Higher Physics

Our Dynamic Universe

Notes

Name……………………………………………………….

Key Area Notes, Examples and Questions

Other Notes

Key Area: Motion - Equations and Graphs

Previous Knowledge

- $s = \bar{v}t$ relationship between displacement, velocity and time.
- $a = \frac{v-u}{t}$ – Definition of acceleration.
- Properties of scalar and vector quantities.
  Scalar – a quantity with magnitude (size) only.
  Vector – a quantity with magnitude and direction
- Displacement is given by the area under a velocity time graph.
- Symbols and units of displacement, velocity, acceleration and time.
- How to find the gradient of a graph - $\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

Success Criteria

1.1 I can use the kinematic relationships

$$s = \bar{v}t \qquad v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

1.2 I can interpret displacement time graphs.

1.3 I can find velocity from the gradient of a displacement time graph.

1.4 I can interpret velocity time graphs.

1.5 I can describe an experiment to measure acceleration down a slope.

1.6 I can find acceleration from velocity time graph.

1.7 I can find displacement from a velocity time graph.

1.8 I can interpret acceleration time graphs.

1.9 I can identify and interpret motion time of objects travelling vertically under the influence of gravity and bouncing objects.

1.1 I can use the kinematic relationships $s = \bar{v}t$, $v = u + at$, $s = ut + \frac{1}{2}at^2$, $s = \frac{1}{2}(u + v)$ and $v^2 = u^2 + 2as$.

Motion in a straight line can be described using the following quantities; displacement, velocity, acceleration and time. The following give relationships between these quantities.

$s = \bar{v}t$ — Where $\bar{v}$ is the average velocity. This relationship is used when the acceleration is zero.

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

Where;

$t = \text{time (s)}$. The time interval between $\begin{cases} u = \text{Initial velocity } (\text{ms}^{-1}) \\ v = \text{Final velocity } (\text{ms}^{-1}) \end{cases}$

$s = \text{Displacement in the time interval (m)}$

$a = \text{The constant acceleration } (\text{ms}^{-2})$

These relationships only apply when we are dealing with a constant acceleration in a straight line.

Kinematic Relationships Background (not essential)

$s = \bar{v}t$

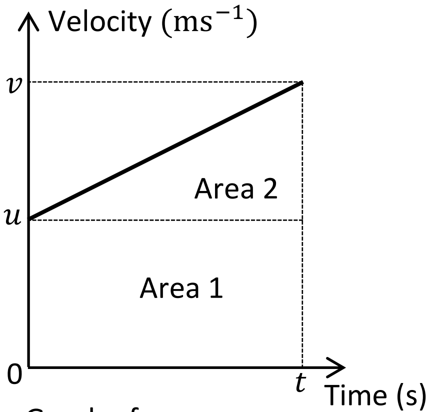
The definition of average velocity is $\bar{v} = \frac{s}{t}$ which rearranges to give $s = \bar{v}t$.

$v = u + at$

The definition of acceleration is $a = \frac{v-u}{t}$, which rearranges to give $v = u + at$

The at term in this relationship is the change in velocity.

$s = ut + \frac{1}{2}at^2$



Graph of an accelerating object.

Displacement = Area under the velocity time graph.

$s = \text{Area 1} + \text{Area 2}$

$$s = ut + \frac{1}{2}(v - u)t$$

Substituting in $v = u + at$ gives

$$s = ut + \frac{1}{2}(u + at - u)t$$

Which simplifies to $s = ut + \frac{1}{2}at^2$

This relationship consists of two parts

- ut which is the displacement that would occur if the acceleration was zero
- $\frac{1}{2}at^2$ which is the increase in displacement due to the acceleration.

$\boldsymbol{s = \frac{1}{2}(u+v)t}$

Substituting $a = \dfrac{v-u}{t}$ into $s = ut + \frac{1}{2}at^2$ gives

$$s = ut + \frac{1}{2}\left(\frac{v-u}{t}\right)t^2$$

$$s = ut + \frac{1}{2}vt - \frac{1}{2}ut$$

Which simplifies to $s = \frac{1}{2}(u+v)t$

This relationship is the mean velocity multiplied by time.

$\boldsymbol{v^2 = u^2 + 2as}$

Squaring the relationship $v = u + at$ gives

$$v^2 = u^2 + 2uat + a^2t^2$$

Taking a factor of 2a from the last two terms gives

$$v^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$$

As $s = ut + \frac{1}{2}at^2$ the term in brackets is s.

then $v^2 = u^2 + 2as$

Using the Kinematic Relationships

Example (all quantities positive)

A car initially travelling at 4.0ms^{-1} accelerates at 2.0ms^{-2} for 5.0s. Calculate how far the car travels in this time?

Solution

$s = ?$
$u = 4.0\text{ms}^{-1}$
$v =$
$a = 2.0\text{ms}^{-2}$
$t = 5.0\text{s}$

Write down "suvat" and take all the information from the question.
DO NOT SKIP THIS STEP.

$$s = ut + \frac{1}{2}at^2$$

$$s = 4.0 \times 5.0 + \frac{1}{2} \times 2.0 \times 5.0^2$$

$$s = 45\text{m}$$

- Select the kinematic relationship with the known quantities and the quantity you are required to find
- Substitute the known quantities into the relationship
- Write down the answer and unit to the correct number of significant figures.

Question book page 8 questions 1 and 2.

Example (Positive and negative quantities)

An arrow is fired vertically upwards. It reaches a maximum height of 65m.

a. State the velocity of the arrow at its maximum height?

b. Calculate the initial velocity of the arrow.

Solution

a. 0ms^{-1}

b. Positive upward

Define a positive direction. This is frequently upward in problems involving gravity.
The direction can be defined as upward or downward so long as all quantities are consistently given the correct positive and negative signs.

$s = 65\text{m}$
$u = ?$
$v = 0\text{ms}^{-1}$
$a = -9.8\text{ms}^{-2}$
$t =$

Negative as gravitational acceleration is downward.

$v^2 = u^2 + 2as$
$0^2 = u^2 + 2 \times (-9.8) \times 65$
$u^2 = 1274$
$u = 36\text{m}$

The positive square root is the correct answer since the arrow was launched upwards i.e. in the positive direction.

Question book page 8 questions 3 and 4.

Example (Two steps to find time)

A car is initially travelling at 25ms^{-1}. It decelerates at 1.5ms^{-2} over a distance of 50m. Calculate the time the car takes to travel the 50m distance.

Solution

Take the initial direction of motion as positive.

$s = 50\text{m}$
$u = 25\text{ms}^{-1}$
$v =$
$a = -1.5\text{ms}^{-2}$
$t = ?$

It seems that the relationship $s = ut + \frac{1}{2}at^2$ would be the correct one to choose. However, this would lead to a quadratic expression in t which would require a difficult solution using the quadratic formula. Use the simpler two step method shown below.

$v^2 = u^2 + 2as$
$v^2 = 25^2 + 2 \times (-1.5) \times 50$
$v = 21.8\text{ms}^{-1}$

Step one. Calculate the final velocity. Remember to carry an extra significant figure as this is an intermediate answer.

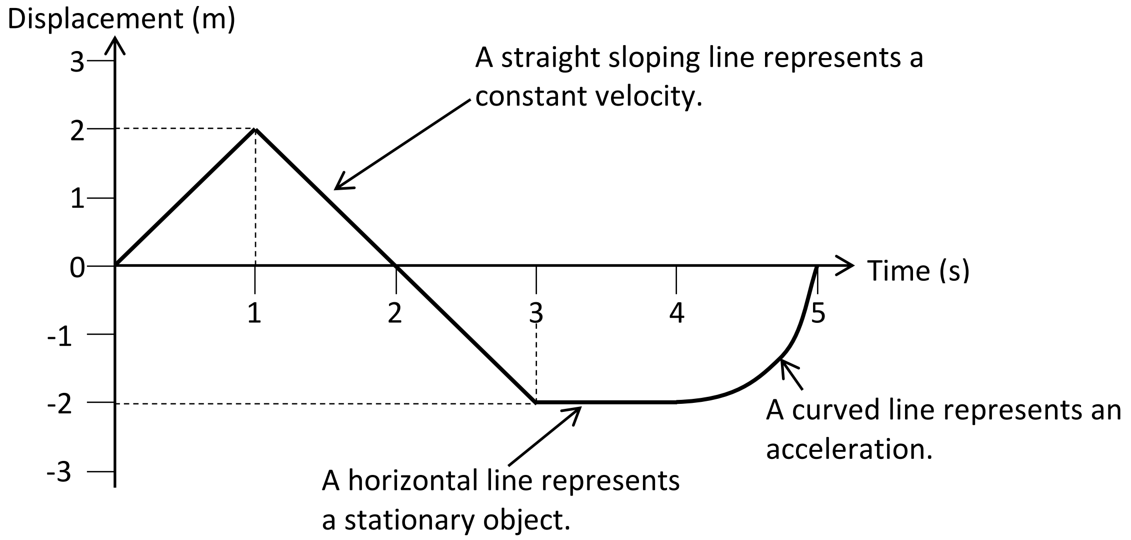
$v = u + at$
$21.8 = 25 + (-1.5)t$
$t = 2.1\text{s}$

Step two. Use the final velocity together with the other quantities to find time.

Question book page 8 questions 5 to 10. *Equations of Motion Homework*

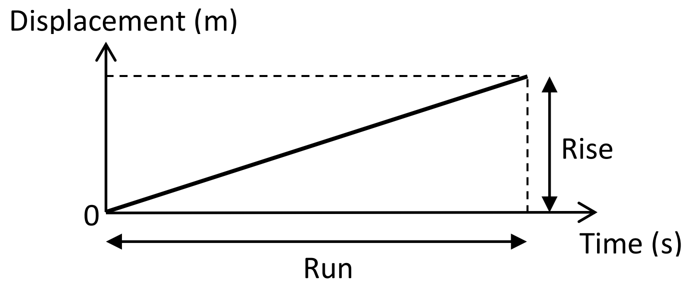
1.2 I can interpret displacement time graphs.

Displacement time graphs are used to present how the displacement of an object varies with time.



1.3 I can find velocity from the gradient of a displacement time graph.

Velocity can be found from a displacement time graph by taking the gradient of the line.

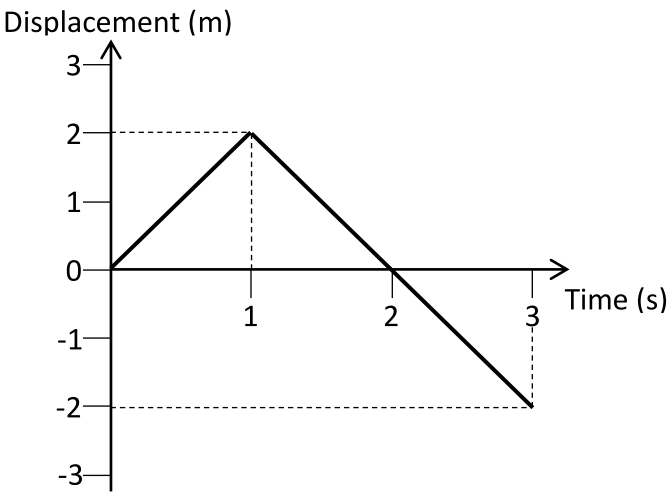


$$\text{velocity} = \text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example

a. Using the displacement time graph shown find the velocity of the object between
   i. 0s and 1s
   ii. 1s and 3s

b. Draw a velocity time graph for the time interval 0s to 3s.

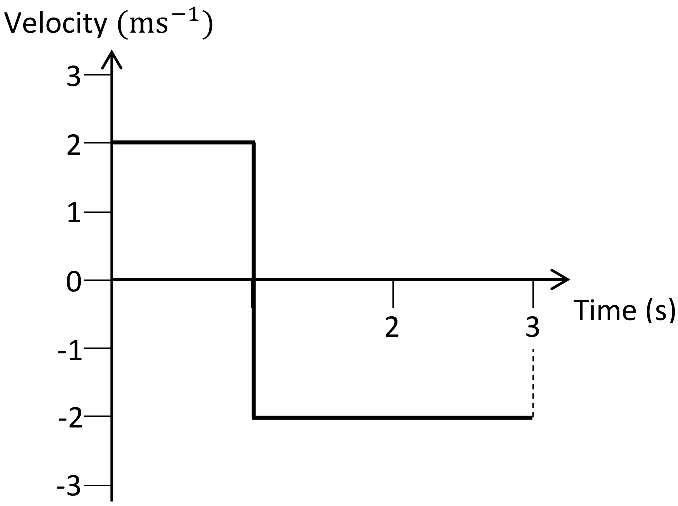


Solution

a.i. $\text{velocity} = \dfrac{\text{rise}}{\text{run}} = \dfrac{2-0}{1-0} = 2\text{ms}^{-1}$

ii. $\text{velocity} = \dfrac{\text{rise}}{\text{run}} = \dfrac{-2-2}{3-1} = -2\text{ms}^{-1}$

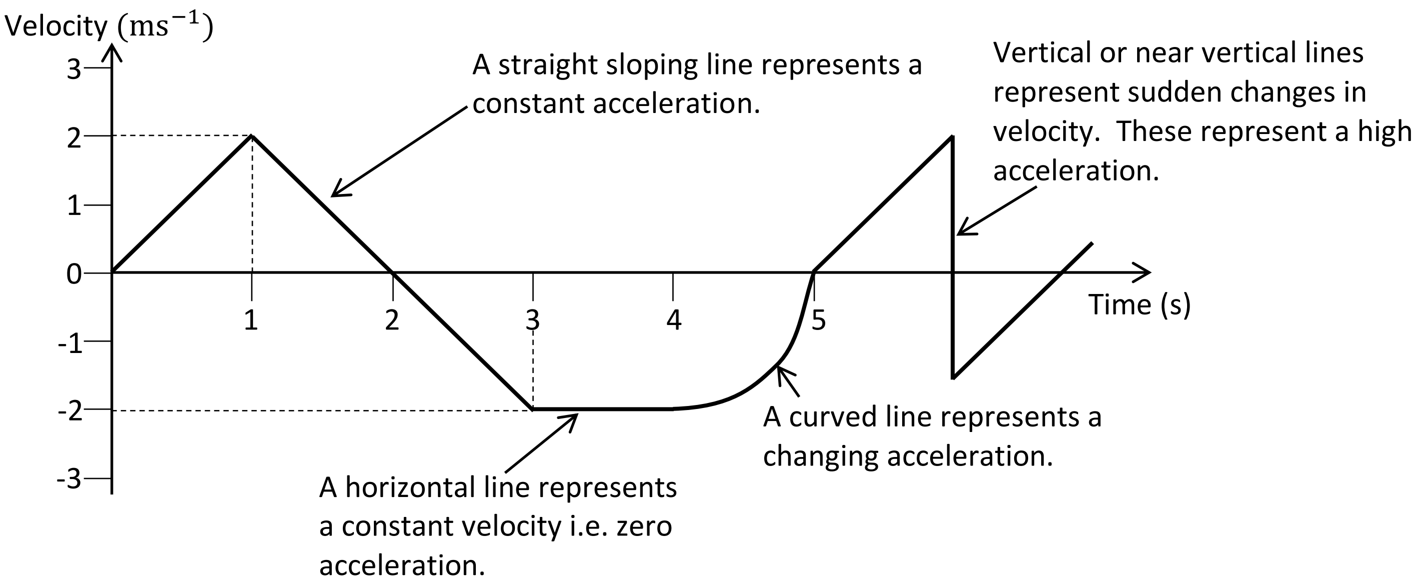
b.



Question book pages 10 to 12 questions 1 to 3.

