- 8. 8.67 m s<sup>-1</sup>.
- 9. 0.6 m s<sup>-1</sup> as initial direction of second vehicle
- 10. 16.7 m s<sup>-1</sup>
- 11. 4 kg
- 12. 0.8 m s-1 in opposite direction
- 13. 1.3 m s<sup>-1</sup>
- 14. 1.58 m s<sup>-1</sup>
- 15. 100 N
- 16. 0.03 s
- 17. 2.67 m s<sup>-1</sup>
- 18. a) -0.39 kg m s<sup>-1</sup> b) 15.6 N
- 19. a) 4 m s-1, 3 m s<sup>-1</sup> b) 0.1 s
- 20. b) 0.2 s c) 20 N d) 4.0 J
- 21. 1250 N
- 22.  $9 \times 10^4 \text{ N}$
- 23.
- 24. a) i)  $4 \text{ m s}^{-1}$  ii)  $4 \text{ kg m s}^{-1}$  iii)  $4 \text{ kg m s}^{-1}$  b) 8 N

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