Equations
$$\frac{Mean}{Average} = \frac{Sum\ of\ the\ results}{No.of\ observations}$$
$$Random\ Uncertainty = \frac{Max - Min}{No.of\ observations}$$
$$Percentage\ Uncertainty = \frac{Random\ Uncertainty}{Mean} \times 100$$
$Analogue\ scale\ reading\ uncertainty \pm\ half\ the\ least\ division\ of\ the\ scale.$ $\%scale\ reading\ uncertainty\ in\ Analogue\ device \pm \frac{half\ the\ least\ division\ of\ the\ scale.}{reading} \times 100\%$
Digital scale reading uncertainty ± 1 in the least significant digit displayed. $\%scale\ reading\ uncertainty\ in\ digital\ device \pm \frac{1\ in\ the\ least\ significant\ digit.}{reading} \times 100\%$

Key Words	Meaning
Uncertainty	Measurement of any physical quantity is liable to error and we are therefore uncertain of the exact value. Repeated measurements of a physical quantity are desirable.
Error	A word often used in place of uncertainty. In Physics we mean a mistake when we refer to an error. Uncertainties are not mistakes.
Accuracy	Results which are accurate have a mean value very close to the correct value or a standard. Accurate results may be spread over a wide range. A result can be accurate but not precise.
Precision	Results which are precise will be spread over a very small range. This does not mean they will be close to the real value. A result can be precise but not accurate!
SI Units	The International System of Units (SI) states the 7 base units that must be used in all calculations. These are: second (time), metre (distance), kilogram (mass), ampere (current), Kelvin (temperature), mole (amount of a substance), and candela (light intensity). All other units are derived from these 7. https://www.npl.co.uk/si-units
Mean average	Mean is the type of average used in physics and equals the sum of the results/ no. of observations. The mean average is the best estimate of the 'true' value of the quantity being measured in an experiments. Each measurement should be taken a number of times and a mean average found.
Random Uncertainties	Random uncertainties occur when an experiment is repeated and slight variations occur. Random uncertainties are present in every experiment. Random uncertainties can be reduced by taking repeated measurements. This is the reason we MUST repeat all measurements.
Why repeat measurements?	Truly random fluctuations in measurements average to zero, and so the way to remove them is to average a large number of measurements,
Scale Reading Uncertainties	The scale-reading uncertainty is a measure of how well an instrument can be read. These are never completely precise.
Scale reading uncertainty in Digital Device	The uncertainty in a digital scale (such as an electronic balance) can be taken as ±1 in the smallest significant figure displayed.
Scale reading uncertainty in Analogue Device	The uncertainty in an analogue scale (such as a metre stick) can be taken as ±½ in the smallest scale division.

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Systematic effects	Where a systematic effect is present the mean value of the measurements will be offset from the true value of the physical quantity being measured. Systematic uncertainties occur when readings taken are either all too small or all too large. They can arise due to measurement techniques or experimental design. A systematic effect is a type of uncertainty that skews all results by the same amount in the same direction. E.g. all values may be 1mm smaller than they should be.
Zero Error	This is the most common systematic effect. It occurs because the zero point of the device is not set perfectly. E.g. the end of a metre stick has worn away. This can be eliminated by starting all measurements from a different value on the metre stick.
Reducing Uncertainty	We can attempt to reduce the random uncertainty by taking multiple observations and calculating a mean from these observations. Up to a point the more measurements the better.
Percentage Uncertainty in a reading	$\boldsymbol{Percentage\ Uncertainty = \frac{Uncertainty}{Mean} \times 100\%}$
Overall Percentage Uncertainty	In an experiment where more than one physical quantity has been measured, the quantity with the largest percentage uncertainty this percentage uncertainty is often a good estimate of the percentage uncertainty in the final numerical result. The result should be in the form: "Value Unit ± percentage uncertainty %"
Absolute Uncertainty	This is the 'total' uncertainty in an experiment. A result including this should be in the form: "(Value ± absolute uncertainty) unit"

WHAT IS THE DIFFERENCE BETWEEN ACCURACY AND PRECISION?

Both accuracy and precision reflect how close a measurement is to an actual value, but they are not the same. Accuracy reflects how close a measurement is to a known or accepted value, while precision reflects how reproducible measurements are, even if they are far from the accepted value. Measurements that are both precise and accurate are repeatable and very close to true values.