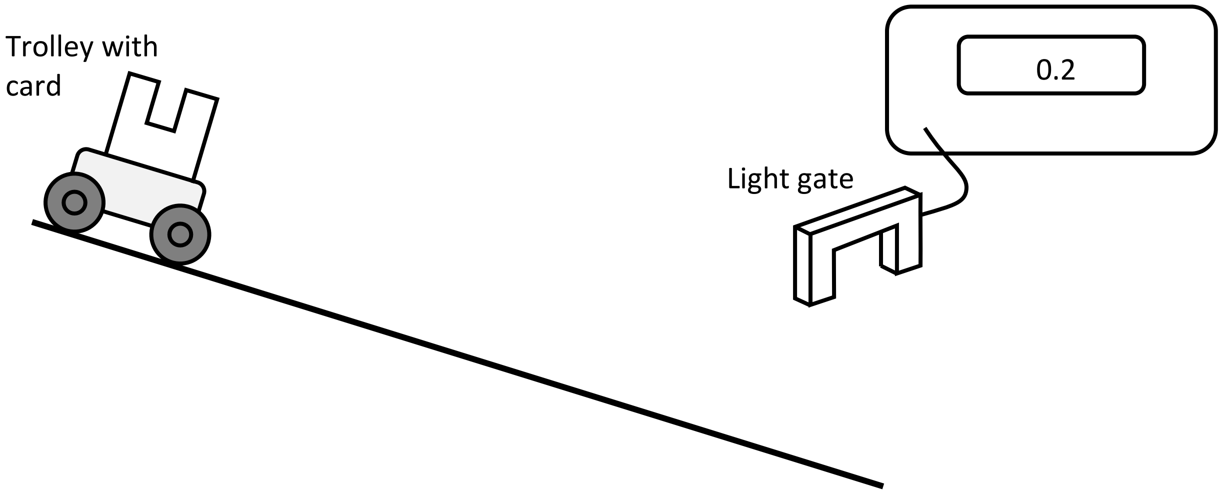
1.4 I can interpret velocity time graphs.

Velocity time graphs are used to present how the velocity of an object varies with time.



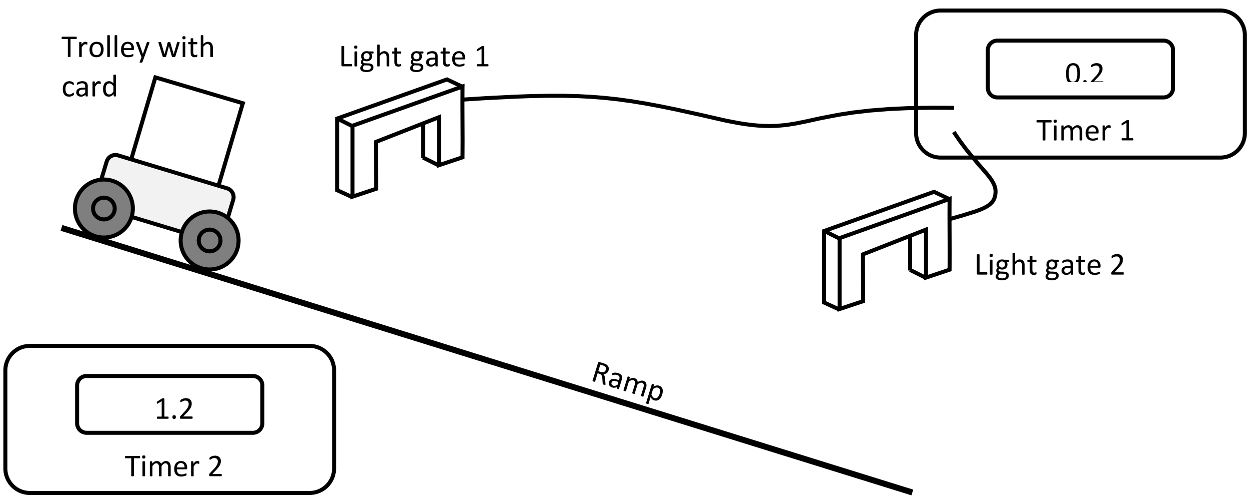
1.5 I can describe an experiment to measure acceleration down a slope.

Acceleration with a single light gate
The equipment is set up as shown below. The double slit on the card allows the data logger to measure two speeds and a time From this the data logger can calculate the acceleration. Allow the trolley to run down the slope through the light gate. The data logger will measure acceleration. This must be done at least five times and a mean value of acceleration calculated.



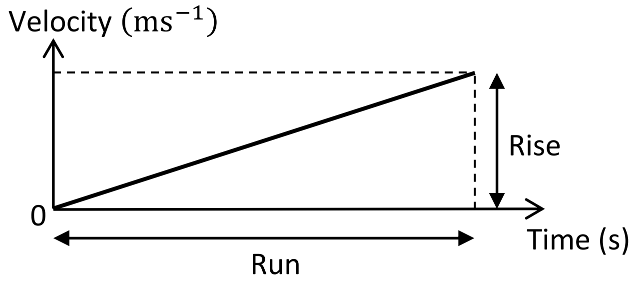
Acceleration with two light gates

- Measure the length of the card
- Run the trolley down through the light gates
- Record both times from timer 1
- Record the time from timer 2 (t_2)
- Use the length of the card and the times from timer 1 to calculate the speeds
- Use the relationship $a = \frac{v-u}{t_2}$ to find the acceleration.
- Repeat at least five times and calculate the mean value of acceleration



1.6 I can find acceleration from velocity time graph.

Acceleration can be found from a velocity time graph by taking the gradient of the line.

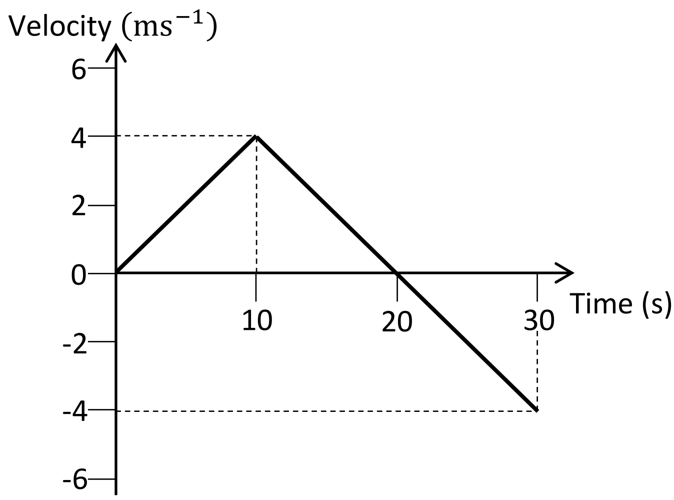


$$\text{Acceleration} = \text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example

a. Using the velocity time graph shown find the acceleration of the object between
   i. 0s and 10s.
   ii. 10s and 30s

b. Draw an acceleration time graph for the time interval 0s to 30s.

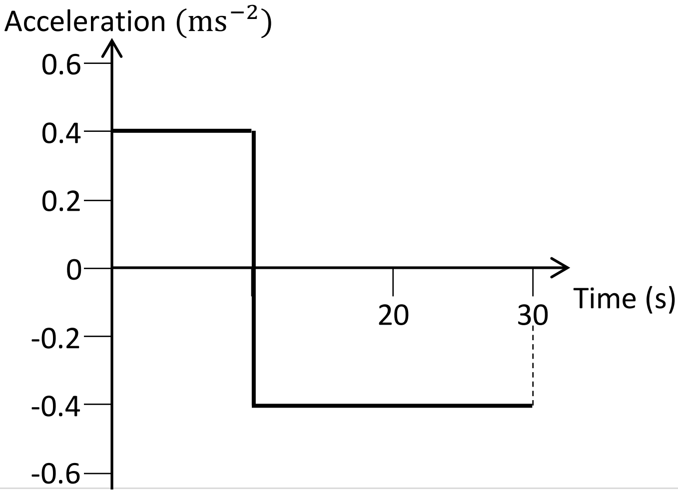


Solution

a.i. $\text{acceleration} = \frac{\text{rise}}{\text{run}} = \frac{4-0}{10-0} = 0.4\mathrm{ms}^{-2}$

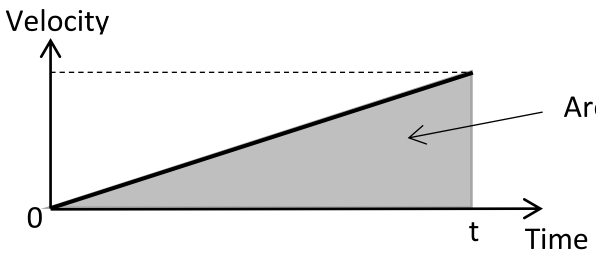
ii. $\text{acceleration} = \frac{\text{rise}}{\text{run}} = \frac{-4-4}{30-10} = -0.4\mathrm{ms}^{-2}$

b.



1.7 I can find displacement from a velocity time graph.

The area under a velocity time graph between the origin and time *t* gives the displacement of the object at time *t*.



If the area is below the time axis the area represents a negative displacement.

Example

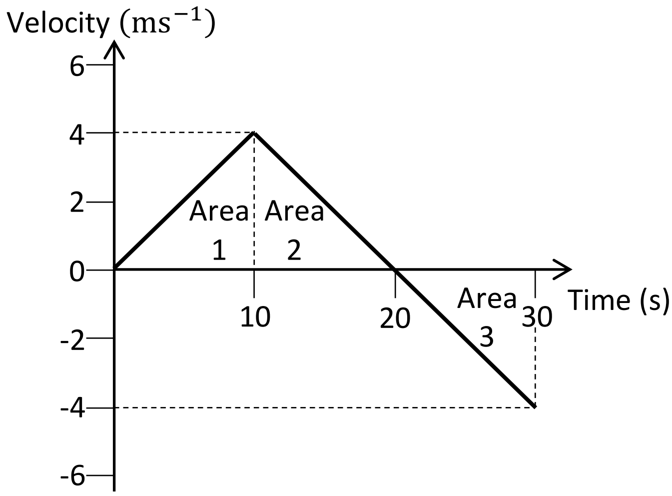
A velocity time graph for the motion of an object is shown.

Find

a. The displacement of the object after 10 seconds.

b. The displacement of the object after 30 seconds.

c. Draw the corresponding displacement time graph.



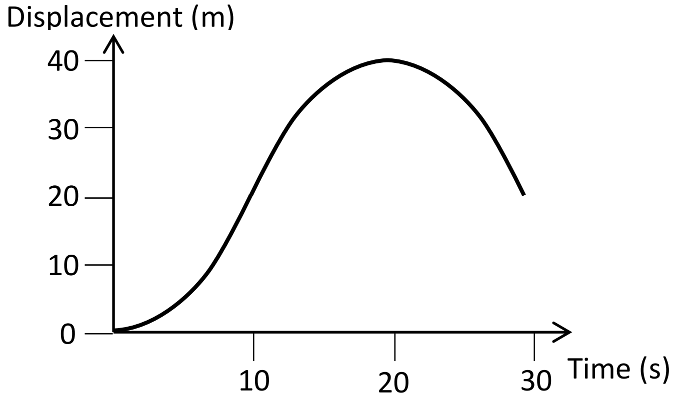
Solution

a. $\text{Displacement} = \text{Area under velocity time graph}$

$\text{Displacement} = \text{Area } 1 = \frac{1}{2} \times 10 \times 4 = 20\text{m}$

b. Displacement=Area 1+Area 2+Area 3=20+20+$\left(\frac{1}{2}\times 10\times(-4)\right)$=20m

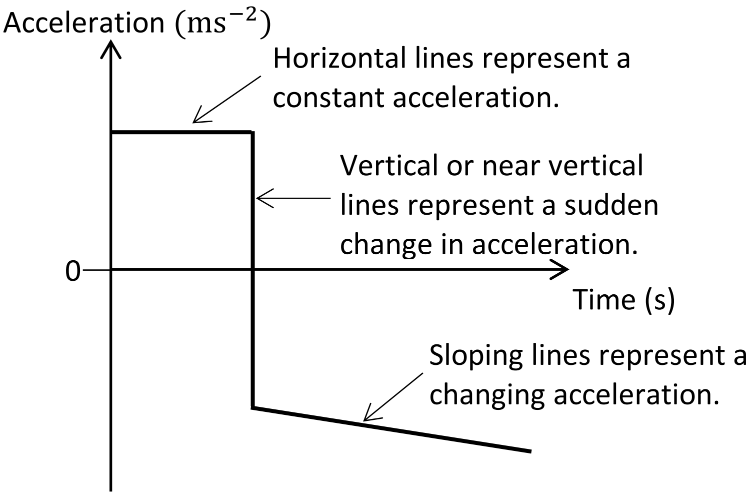
c. The displacement time graph is shown below.



Question book pages 13 to 15 questions 4 to 6.

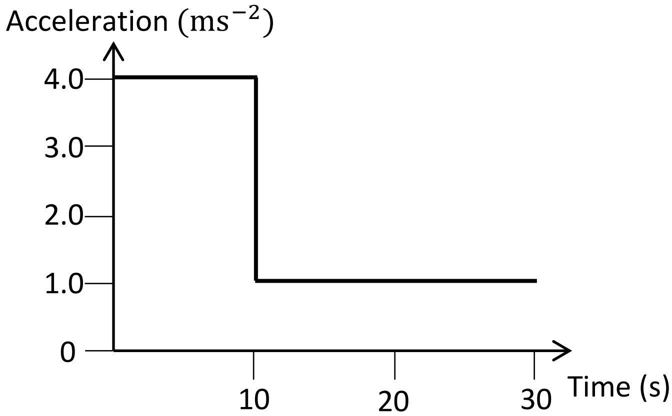
1.8 I can interpret acceleration time graphs.

Acceleration time graphs are used to present how the acceleration of an object varies with time.



Example

The graph below shows the acceleration time graph of an object starting from rest. Draw the corresponding velocity time graph.



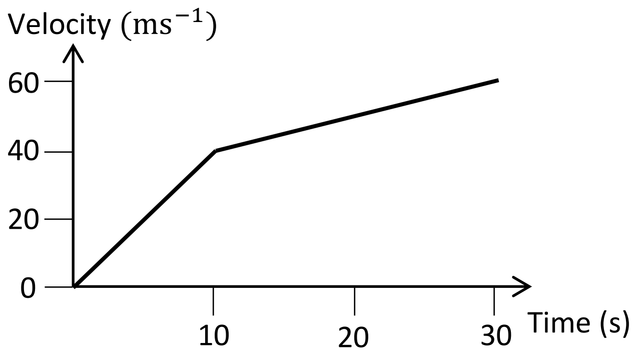
Solution

From 0s to 10s, $a = 4.0$ms^{-2}

After 10s, $v = u + at = 0 + 4.0 \times 10 = 40$ms^{-1}

From 10s to 30s, $a = 1.0$ms^{-2}

After 30s, , $v = u + at = 40 + 1.0 \times 20 = 60$ms^{-1}

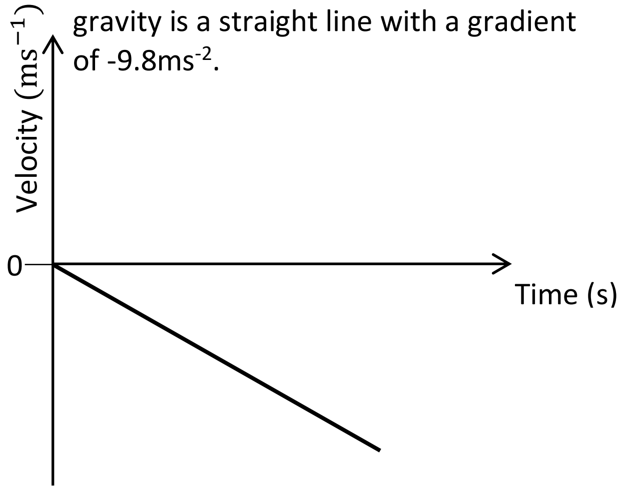


Question book pages 15 question 7.

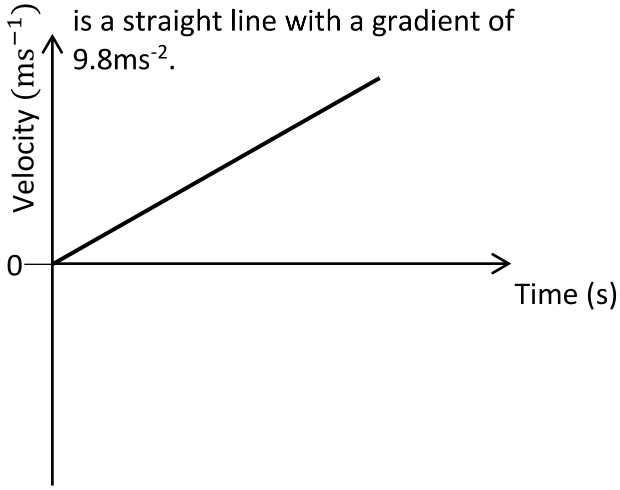
1.9 I can identify and interpret motion time graphs of objects travelling vertically under the influence of gravity and bouncing objects.

The appearance of velocity and acceleration time graphs of an object moving under the influence of gravity will be different depending on which direction is taken as positive.

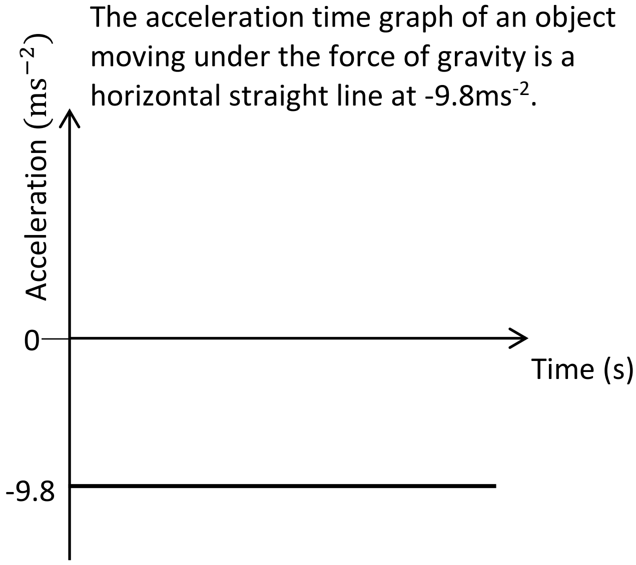
Taking **upwards** as the positive direction. The velocity time graph of an object moving under the force of gravity is a straight line with a gradient of $-9.8ms^{-2}$.



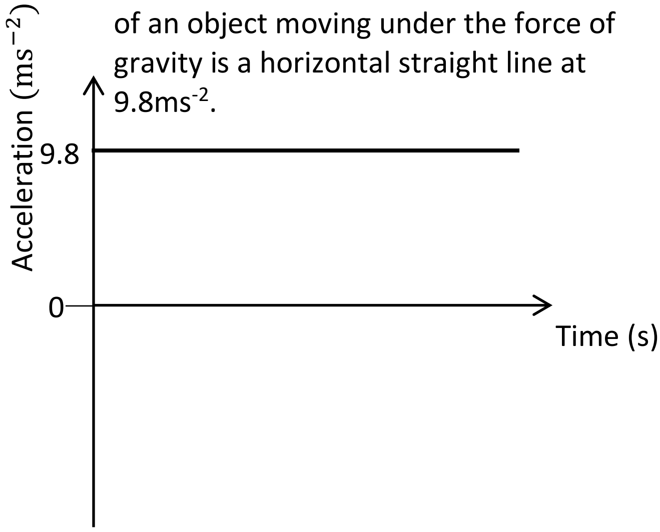
Taking **downwards** as the positive direction. The velocity time graph of an object moving under the force of gravity is a straight line with a gradient of $9.8ms^{-2}$.



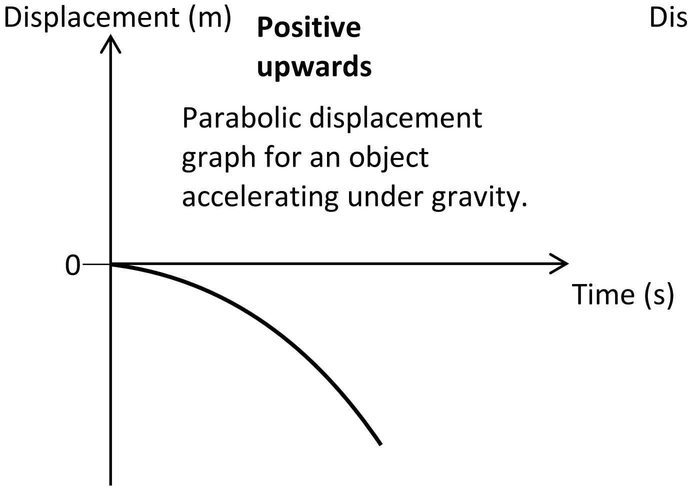
Taking **upwards** as the positive direction. The acceleration time graph of an object moving under the force of gravity is a horizontal straight line at $-9.8ms^{-2}$.

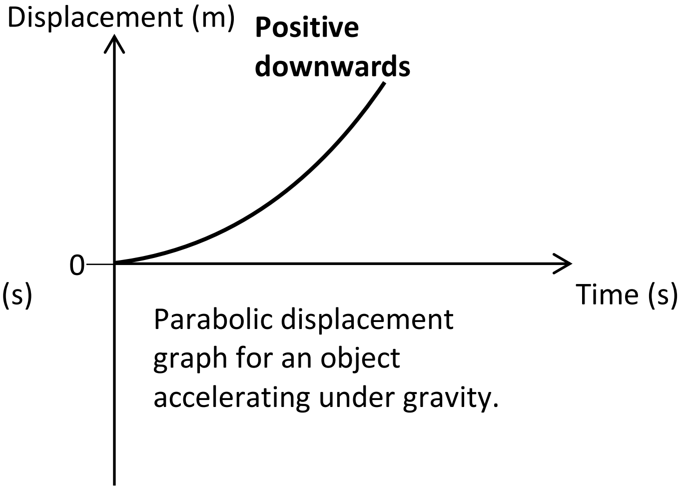


Taking **downwards** as the positive direction. The acceleration time graph of an object moving under the force of gravity is a horizontal straight line at $9.8ms^{-2}$.



Displacement time graphs of objects moving under the force of gravity are always curved.





Example

A ball is thrown vertically upwards, reaches a maximum height and then drops to the same initial height. For the balls motion sketch

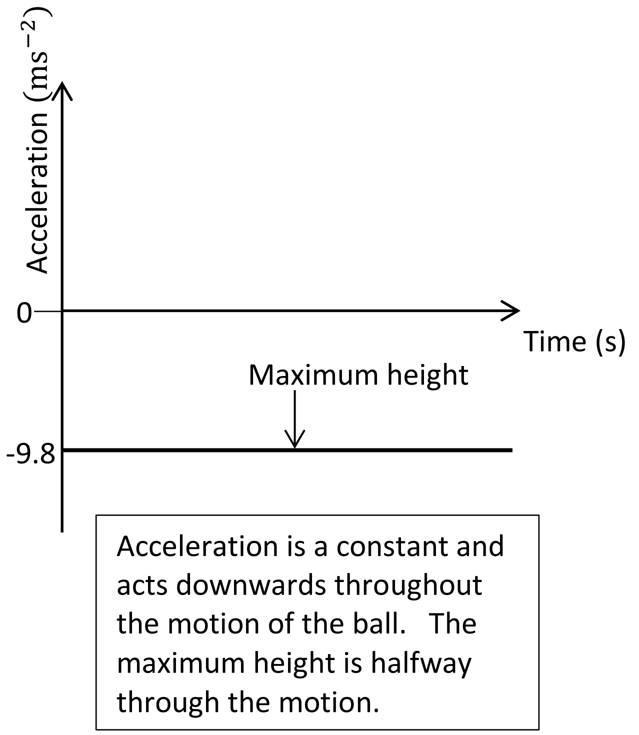
a. An acceleration time graph.

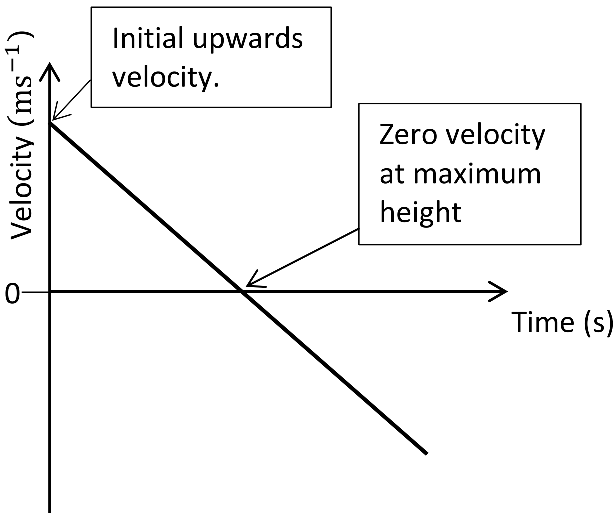
b. A velocity time graph.

c. On each graph label the point of maximum height.

Solution

Taking positive as being upwards.



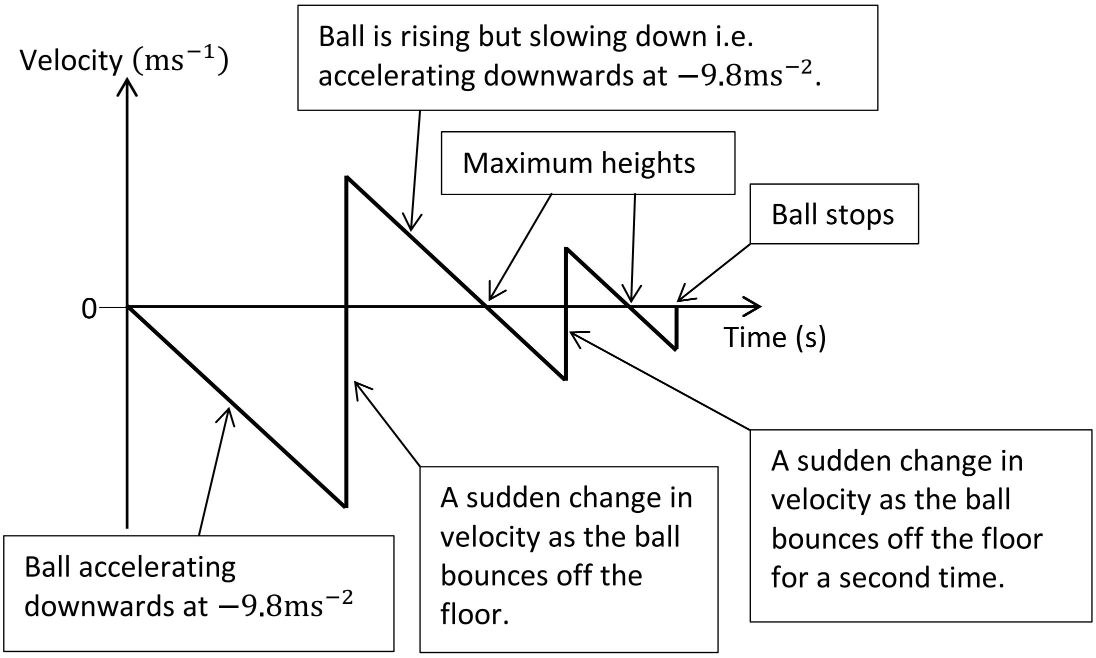


Example

A ball is dropped vertically onto a hard surface and bounces twice before stopping. Draw a velocity time graph for the ball's motion.

Solution

Taking positive as upwards.



Question book pages 16 to 17 questions 8 to 10
Graphs of motion Homework

Key Area: Forces, Energy and Power

Previous Knowledge

- Balanced and unbalanced forces – If the vector sum of the forces acting on an object is zero the forces are balanced if not they are unbalanced.
- Newton's 1st, 2nd and 3rd laws

  1st Law - An object will move with a constant speed in a straight line unless acted upon by an unbalanced force.

  2nd Law - $F_u = ma$ which relates the unbalanced force to the mass and acceleration.

  3rd Law - For every force there is an equal and opposite reaction force.
- How to calculate with sine and cosine.
- $W = mg$ - the relationship between mass, weight and gravitational field strength.
- Friction – The force which opposes motion.
- Energy and power relationships $W = fd$, $E_p = mgh$, $E_k = \frac{1}{2}mv^2$ and $P = \frac{E}{t}$

Success Criteria

2.1 I can use free body diagrams as an aid to solving problems involving forces.

2.2 I can solve problems involving balanced forces, unbalanced forces, mass, acceleration, gravitational field strength and friction.

2.3 I can explain terminal velocity in terms of the forces acting on an object.

2.4 I can draw and interpret a velocity time graph of a falling object taking air resistance into account.

2.5 I can analyse the motion of a rocket with constant thrust and a varying mass of fuel.

2.6 I can analyse the forces on and the motion of objects moving vertically.

2.7 I can find the internal forces on connected objects being accelerated, including friction.

2.8 I can analyse the forces on and motion of an object where the pulling force is exerted by a string or cable.

2.9 I can analyse the forces on and the motion of an object by resolving the force into two perpendicular components.

2.10 I can analyse the forces on and the motion of objects sliding down slopes.

2.11 I can use the principle of conservation of energy to solve problems involving work done, potential energy, kinetic energy and power.

2.1 I can use free body diagrams as an aid to solving problems involving forces.

The Free Body Diagram

A free body diagram shows the relevant forces acting on an object and nothing else. It is used to aid the understanding of the forces acting on the object and to find the unbalanced force.

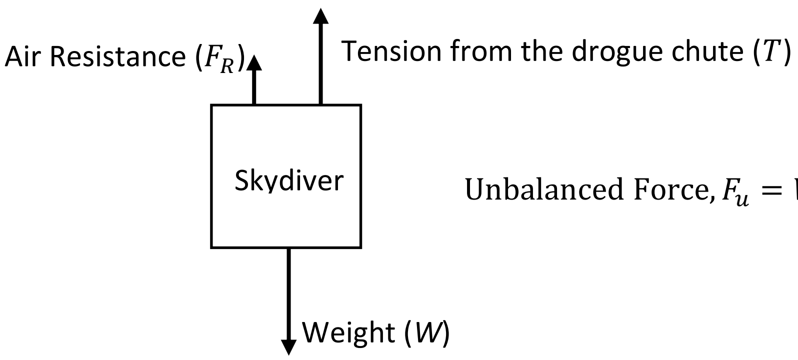
- Draw the object as a box
- Draw all the relevant forces as arrows approximately the correct scaled size and direction
- Find the unbalanced force by taking the vector sum of all the forces.

Example

a. Draw a free body diagram for a skydiver accelerating after they jump out of an aeroplane using a drogue chute to slow their descent.

b. Find and expression for the unbalanced force on the skydiver.

Solution



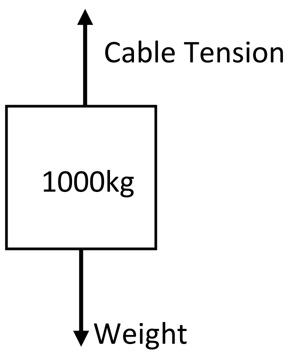
Unbalanced Force, $F_u = W - F_R - T$

2.2 I can solve problems involving balanced forces, unbalanced forces, mass, acceleration, gravitational field strength and friction.

Example with balanced forces

A load of bricks of mass 1000kg is being lifted by a crane at constant speed. Calculate the tension in the crane cable.

Solution



The bricks are moving at a constant speed so the forces are balanced.

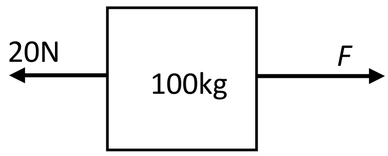
Cable Tension = Weight

Cable Tension $= W = mg = 1000 \times 9.8 = 9800\text{N}$

Example with an unbalanced force, friction and $\boldsymbol{F_u = ma}$

A cyclist of mass 100kg is accelerating at 1.0ms^{-2} along a level road. The total frictional force due to air resistance and rolling resistance is 20N. Calculate the forward force produced by the cyclist.

Solution



From the free body diagram the unbalanced force is given by

$F_u = F - 20$

Using Newton's second law

$F_u = ma = 100 \times 1.0 = 100\text{N}$

Equating the two expression for F_u gives

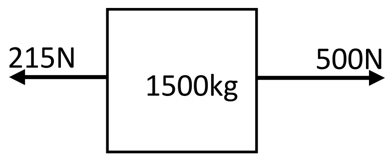
$F - 20 = 100$

$F = 120\text{N}$

Example with an unbalanced force, Friction, $\boldsymbol{F_u = ma}$ and SUVAT

In an acceleration test, a car of mass 1500kg passes a marker on a straight, level section of road at 20ms^{-1}. The engine is producing a constant force of 500N and there is a constant frictional force of 215N. Find the speed of the car 3.0s after the car passes the marker.