SCALE READING UNCERTAINTY

Analogue scale reading uncertainty± half the least division of the scale.

$$\begin{matrix}\%scale\ reading\ uncertainty \\ in\ Analogue\ device\end{matrix} \pm \frac{half\ the\ least\ division\ of\ the\ scale.}{reading} \times 100\%$$

Digital scale reading uncertainty ± 1 in the least significant digit displayed.

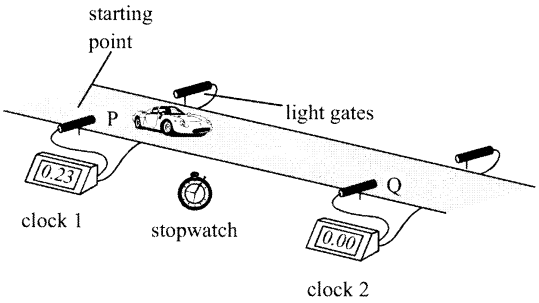
$$\begin{matrix}\%scale\ reading\ uncertainty \\ in\ digital\ device\end{matrix} \pm \frac{1\ in\ the\ least\ significant\ digit.}{reading} \times 100\%$$

So find the overall scale reading uncertainty in each result, find the approximate reading uncertainty in each measurement. The largest of these will give an indication of the overall uncertainty in your experiment.

Example

A student sets up the apparatus in the diagram to measure the average acceleration of a model car as it travels between P and Q

For one run, the following measurements were recorded along with their estimated uncertainty.
clock 1 reading = 0.23 s ± 0.01 s
clock 2 reading = 0.12 s ± 0.01 s
stopwatch reading = 0.95 s ± 0.20 s
length of car = 0.050 m ± 0.002 m
distance PQ = 0.30 m ± 0.01 m



The measurement which gives the largest percentage uncertainty is the	$t_1 = \frac{0.01}{0.23} = 4\ \%$
a) reading on clock 1	$t_2 = \frac{0.01}{0.12} = 8\ \%$
b) reading on clock 2	$t = \frac{0.20}{0.95} = 21\ \%$
c) reading on the stopwatch	$l = \frac{0.002}{0.050} = 4\%$
d) length of the car	$d = \frac{0.01}{0.30} = 3\%$
e) distance PQ.	

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During an experiment to measure the specific heat capacity of a liquid the relationship $VIt = mc\Delta T$ is used

The following quantities are measured
V= 12.0 ± 0.1 V
I = 4.2 ± 0.1 A
t =300 ± 1s
m=500± 2 g
ΔT = 15 ± 1 °C

$V = \frac{0.1}{12} = 0.8\%$

$I = \frac{0.1}{4.2} = 2\ \%$

$t = \frac{1}{300} = 0.3\%$

$m = \frac{2}{500} = 0.4\%$

$\mathrm{D}T = \frac{1}{15} = 7\%$

Which quantity will contribute the largest uncertainty to the final answer for the specific heat capacity, c?

A Voltage
B Current
C Time
D mass
E temperature difference

QUANTIFYING RANDOM UNCERTAINTIES

FIND THE MEAN

$$\text{mean value} = \frac{\Sigma\text{results}}{\text{no. of observation}}$$

This is the best estimate of the "true" value but not necessary the "true" value

FIND THE APPROXIMATE RANDOM UNCERTAINTY IN THE MEAN (ABSOLUTE UNCERTAINTY)

$$\text{approximate random uncertainty in mean} = \frac{\max value - \min value}{\text{no. of observation}}$$

$$\Delta R = \frac{R_{max} - R_{min}}{n}$$

This can be written as $= \frac{\text{range}}{\text{no. of observation}}$ **and it is sometimes referred to as *average deviation or absolute uncertainty*.**

stop here and quote the result with an approx. random uncertainty

MEAN±approx. random uncertainty (UNIT)

FIND THE PERCENTAGE UNCERTAINTY.

$$\text{percentage uncertainty} = \frac{\text{approx. random uncertainty}}{\text{mean value}} \times 100\%$$

$$\text{percentage uncertainty} = \frac{\frac{\text{range}}{\text{n}}}{\text{mean value}} \times 100\%$$

stop here and quote the result with a percentage uncertainty

MEAN (UNIT)± percerntage uncertainty (%)