Solution



From the free body diagram

$F_u = 500 - 215 = 285\text{N}$

Using Newton's second law $F_u = ma$

$$a = \frac{F_u}{m} = \frac{285}{1500} = 0.19\text{ms}^{-2}$$

$s =$

$u = 20\text{ms}^{-1}$

$v = ?$

$a = 0.19\text{ms}^{-2}$

$t = 3.0\text{s}$

$v = u + at$

$v = 20 + 0.19 \times 3.0$

$v = 21\text{ms}^{-1}$

As the time and initial velocity are to two significant figures.

Question book pages 19 questions 1 to 3

Question book pages 20 questions 5 to 8

2.3 I can explain terminal velocity in terms of the forces acting on an object.

Terminal velocity occurs when the forces acting on an object in the direction of motion are balanced by the frictional forces acting on the object.

Example

Two parachutists each of mass 75kg jump from a plane. One has a parachute of area $30 m^2$ and the other $20 m^2$. Describe in terms of forces the motion of each of the parachutists after they leave the plane.

Solution

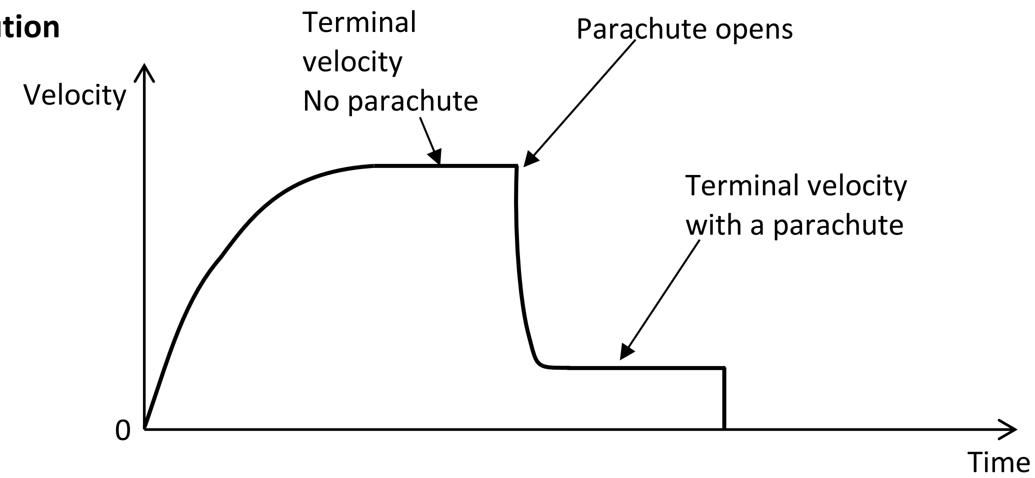
As the parachutists leave the plane, their weight will provide an unbalanced force causing them to accelerate downwards.
As their speed increases, the frictional force due to air resistance will increase until it balances their weight. They will then continue to fall at a constant terminal velocity.
When the parachutes are opened, there is a larger frictional force on each parachutist. They will both decelerate until the frictional force balances their weight. They will then continue to fall at a reduced terminal velocity.
The parachutist with the $20 m^2$ parachute will fall at a higher terminal velocity. Once the parachute is opened, the smaller area will only produce a sufficient frictional force to balance their weight at a higher speed than the parachutist with the $30 m^2$ parachute.

2.4 I can draw and interpret a velocity time graph of a falling object taking air resistance into account.

Example

Draw a velocity time graph of a parachutist jumping from a plane, reaching terminal velocity, opening their parachute then descending until they reach the ground. Mark on the graph points where the parachutist is moving at terminal velocity and the point where the parachute is opened.

Solution



Question book page 20 question 4

2.5 I can analyse the motion of a rocket with constant thrust and a varying mass of fuel.

Example

A rocket of mass 1000kg produces a constant thrust of 20,000N.

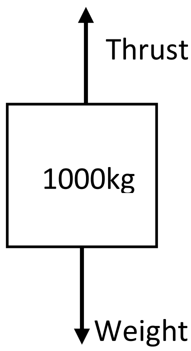
a. Calculate the initial acceleration of the rocket.

b. Sketch a velocity time graph of the rocket assuming air resistance is negligible and g is constant.

c. Explain the shape of the graph.

Solution

a.



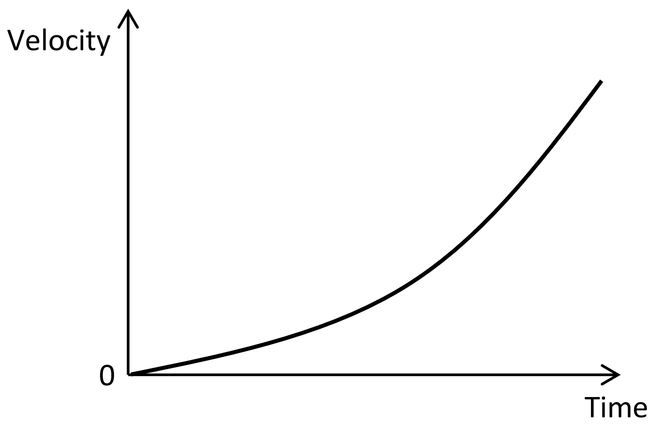
$W = mg = 1000 \times 9.8 = 9{,}800\text{N}$

$F_u = 20{,}000 - 9800 = 10{,}200\text{N}$

$a = \dfrac{F_u}{m} = \dfrac{10{,}200}{1{,}000} = 10.2\text{ms}^{-2}$

$a = 10\text{ms}^{-2}$ to two significant figures.

b.



c. The mass of the rocket will decrease as the fuel is used. From Newton's Second Law $a = \dfrac{F_u}{m}$ as the mass decreases the acceleration increases. As the acceleration is increasing the gradient of the line will increase.

Question book page 21 questions 9 and 10

2.6 I can analyse the forces on and the motion of objects moving vertically.

Example

A 60kg student is standing in a lift of mass 200kg. Calculate the tension in the lift cable when

a. the lift is stationary

b. the lift is moving downwards at a constant speed.

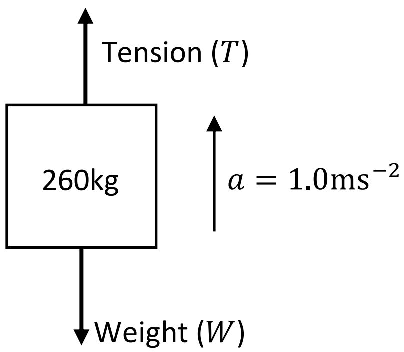
c. the lift is moving downwards and decelerating at 1.0ms^{-2}.

Solution

a. $T = W = mg = (60 + 200) \times 9.8 = 2548\text{N}$ (2500N to two significant figures)

b. 2500N as the lift is moving at a constant speed the forces are balanced.

c. Taking positive upwards.



$F_u = T - W = T - 2548$

and

$F_u = ma = 260 \times 1.0 = 260\text{N}$

Equating both expressions

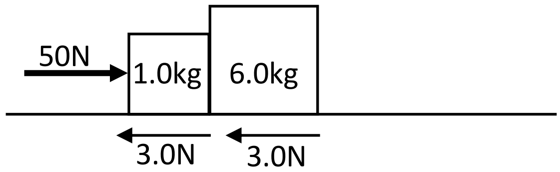
$T - 2548 = 260$

$T = 2808\text{N}$

Question book page 21 questions 11 to 15.

2.7 I can find the internal forces of connected objects being accelerated, including friction.

Example



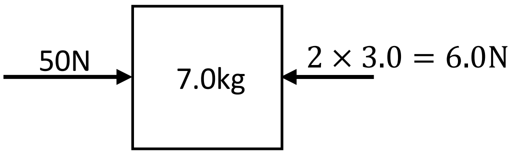
A student is moving two boxes of mass 1.0kg and 6.0kg. He pushes them along the floor by applying a 50N force to the 1.0kg box. There is a 3.0N frictional force on each box.

a. Calculate the acceleration of the boxes.

b. Calculate the force the 1kg block applies to the 6kg block.

Solution

a. Find the acceleration of the whole system

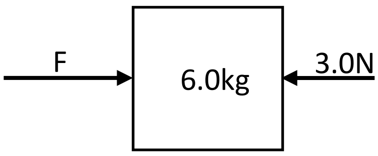


$$F_u = 50 - (2 \times 3.0) = 44\text{N}$$

$$a = \frac{F_u}{m} = \frac{44}{7.0} = 6.29\text{ms}^{-2}$$

$a = 6.3\text{ms}^{-2}$ to two significant figures.

b. Use the acceleration from part a. and the forces on the 6kg block to find the force from the 1.0kg block on the 6.0kg block.



$$F_u = F - 3.0$$

$$F_u = ma = 6.0 \times 6.29 = 37.7\text{N}$$

Equating both expressions

$$F - 3.0 = 37.7$$

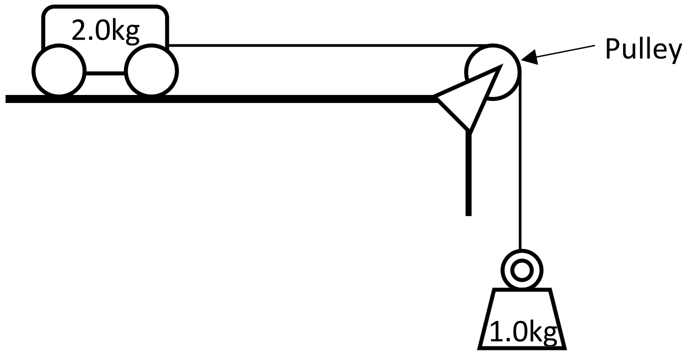
$$F = 40.7\text{N}$$

$F = 41\text{N}$ to two significant figures.

2.8 I can analyse the forces on and the motion of an object where the pulling force is exerted by a string or cable.

Example

A trolley of mass 2.0kg is accelerated by a mass of 1.0kg using the arrangement shown below. Calculate the acceleration of the trolley.



Solution

The force (weight) acting on the 1.0kg mass will accelerate both the trolley and the 1.0kg mass.

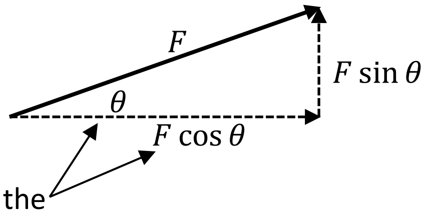
$$W = mg = 1.0 \times 9.8 = 9.8\text{N}$$

$$a = \frac{F_u}{m} = \frac{9.8}{(2.0 + 1.0)} = 3.3\text{ms}^{-2}$$

Question book page 23 and 24 questions 16 to 20.
Forces Homework

2.9 I can analyse the forces on and the motion of an object by resolving the forces into two perpendicular components.

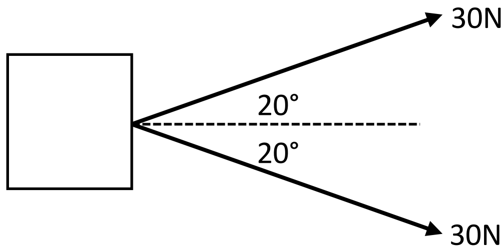
Any force can be resolved into two perpendicular components. As each of these components behave as if they are independent forces they can be analysed separately. This can simplify problems where forces are at an angle to each other.



The component with the angle is always the cosine.

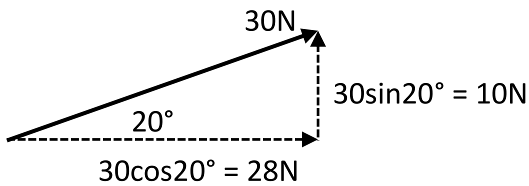
Example

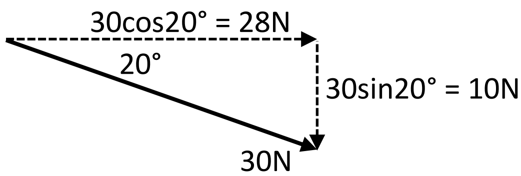
Resolve the forces below into horizontal and vertical components to find the resultant force on the object.



Solution

Split each force into a horizontal and a vertical component.

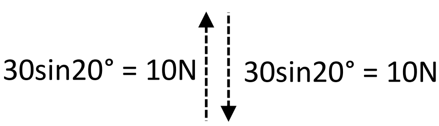




Horizontal components

These combine to give the single vector.

Vertical components



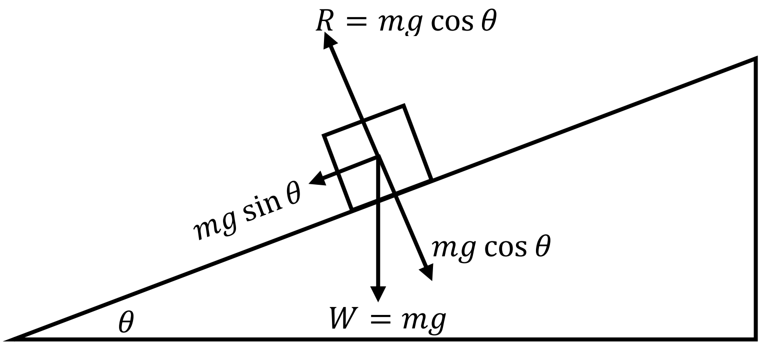
These vectors cancel.

Resultant force = 56N to the right.

Question book page 25 questions 4 and 5.

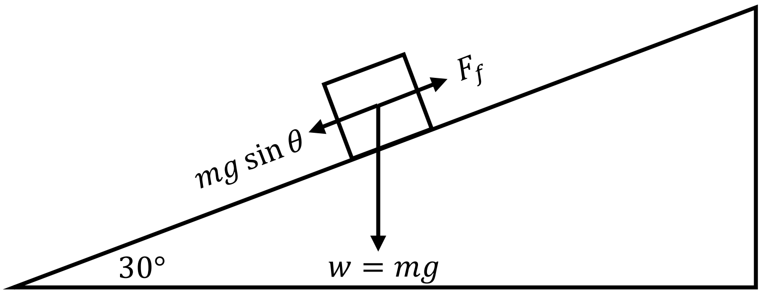
2.10 I can analyse the forces on and the motion of objects sliding down slopes.

The weight of the object on a slope can be resolved into two perpendicular components. The component parallel to the slope is $mg \sin\theta$. The component perpendicular to the slope is $mg \cos\theta$. The reaction force, R, will have the same magnitude as the perpendicular component of the weight.



Example - Constant speed with friction

A block of mass 3.0kg is sliding down a 30° slope at a constant speed. Find the magnitude of the frictional force on the block.



Solution

$F_f = mg \sin\theta$

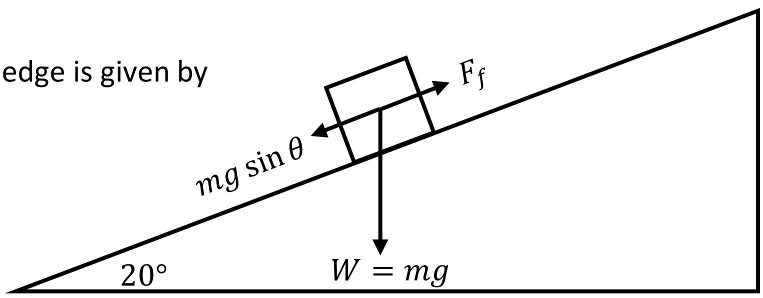
$F_f = 3.0 \times 9.8 \times \sin 30°$

$F_f = 14.7\text{N}$

Question book page 26 questions 6 and 7.

Example – acceleration with friction

A boy on a sledge of total mass of 55kg is sliding down a 20° slope. The sledge accelerates at 3.0ms^{-2}. Find the frictional force on the sledge.



Solution

The unbalanced force on the boy and sledge is given by

$F_u = mg \sin\theta - F_f$

And

$F_u = ma$

Equating both gives

$mg \sin\theta - F_f = ma$

$F_f = mg \sin\theta - ma$

$F_f = 55 \times 9.8 \times \sin 20° - 55 \times 3.0$

F_f =19N

Question book page 26 questions 8 and 9.

Forces at an Angle Homework.

2.11 I can use the principle of conservation of energy to solve problems involving work done, potential energy, kinetic energy and power.

The energy and power relationships are

Work Done	$W = fd$ or $E_w = fd$	Kinetic Energy	$E_k = \frac{1}{2}mv^2$
Gravitational Potential Energy	$E_p = mgh$	Power	$P = \frac{E}{t}$

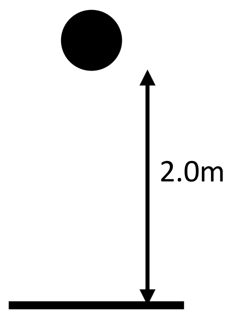
The principle of conservation of energy.

In an isolated system, energy can change from one form to another but its total value is always constant.

The principle of conservation of energy, the energy relationships together with the power relationships can be used to solve problems.

Example

A ball is dropped from a height of 2.0m and bounces to a height of 1.8m



a. Write down the energy changes as

i. the ball drops from 2.0m.

ii. the ball bounces on the ground.

iii the ball rises to 1.8m.

b. Calculate the speed of the ball as it hits the ground.

c. Calculate the speed of the ball as it leaves the ground.

Solution

a.i. Gravitational potential energy → kinetic energy

ii. Kinetic energy → kinetic energy, heat and sound.

iii. Kinetic → gravitational potential energy.

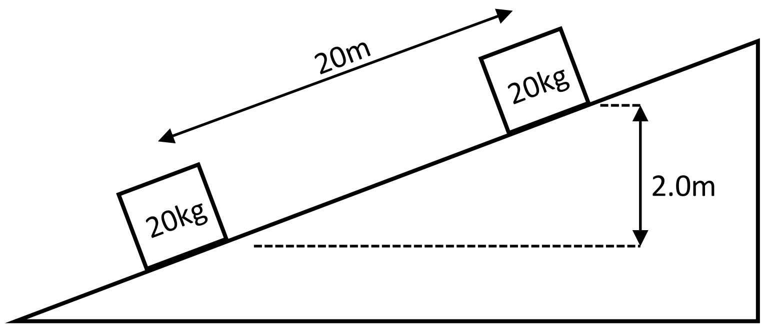
b. $E_p = mgh$ and $E_k = \frac{1}{2}mv^2$. As energy is conserved

$$mgh = \frac{1}{2}mv^2$$
$$gh = \frac{1}{2}v^2$$
$$v = \sqrt{2gh}$$
$$v = \sqrt{2 \times 9.8 \times 2.0}$$
$$v = 6.3\text{ms}^{-1}$$

c.
$$v = \sqrt{2gh}$$
$$v = \sqrt{2 \times 9.8 \times 1.8}$$
$$v = 5.4\text{ms}^{-1}$$

Example

A winch pulls a 20kg load 20.0m up a slope through a height of 2.0m in 10s. There is a frictional force of 20N on the box. Calculate the power of the winch.



Solution

Energy transferred from the winch $E = mgh + fd$

$$E = (20 \times 9.8 \times 2.0) + (20 \times 20)$$

$$E = 792\text{J}$$

Power $P = \dfrac{E}{t}$

$$P = \frac{792}{10} = 79.2\text{W}$$

$P = 79\text{W}$ to 2 significant figures

Question book pages 27 and 28 questions 1 to 6
Conservation of Energy Homework

Key Area: Collisions, Explosions and Impulse

Previous Knowledge

- Energy $E_k = \frac{1}{2}mv^2$
- Newton's Third Law.

Success Criteria

3.1 I can find momentum of an object using $p = mv$.

3.2 I can define the law of conservation of momentum.

3.3 I can use the law of conservation of momentum to solve problems involving collisions.

3.4 I can understand and the terms elastic and inelastic when applied to collisions.

3.5 I can determine whether a collision is elastic or inelastic by calculating the kinetic energy before and after the collision.

3.6 I can use conservation of momentum to solve problems involving explosions.

3.7 I can use Newton's third law to explain the motion of objects involved in interactions.

3.8 I can define impulse of a force as the average force on an object during an interaction multiplied by the time of interaction.

3.9 I can state that impulse is equal to the change in momentum of an object and use the relationship $Ft = mv - mu$ to solve problems.

3.10 I can state that impulse is equal to the area under a force time graph.

3.11 I can use information from a force time graph to solve problems.

3.12 I can interpret force-time graphs during the contact of interacting objects.

3.1 I can find momentum of an object using $p = mv$

The momentum of an object is defined as $p = mv$ Where p is momentum (kgms^{-1}).

Momentum is a vector quantity so has a magnitude and direction. It is a useful quantity when dealing with collisions between objects.

Question book page 29 question 1.

3.2 I can define the law of conservation of momentum.

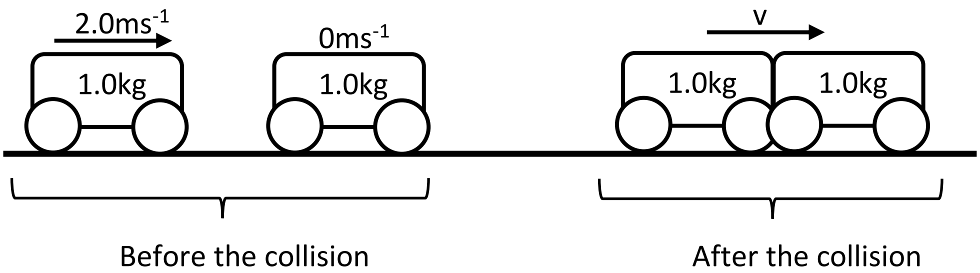
The Law of conservation of momentum states that "in the absence of external forces momentum before a collision is equal to the momentum after the collision".

Friction is an example of an external force. The Law of conservation of momentum cannot be used for problems where friction is a significant factor.

3.3 I can use the law of conservation of momentum to solve problems involving collisions.

Example

A trolley of mass 1.0kg moving at 2.0ms^{-1} collides with a stationary trolley of the same mass. Both trolley stick together. Calculate the velocity of the trolleys after the collision.



Solution

Define a direction as positive. Positive in this case is taken as to the right.
Find values or expressions for the momentum before and after the collision.

Before the Collision	**After the Collision**
$p = mv$	$p = mv$
$p = (1.0 \times 2.0) + (1.0 \times 0) = 2.0\text{kgms}^{-1}$	$p = (1.0 + 1.0)v$
	$p = 2.0v$

As momentum before the collision is equal to the momentum after the collision

$2.0v = 2.0$

So $v = 1.0\text{ms}^{-1}$ which is positive so is to the right.

Question book pages 29 and 30 questions 3, 4, 6, 8, 9.

3.4 I can understand and the terms elastic and inelastic when applied to collisions.

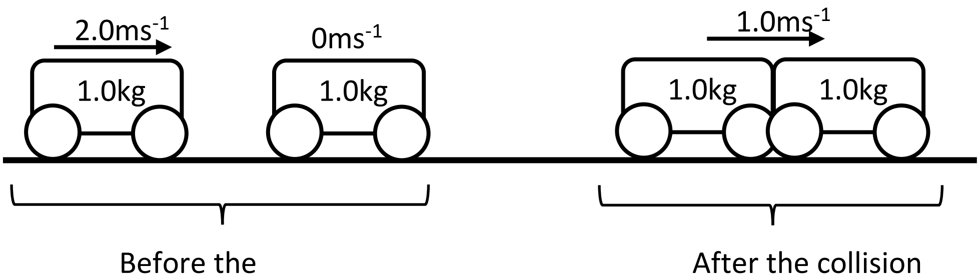
In an **elastic collision** the total kinetic energy of the objects before the collision **is equal** to the total kinetic energy after the collision.

In an **inelastic collision** the total kinetic energy of the objects before the collision **is not equal** to the total kinetic energy after the collision.

3.5 I can determine whether a collision is elastic or inelastic by calculating the total kinetic energy before and after the collision.

Example

A trolley of mass 1.0kg moving at 2.0ms^{-1} collides with a stationary trolley of the same mass. Both trolleys stick together and move off at 1.0ms^{-1}. Find whether the collision is elastic or inelastic.



Solution

Define a direction as positive. Positive in this case is taken as to the right.
Find values for the kinetic energy before the collision and after the collision

Before the Collision

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \left(\frac{1}{2} \times 1.0 \times 2.0^2\right) + \left(\frac{1}{2} \times 1.0 \times 0^2\right)$$

$$E_k = 2.0\text{J}$$

After the Collision

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2} \times (1.0 + 1.0) \times 1.0^2$$

$$E_k = 1.0\text{J}$$

As the total kinetic energy before the collision does not equal the total kinetic energy after the collision the collision is inelastic.

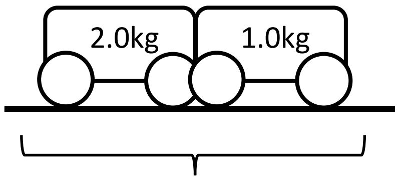
Question book pages 29 and 30 questions 2, 5 and 7.

3.6 I can use conservation of momentum to solve problems involving explosions.

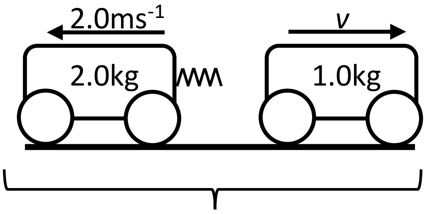
In an explosion the total momentum before and after the explosion is zero. Explosions are always inelastic as the kinetic energy is always higher after the explosion.

Example

Two stationary trolleys are held together with a compressed spring between them. They are released and spring apart. The left trolley moves off with a speed of $2.0ms^{-1}$. Calculate the speed of the right 1.0kg trolley.



Before the explosion



After the explosion

Solution

Before the Explosion	After the Explosion
$p = 0\text{kgms}^{-1}$ as the trolleys are stationary	$p = mv$ $p = \left(2.0 \times (-2.0)\right) + (1.0 \times v)$ $p = -4.0 + v$

As momentum before the explosion is equal to the momentum after the explosion.

$0 = -4.0 + v$

So $v = 4.0\text{ms}^{-1}$

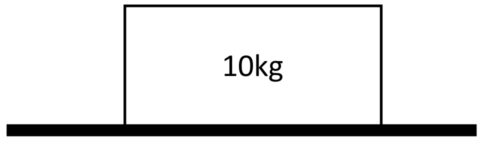
Question book page 31 questions 10 to 13.
Momentum Homework

3.7 I can use Newton's third law to explain the motion of objects involved in interactions.

Newton's Third Law – For every action there is an equal and opposite reaction.
This means that forces always occur in pairs, each acting in opposite directions.

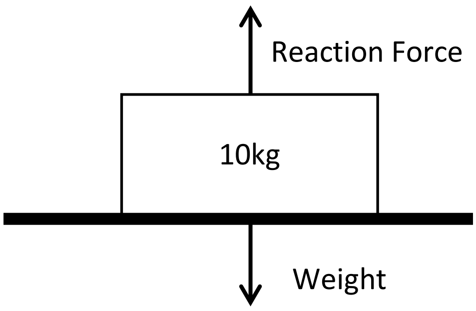
Example

A 10kg box is stationary on the floor. Explain why the weight of the box does not cause the box to accelerate downwards.



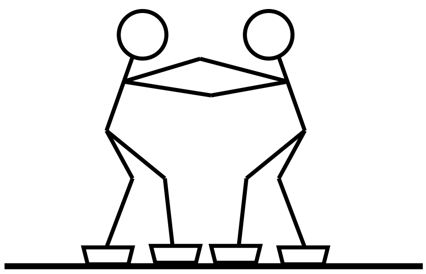
Solution

The weight of the object on the floor produces a small deformation of the floor. The deformation produces a reaction force in the opposite direction to the weight. This deformation increases until it produces a force which balances the weight. The unbalanced force on the box is zero so the box remains at rest.



Example

Two ice skaters are facing each other. The left skater pushes the right skater. Describe what happens to the motion of both skaters.



Solution

The left skater applies an unbalanced force to the right skater causing him to accelerate to the right.
Newton's third law states that there will be an equal and opposite reaction force on the left skater. This unbalanced force will cause the left skater to accelerate towards the left.

3.8 I can define impulse of a force as the average force on an object during an interaction multiplied by the time of interaction.

The impulse of a force is the average force multiplied by time.

$\text{impulse} = Ft$ Where F is the average force.

Impulse is a vector quantity with units of Newton seconds (Ns) and acts in the same direction as the force.

3.9 I can state that impulse is equal to the change in momentum of an object and use the relationship $\boldsymbol{Ft = mv - mu}$ to solve problems.

The impulse of a force is also given by the change in momentum during the time the force is applied

$\text{impulse} = Ft = mv - mu$ Where F is the average force.

Example

A 45g golf ball is struck with a club with an average force of 3800N and moves off with a velocity of 45ms^{-1}. For how long was the ball in contact with the club.

Solution

$u = 0\text{ms}^{-1}$
$v = 45\text{ms}^{-1}$
$F = 3800\text{N}$
$m = 45\text{g} = 0.045\text{kg}$
$t = ?$

$Ft = mv - mu$
$3800 \times t = (0.045 \times 45) - (0.045 \times 0)$
$t = 5.3 \times 10^{-3}\text{s}$

Example

A 200g hammer is used to drive nail into a wooden frame. The frictional force between the nail and the wood is 300N. If the hammer is in contact with the nail for 4.0ms, calculate the minimum impact speed required to hammer a nail into the wood. Assume the hammer comes to a complete stop when it hits the nail.

Solution

Taking positive in the same direction as velocity.

$u = ?$
$v = 0\text{ms}^{-1}$
$F = -300\text{N}$
$m = 200\text{g} = 0.200\text{kg}$
$t = 4.0\text{ms} = 4.0 \times 10^{-3}\text{s}$

$Ft = mv - mu$
$-300 \times 4.0 \times 10^{-3} = (0.200 \times 0) - (0.200 \times u)$
$u = 6.0\text{ms}^{-1}$

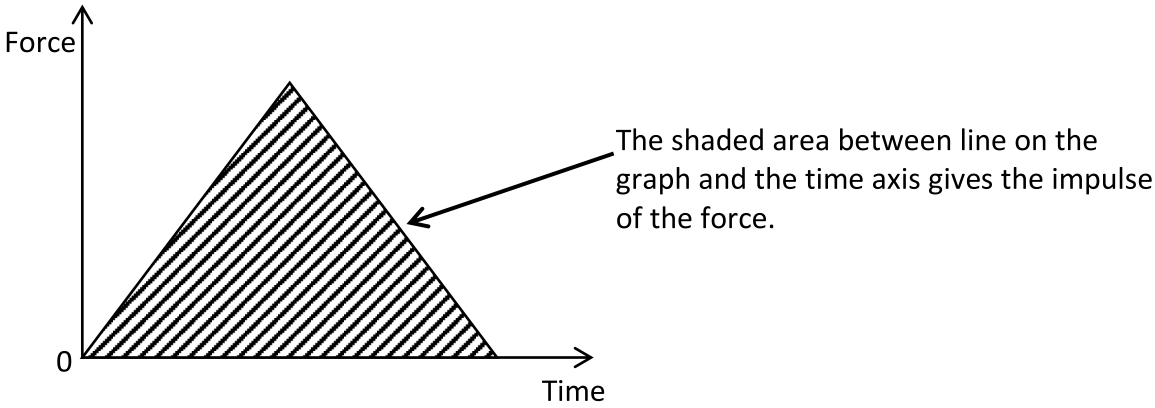
Note: F is negative as the force on the hammer causing it to decelerate will be in the opposite direction to the velocity.

Question book page 31 questions 14, 15, 16, 18, 19

3.10 I can state that impulse is equal to the area under a force time graph.

The impulse of a force is also given by the area under a force time graph.

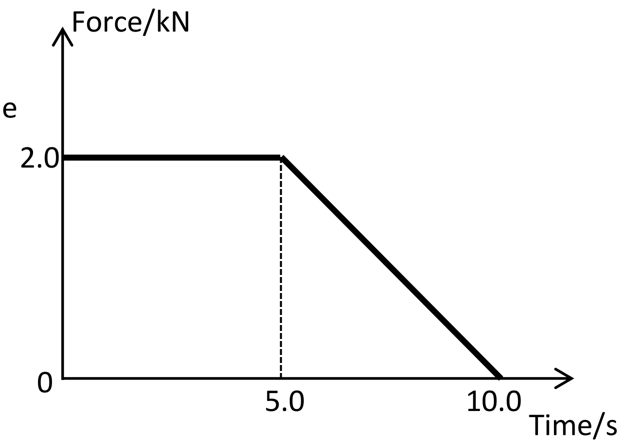
$$\text{impulse} = Ft = mv - mu = \text{Area under force time graph}$$



3.11 I can use information from a force time graph to solve problems.

Example

A car moving at 25ms^{-1} slows to 10ms^{-1}. The braking force applied during braking is shown on the force time graph shown. Find the mass of the car.



Solution

Impulse $= mv - mu =$ Area under force time graph

$m(v-u) =$ Area under force time graph

$$m(25-10) = (5.0 \times 2.0 \times 10^3) + \left(\frac{1}{2} \times 5.0 \times 2.0 \times 10^3\right)$$

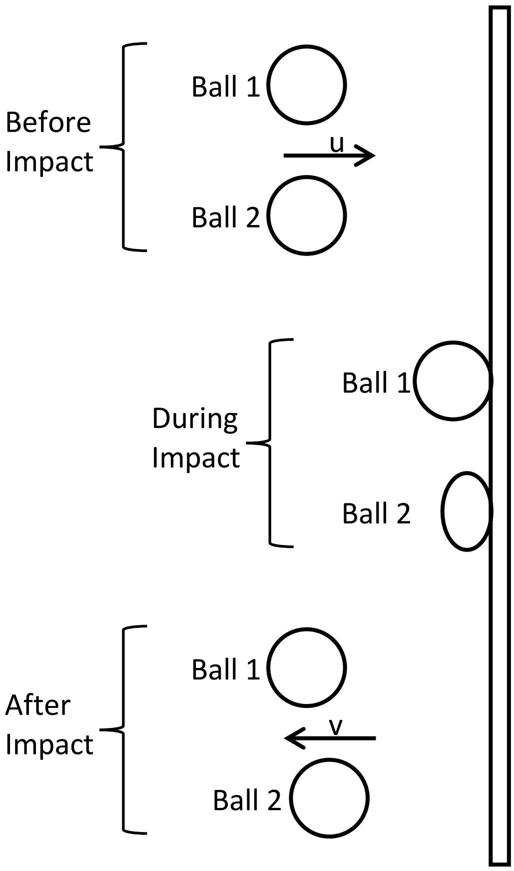
$$15m = 15000$$

$$m = 1000\text{kg}$$

3.12 I can interpret force-time graphs during the contact of interacting objects.

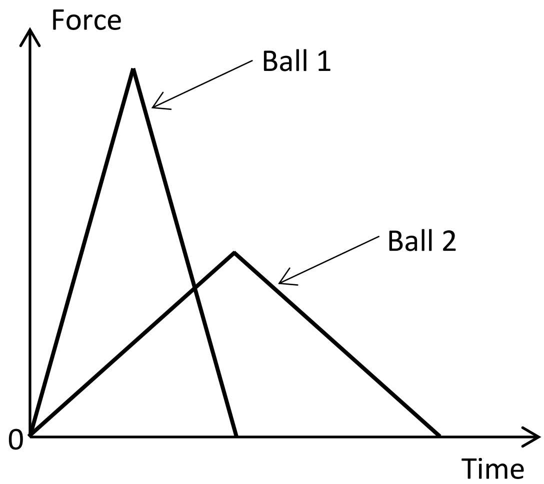
Example

Two balls of the same mass and diameter are thrown at a wall at the same velocity. Ball 1 is made of a harder material than ball 2. They both rebound with the same velocity. Sketch a force time graph for both balls during the time of contact with the wall.



Solution

As both balls have the same mass, initial velocities, final velocities and their change in momentum is the same, the area under the force time graph will be the same for both balls.
As ball 2 is softer than ball 1 its contact time will be longer. The peak force on ball 2 must be reduced to give the same area under the graph.



Question book page 31 question 17, page 32 question 20
Momentum and Impulse Homework

Key Area: Gravitation

Previous Knowledge

Kinematic relationships

$$s = \bar{v}t$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

How to split a vector into horizontal and vertical components.

Success Criteria

4.1 I can describe an experiment to measure the acceleration of a falling object.

4.2 I can state that projectiles will follow a parabolic path.

4.3 I can state that during projectile motion the only force acting on an object is the gravitational force.

4.4 I can split projectile motion into a horizontal component which has constant velocity and a vertical component which has constant acceleration.

4.5 I can solve problems involving projectile motion.

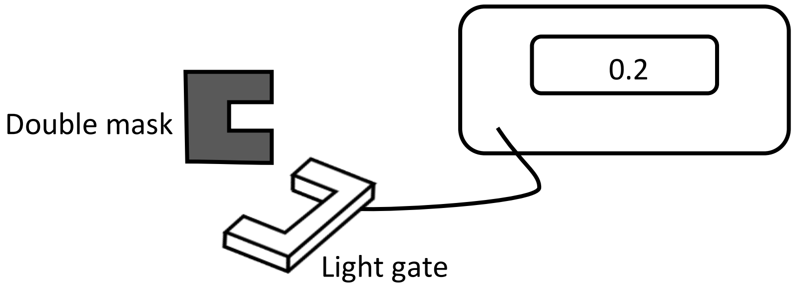
4.6 I can state that satellites are in free fall around a planet or star.

4.7 I can use of Newton's Law of Universal Gravitation to solve problems involving, force, masses and their separation.

4.1 I can describe an experiment to measure the acceleration of a falling object.

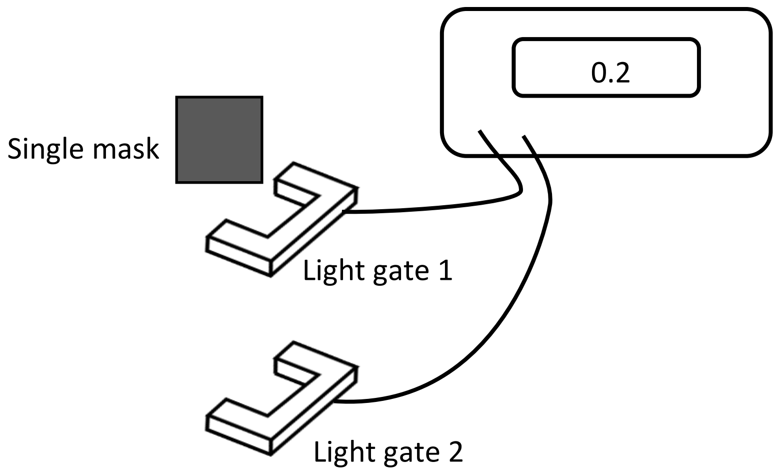
Acceleration with a single light gate

The equipment is set up as shown below. The double slit on the mask allows the data logger to measure two speeds and a time and from this it can calculate the acceleration. Allow the mask to fall through the light gate. The data logger will measure acceleration. This must be done at least five times and a mean value of acceleration calculated.



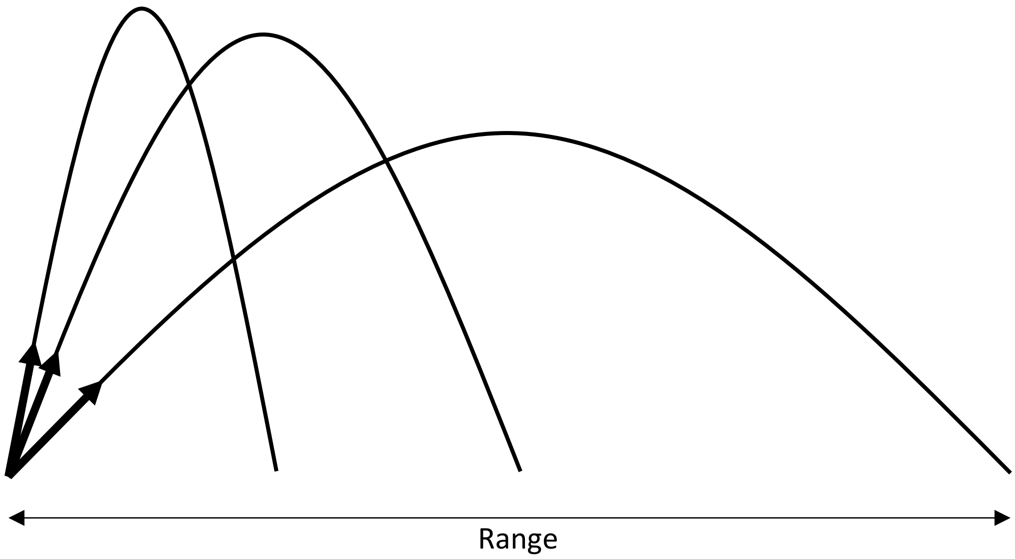
Acceleration with two light gates

- Measure the length of mask to be dropped
- Drop the mask through the light gates
- Record both times recorded from light gates 1 and 2.
- Use the length of the mask and the times from timer to calculate the speeds
- Use the relationship $a = \frac{v-u}{t_2}$ to find the acceleration.
- Repeat at least five times and calculate the mean value of acceleration



4.2 I can state that projectiles will follow a parabolic path.

An object launched at an angle other than vertically will follow a parabolic path. The maximum height and the range of the object's motion are determined by the initial velocity, its angle of launch and the acceleration due to gravity.



4.3 I can state that during projectile motion the only force acting on an object is the gravitational force.

The only force acting on a projectile is the gravitational force. When other forces are acting e.g. air resistance, thrust, lift etc. the motion will not be parabolic and so the object will not be a projectile. Objects moving through the atmosphere can be treated as a projectile only when their interaction with the air is small.

4.4 I can split projectile motion into a horizontal component which has constant velocity and a vertical component which has constant acceleration.

The motion of a projectile can be split into two components which can be dealt with separately;

- **A horizontal component** where the velocity is constant. This kinematic relationship $s = \overline{v}t$ applies to the horizontal motion.

- **A vertical component** where the acceleration is constant and equal to the acceleration due to gravity. The kinematic relationships $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$ and $s = \frac{1}{2}(u + v)t$ can be applied to the vertical motion.

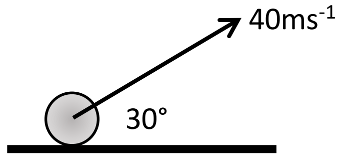
The horizontal and vertical displacement, velocity and acceleration are independent. However, at any point in the motion the horizontal and vertical times are the same.

Example

A golf ball is struck and moves off with an initial velocity of 40ms^{-1} at an angle of 30° to the horizontal. Find

a. The horizontal component of the velocity.

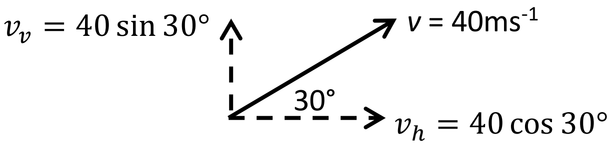
b. The vertical component of the velocity.



Solution

a. $v_h = 40\cos 30° = 35\text{ms}^{-1}$

b. $v_v = 40\sin 30° = 20\text{ms}^{-1}$

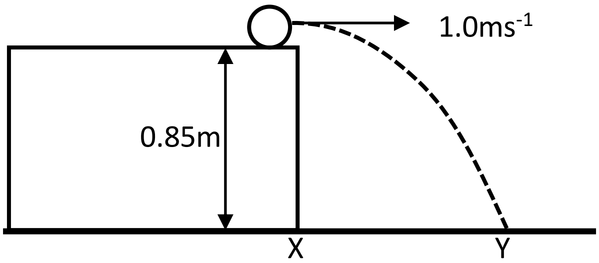


Question book page 35 question 4

4.5 I can solve problems involving projectile motion.

Example

A ball is projected horizontally from a 0.85m high bench at 1.0ms^{-1}. It leaves the bench above the point X and lands at the point Y. Find the distance XY



Solution

The horizontal and vertical motions can be treated separately. The time at which the ball leaves point X and lands at point Y must be the same for both components of the motion.

<u>Vertical Motion (positive upwards)</u>

Use the vertical component of the motion to find the time.

$s = -0.85\text{m}$

$u = 0\text{ms}^{-1}$

$v =$

$a = -9.8\text{ms}^{-2}$

$t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$-0.85 = 0 \times t + \frac{1}{2}(-9.8)t^2$$

$t = 0.416\text{s}$

Remember to use an extra significant figure as this is an intermediate answer.

<u>Horizontal Motion</u>

The horizontal velocity is constant

$s = \text{Distance XY}$

$\bar{v} = 1.0\text{ms}^{-1}$

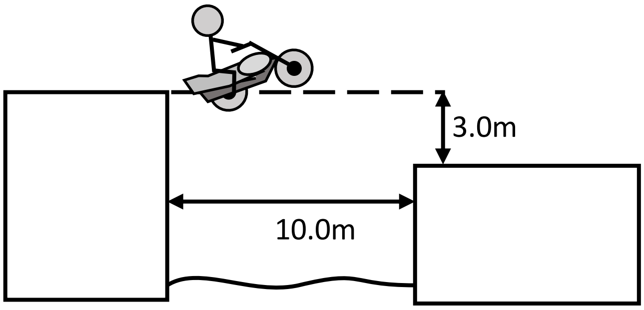
$t = 0.416\text{s}$

$s = 1.0 \times 0.416$

$s = 0.42\text{m}$

Example

A stuntman on a motorcycle is attempting to jump a river which is 10.0m wide. He needs to land on the edge of the far bank which is 3.0m lower than the bank from which he takes off. Calculate the speed the motorcyclist needs to travel at to jump the river.



Solution

The time at which he launches from one bank and lands on the other must be the same for both components of the motion.

<u>Vertical Motion (positive upwards)</u>

Use the vertical component of the motion to find the time.

$s = -3.0\text{m}$

$u = 0\text{ms}^{-1}$

$v =$

$a = -9.8\text{ms}^{-2}$

$t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$-3.0 = 0 \times t + \frac{1}{2}(-9.8)t^2$$

$t = 0.782\text{s}$

Remember to use an extra significant figure as this is an intermediate answer.

<u>Horizontal Motion</u>

The horizontal velocity is constant.

$s = 10.0\text{m}$

$\bar{v} = ?$

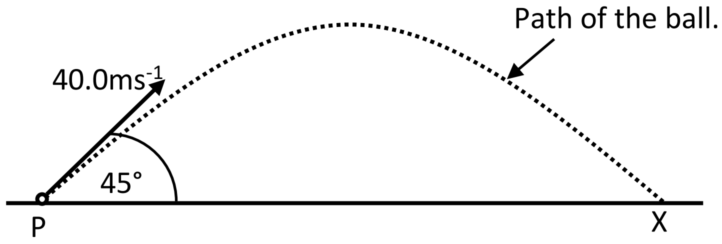
$t = 0.782\text{s}$

$10.0 = v \times 0.782$

$v = 13\text{ms}^{-1}$

Example

A golf ball is struck at point P at a velocity of 40.0ms^{-1} at an angle of 45° to the ground. Find the range of the golf ball from point P to where it lands at point X. Assume that air resistance is negligible.



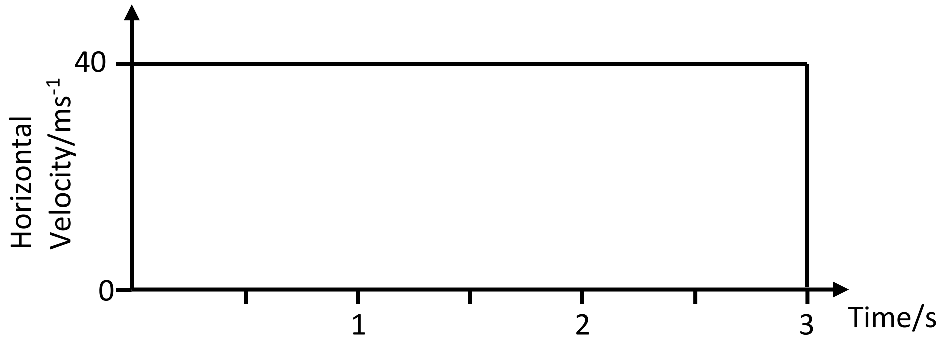
Solution

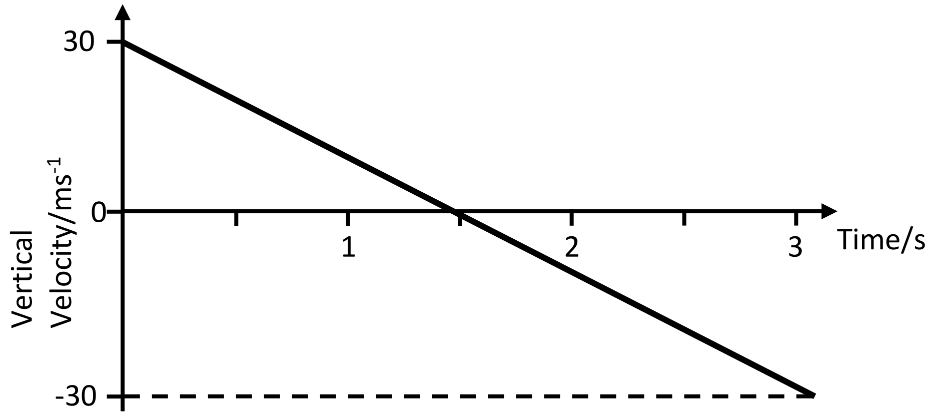
The easiest solution is to find the time the ball takes to reach its maximum height then double this to give the total flight time of the ball.

Vertical Motion (positive upwards)	Horizontal Motion
Use the vertical component of the motion to find the time. $s =$ $u = 40.0 \sin 45° = 28.3 \text{ ms}^{-1}$ $v = 0\text{ms}^{-1}$ $a = -9.8\text{ms}^{-2}$ $t = ?$ $v = u + at$ $0 = 28.3 + (-9.8)t$ $t = 2.89\text{s}$ Total flight time $= 2 \times 2.89 = 5.78\text{s}$ Remember to use an extra significant figure as this is an intermediate answer.	The horizontal velocity is constant $s = ?$ $\bar{v} = 40.0 \cos 45° = 28.3\text{ms}^{-1}$ $t = 5.78s$ $s = 28.3 \times 5.78$ $s = 160\text{m}$ to two significant figures.

Example

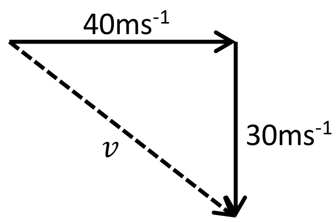
The graphs below show the horizontal and vertical components of a projectile during its 3 second motion. Find the magnitude of the velocity of the ball at the time of 3 seconds.





Solution

Read the horizontal and vertical components of velocity from the graph at a time of 3 seconds. Add both components as vectors.

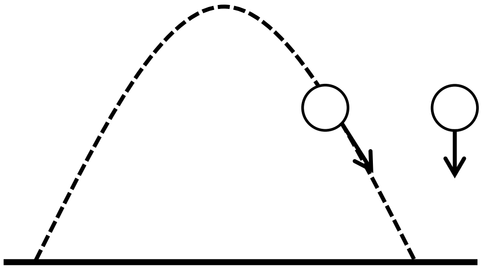


$$v = \sqrt{40^2 + (-30)^2}$$
$$v = 50\text{ms}^{-1}$$

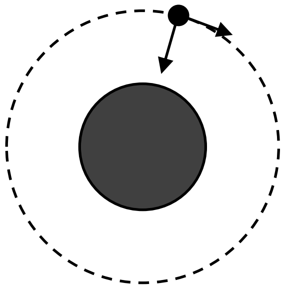
Question book page 35 to page 38 all questions except question 4
Projectiles Homework

4.6 I can state that satellites are in free fall around a planet or star.

For a projectile, the vertical component of the object's motion is the same as if it were in free fall.



An object in orbit is also a projectile as the only a gravitational force is acting on it. The vertical component of the object's orbital motion is the same as if it were in free fall.



4.7 I can use of Newton's Law of Universal Gravitation to solve problems involving, force, masses and their separation.

The gravitational force produced between two masses is given by Newton's Law of Universal Gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

Where

- F is the gravitational force between the two objects in Newtons
- m_1 and m_2 are the masses of the objects in kilograms
- r is the distance between the centre of mass of the objects in metres
- $G = 6.67 \times 10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$, the universal constant of gravitation

Example

Using the data given calculate the average gravitational force between the Earth and the Moon.

Data

Mass of the Earth $= 5.97 \times 10^{24}\mathrm{kg}$

Mass of the Moon $= 7.34 \times 10^{22}\mathrm{kg}$

Average Earth to Moon distance $384400\mathrm{km}$ = $384400 \times 10^3\mathrm{m}$

$G = 6.67 \times 10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$

Solution

Substitute all the values into Newton's Law of Universal Gravitation.

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.34 \times 10^{22}}{(384400 \times 10^3)^2} = 1.98 \times 10^{20}\mathrm{N}$$

Example

The relationships $F = \frac{Gm_1m_2}{r^2}$ and $W = mg$ both give the gravitational force on an object on the surface of the Earth. Show that $g = \frac{Gm_1}{r^2} = 9.8\mathrm{Nkg}^{-1}$

Where G is the Universal gravitation constant $= 6.67 \times 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$
m_1 is the mass of the Earth $= 5.97 \times 10^{24}\mathrm{kg}$
r is the radius of the Earth $=$ $6{,}371\mathrm{km} = 6371 \times 10^3\mathrm{m}$

Solution

$$g = \frac{Gm_1}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6371 \times 10^3)^2} = 9.8\mathrm{Nkg}^{-1}$$

Question book page 38 and page 39 questions 1 to 6.
Gravitation Homework

Key Area: Special Relativity

Previous Knowledge

Speed of light $= 3.0 \times 10^{8} \mathrm{ms}^{-1}$
Vector addition.

Success Criteria

5.1 I understand the term “frame of reference”.

5.2 I can, using Newtonian relativity, calculate the relative velocities of objects moving at less than 0.1c.

5.3 I can state that the speed of light in a vacuum is the same for all observers.

5.4 I can convert speeds in metres per second into fractions of the speed of light.

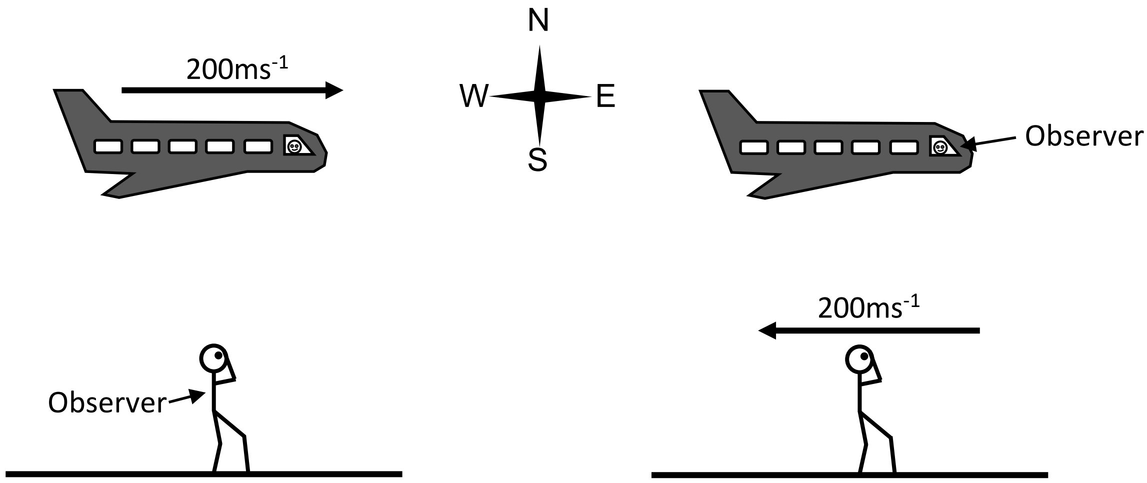
5.5 I understand that measurements of time and distances in space for a moving observer are changed relative to those for a stationary observer, giving rise to time dilation.

5.6 I can use an appropriate relationship to solve problems involving time dilation and speed.

5.7 I can use an appropriate relationship to solve problems involving length contraction and speed.

5.1 Understand the term "frame of reference".

A frame of reference is the location from which an observer measures the positions and velocities of other objects. The observer is stationary (at rest) in their frame of reference. The motion of an object will change depending on the choice of frame of reference. All frames of reference we will be dealing with will be non-accelerating and non-rotating.



From the frame of reference of an observer on the ground the observer is stationary and the plane is travelling at $200ms^{-1}$ towards the east.

From the frame of reference of an observer on the plane the observer is stationary and the ground is travelling at $200ms^{-1}$ towards the west.

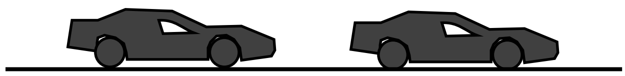
> **Important Note**
> The term "stationary" means that an object or observer has zero velocity in the frame of reference being considered. It does not mean that the object or observer is not moving. When observed from another frame of reference a "stationary" object or observer can have a non-zero velocity.

5.2 I can, using Newtonian relativity, calculate the relative velocities of objects moving at less than 0.1c.

In Newtonian relativity, times and distances between events in moving frames of reference are not affected by their relative speed. Times and distances measured in any non-accelerating, non-rotating frame of reference will be the same for all observers. Newtonian relativity only applies to objects moving at relative speed of less than 0.1c. At higher relative speeds the rules of special relativity need to be applied.

Example

Two cars are travelling along a motorway at a constant speed. The speedometer in each car reads $30ms^{-1}$ (~70miles per hour).



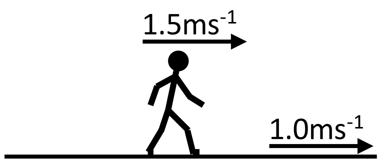
a. Taking the frame of reference to be the rear car, what is the speed of the front car?
b. Taking the reference frame to be the front car, what is the speed of the rear car?
c. Taking the reference frame as the front car what is the speed of the ground?
d. A car is travelling at $30ms^{-1}$ on the opposite carriageway. Using the frame of reference of this car, what is the speed of the two cars?

Solution

a. Zero as both cars are moving with the same speed and direction.
b. Zero as both cars are moving with the same speed and direction.
c. $-30ms^{-1}$.
d. $30 - (-30) = 60ms^{-1}$

Example

A traveller is walking along a moving walkway at an airport terminal at $1.5ms^{-1}$. The walkway is moving at $1.0ms^{-1}$.



Find

a. The speed of the walker in the frame of reference of the airport terminal.
b. If the walker walked in the opposite direction on the moving walkway at what speed would the airport terminal be moving in his frame of reference?

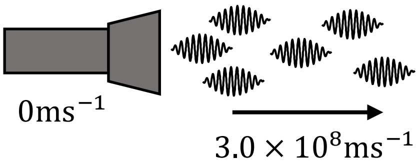
Solution

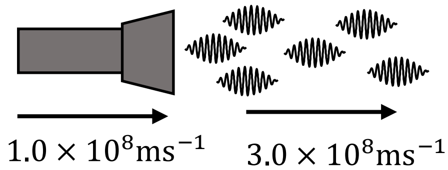
a. $1.5 + 1.0 = 2.5ms^{-1}$
b. $1.5 – 1.0 = 0.5ms^{-1}$

Question book pages 40 and page 41 questions 1, 2, 3, 5.

5.3 I can state that the speed of light in a vacuum is the same for all observers.

The speed of light in a vacuum is defined as 299,792,458ms^{-1} which we usually approximate to $3.0 \times 10^8 \mathrm{ms}^{-1}$. This value is the same in all reference frames. This been confirmed in many experiments.





Even when the light source is moving at high speed the light emitted will still be travelling at $3.0 \times 10^8 \mathrm{ms}^{-1}$. This has consequences for the measurements of time and distances.

Question book pages 41 and 42 questions 7, 8 and 9.

5.4 I can convert speeds in metres per second into fractions of the speed of light.

The speed of light is usually given the symbol c. Remember that $\mathrm{c} = 3.0 \times 10^8 \mathrm{ms}^{-1}$.

Example

A space ship is travelling toward Alpha Centauri at $7.5 \times 10^7 \mathrm{ms}^{-1}$. Calculate its speed as a fraction of the speed of light.

Solution

$$\frac{7.5 \times 10^7}{3.0 \times 10^8} = 0.25$$

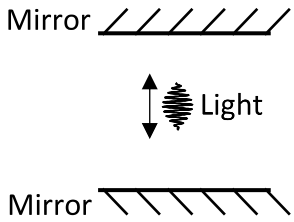
So $7.5 \times 10^7 \mathrm{ms}^{-1} = 0.25\mathrm{c}$

Question book page 42 questions 10 and 11.

5.5 I understand that measurements of time and distances in space for a moving observer are changed relative to those for a stationary observer, giving rise to time dilation.

Einstein's thought Experiment

Imagine a clock where the time interval is measured by a light pulse being reflected back and forth between two parallel mirrors. The time interval measured is the time it takes the pulse to move from one mirror to the other.

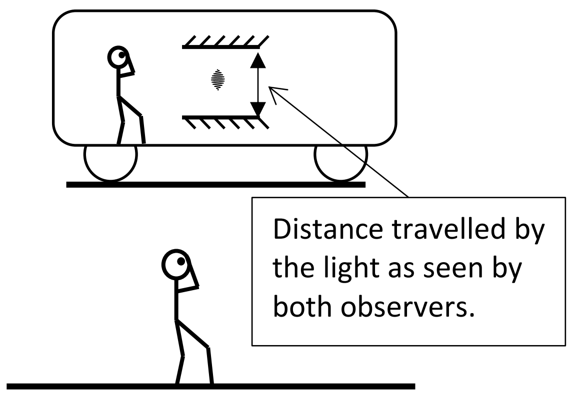


Stationary Train

We put our light clock in a train which is stationary in the frame of reference of the observer on the ground.
The distance travelled by the light pulse and the speed of light is identical for both observers.

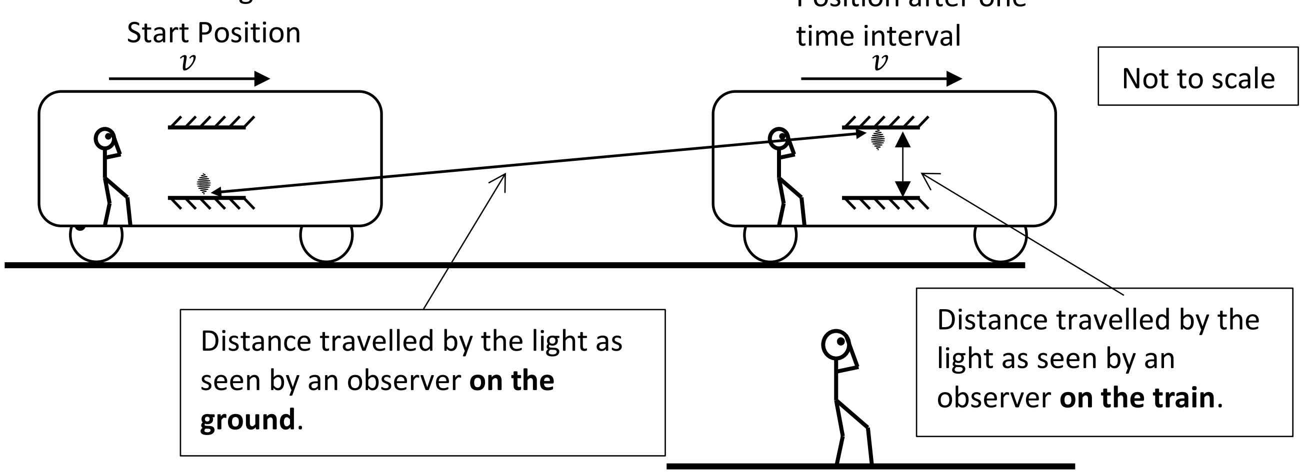
$$\text{time inverval} = \frac{\text{distance travelled by the light pulse}}{\text{speed of light}}$$

The time interval measured by both observers is identical.



Moving Train/Stationary Ground

Now we put our light clock on a train which is moving at speed v in the frame of reference of the observer on the ground.



The observer on the ground sees the light travel a greater distance than the observer on the train. However, the speed of light is the same for both observers.

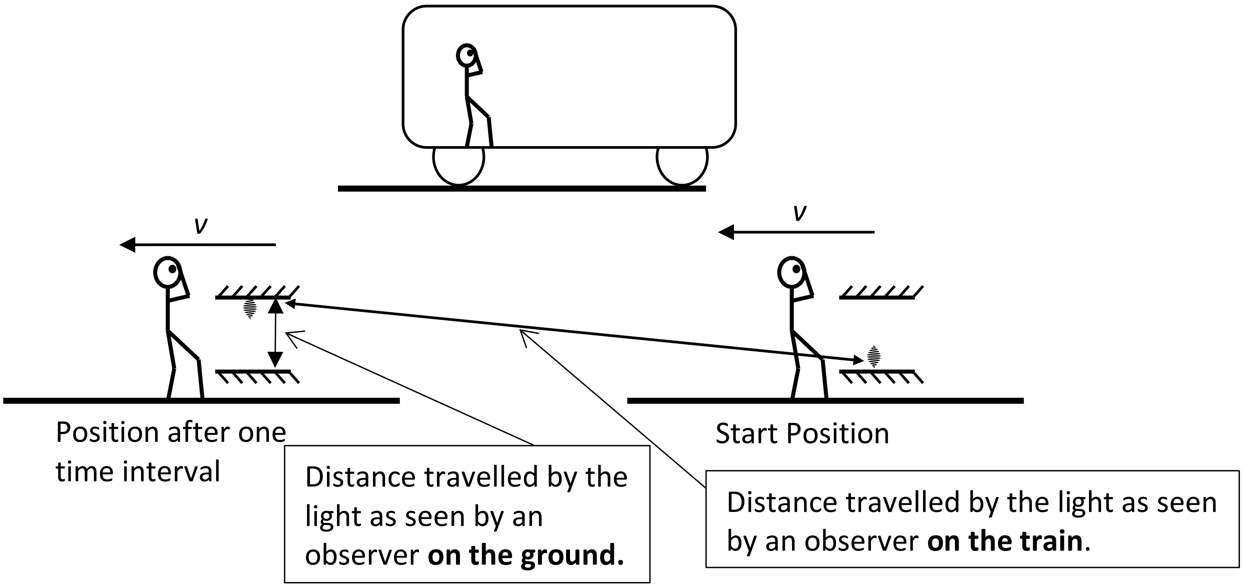
$$\text{time inverval} = \frac{\text{distance travelled by the light pulse}}{\text{speed of light}}$$

The time interval seen by the observer in the stationary reference frame of the ground is greater than the time interval seen by the observer in the moving reference frame of the train. Therefore the observer on the ground sees the pulse take a longer time to travel betewen the two mirrors.

- The observer on the ground sees that time has slowed down on the moving train.
- The observer on the train sees the same time interval as if the train was stationary.

<u>Moving ground/Stationary train</u>

Now we put our light clock on the ground which is moving at speed v in the frame of reference of the observer on the train.



The observer on the train sees the light travel a greater distance than the observer on the ground. However, the speed of light is the same for both observers.

$$\text{time inverval} = \frac{\text{distance travelled by the light pulse}}{\text{speed of light}}$$

The time interval seen by the observer in the stationary frame of reference of the train is greater than the time interval seen by the observer in the moving reference frame of the ground. Therefore the observer on the train sees the pulse take a longer time to travel betewen the two mirrors.

- The observer on the train sees that time has slowed down on ground.
- The observer on the ground sees the same time interval as if the ground was stationary.

Notice that the observer on the train **and** the observer on the ground both see the others time slowed down.
All of the above is summarised in the statement

“Moving clocks run slow”

Although it is time which has slowed not just clocks.
This time difference is called **time dilation** and is present when any object is moving relative to another.
As shown in section 5.6, time dilation is very small for low speeds. Its effects can be ignored when speeds are below 0.1c.

5.6 I can use an appropriate relationship to solve problems involving time dilation and speed.

In special relativity an **event** is anything for which we can define a position in space and a time e.g. the position in space and the time which light reflects off the mirror in the light clock in section 5.5. The difference in time between two events is called a **time interval**.

The relationship below relates time intervals between events as observed from two different frames of reference.

- t' is called the **dilated time.** This is the time interval measured by an observer in the stationary frame of reference.
- t is called the **proper time.** This is the time interval measured by an observer in the frame of reference moving (at speed v) in which the events are occurring.

$$\underbrace{t'}_{\text{Dilated time (s)}} = \frac{\overbrace{t}^{\text{Proper time (s)}}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Proper time (s)

Dilated time (s)

Speed of the moving frame of reference (ms^{-1})

Speed of light (ms^{-1})

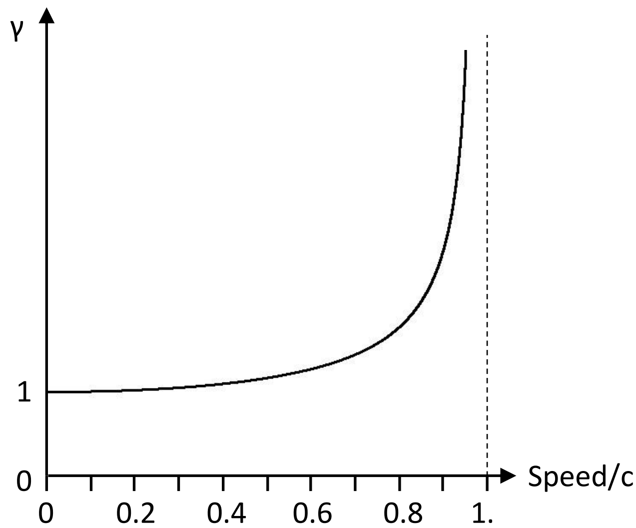
When solving problems using this relationship, your first step must be to establish
- which frame of reference you are considering as stationary, (t').
- which frame of reference is moving with the events(s), (t).

To check your answers note that t' is always larger than t.

The term $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is known as the Lorentz factor and given the symbol γ (gamma). The graph shows that γ has the value of 1 below 0.1c and rapidly increases as the speed of light is approached.
Below 0.1c

$$t' = t$$

The effects of special relativity are negligible for speeds below 0.1c.



Example

A spaceship is travelling away from Earth at $2.9 \times 10^8 \mathrm{ms}^{-1}$. A beacon on the spaceship is set to flash once every 1.0s. How often does the beacon flash when seen from Earth?

Solution

Take the frame of reference being considered stationary as the Earth. The moving frame of reference is the spaceship. Therefore t' is the time interval of the flashes as measured on Earth and t is the time interval as measured on the moving spaceship.

$t' =?$

$t = 1.0\mathrm{s}$

$v = 2.9 \times 10^8 \mathrm{ms}^{-1}$

$c = 3.0 \times 10^8 \mathrm{ms}^{-1}$

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1.0}{\sqrt{1 - \frac{(2.9 \times 10^8)^2}{(3.0 \times 10^8)^2}}} = 3.9\mathrm{s}$$

Example

A spaceship flying past a space station is unsure of their speed relative to the space station. They know that a beacon on the space station sends out radio pulses every 0.1s. As they pass the space station they record radio pulses every 0.2s. Calculate the speed of the spaceship.

Solution

The frame of reference being considered stationary is the spaceship. The moving frame of reference is the space station. Therefore t' is the time interval of the flashes as measured on spaceship and t is the time interval as measured on the moving space station.

Or

Just remember that t' is always bigger than t, so $t' = 0.2\mathrm{s}$ and $t = 0.1\mathrm{s}$.

$t' = 0.2\mathrm{s}$

$t = 0.1\mathrm{s}$

$v =?$

$c = 3.0 \times 10^8 \mathrm{ms}^{-1}$

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$0.2 = \frac{0.1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{0.1}{0.2}$$

$$v = c\sqrt{1 - \left(\frac{0.1}{0.2}\right)^2} = 0.866c = 2.6 \times 10^8 \mathrm{ms}^{-1}$$

Example

A spaceship is passing Earth at 0.9c. An observer on Earth looking at the spacecraft sees that the time on the spaceship's clock reads 2 o'clock the same time as his clock own. After a further 10 minutes passes on Earth what will the time read on the spaceship's clock?

Solution

Taking the Earth to be the stationary frame of reference and the moving frame of reference as the spaceship. $t' = 10$ minutes is the time measured on Earth and t is the time measured on the spaceship

$t' = 10\text{minutes}$

$t = ?$

$v = 0.9c$

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$10 = \frac{t}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}}$$

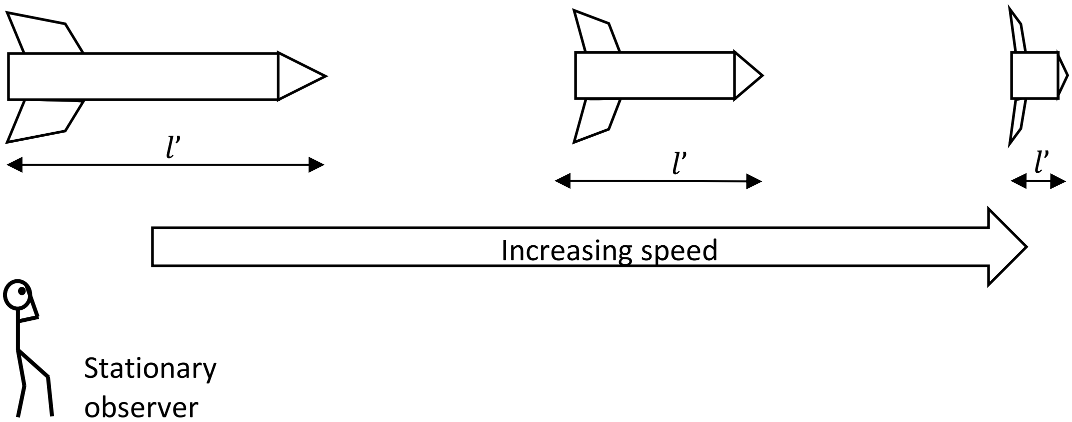
$$10 = \frac{t}{0.435}$$

$$t = 4.4\text{minutes}$$

Question book pages 42 and 43 questions 2, 4 to 9, 3.

5.7 I can use an appropriate relationship to solve problems involving length contraction and speed.

As with time, the measured length of an object is affected by its relative motion. In this case the motion of an object is affected in the direction of motion only. Its dimensions in other directions are not affected. This effect is termed **length contraction**.



An observer moving with the rocket will measure the length of the rocket as l. This will remain constant regardless of the speed of the rocket.
An observer who sees the rocket moving will measure the length of the rocket as l'. The length will reduce as the rocket's speed increases.

The relationship below relates measured lengths as observed from two different frames of reference.

- l' is the **contracted length.** This is the length measured by an observer in the stationary frame of reference.
- l is the **proper length.** This is the length measured by an observer in the frame of reference moving (at speed v) with the object whose length is being measured.

Proper length (m)

Speed of the moving frame of reference

Contracted length (m) $l' = l\sqrt{1-\frac{v^2}{c^2}}$

Speed of light (ms^{-1})

When solving problems using this relationship, your first step must be to establish
- which frame of reference you are considering as stationary.
- which frame of reference is moving with the object whose length being measured.

To check your answers note that l' is always smaller than l.

Example

A rocket of is flying past a moon at 0.95c. An observer on the rocket measures the length of the rocket as 90m. Find the length of the rocket as measured by the observer on the moon.

Solution

Taking the stationary frame of reference as the moon and the moving frame travelling with the rocket.

$l' = ?$
$l = 90\text{m}$
$v = 0.95c$

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}$$

$$l' = 90 \times \sqrt{1 - \frac{(0.95c)^2}{c^2}}$$

$$l' = 28\text{m}$$

Example

The rocket in the previous example slows down and passes the moon once more. This time the observer on the moon measures the length of the rocket as half the length measured by an observer on the rocket. Find the relative speed of the rocket as a fraction of *c*.

Solution

From the question $l' = \frac{1}{2}l$.

Substituting this into $l' = l\sqrt{1 - \frac{v^2}{c^2}}$ gives

$$\frac{1}{2}l = l\sqrt{1 - \frac{v^2}{c^2}}$$

$$\frac{1}{2} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$v = c\sqrt{1 - \left(\frac{1}{2}\right)^2}$$

$$v = 0.87c$$

Example

A spacecraft is traveling from Earth to Proxima Centauri. As measured from Earth, the spacecraft is travelling at a speed of 0.90c and the distance it travels 4.25 light years. Find how far a traveller on the spacecraft would measure the Earth to Proxima Centauri distance.

Solution

Taking the stationary frame of reference to be travelling with the rocket.

$l' = ?$
$l = 4.25$ light years
$v = 0.90c$

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}$$

$$l' = 4.25 \times \sqrt{1 - \frac{(0.90c)^2}{c^2}}$$

$$l' = 3.8 \text{ light years}$$

Question book pages 44 and 45 questions 1 to 9.
Further relativity questions on pages 45 to 46 questions 1 to 9
Special Relativity Homework

Key Area: The Expanding Universe

Previous Knowledge
Emission and Absorption spectra from Particles and Waves unit

Success Criteria
6.1 I aware of the main ideas of the big bang model of the universe.

6.2 I know that the doppler effect causes shifts in wavelengths of sound and light.

6.3 I can use the relationship $f_o = f_s\left(\frac{v}{v \pm v_s}\right)$ to solve problems involving the observed frequency, source frequency, source speed and wave speed.

6.4 I can define the terms **redshift** for the light from object moving away and **blueshift** for the light from an object moving closer.

6.5 I know that the redshift, z, of a galaxy is given by $z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$ and for galaxies moving less than 0.1c the redshift is given by $z = \frac{v}{c}$.

6.6 I can use Hubble's Law $v = H_o d$ to solve problems involving the Hubble Constant, the recession velocity of a galaxy and its distance from us.

6.7 I know that Hubble's Law allows us to estimate the age of the Universe.

6.8 I know that the mass of a galaxy can be estimated from the orbital speed of its stars.

6.9 I know that evidence supporting the existence of dark matter comes from estimations of the mass of galaxies.

6.10 I know that evidence supporting the existence of dark energy comes from the accelerating rate of expansion of the Universe.

6.11 I know that the temperature of stellar objects is related to the distribution of emitted radiation over a wide range of wavelengths.

6.12 I know that the peak wavelength of the radiation emitted is shorter for hotter objects than for cooler objects.

6.13 I am aware of the qualitative relationship between radiation emitted per unit surface area per unit time and the temperature of a star.

6.14 I am aware of evidence supporting the big bang theory.

6.1 I aware of the main ideas of the Big Bang Model of the Universe.

The origins of the big bang model of the universe lies in Albert Einstein's theory of gravity called the general theory of relativity, which was published in 1915. When this theory was applied to the whole universe by Friedman and Lemaître it predicted that the universe should be expanding. This indicates that at some point in the past the universe must have started from a single point.
The main points of the Big Bang Theory are

- In the beginning, all of space and time originated at a single small, hot, dense point called a singularity. The universe then expanded out from this single point forming all the galaxies and all of space.

- The universe started at a high temperature and high density which cooled as it expanded. (For evidence see – Hubble's Law, redshift of galaxies)

- As the universe cooled, it reached a temperature where protons and neutrons could form. These then underwent nuclear fusion producing helium (and a tiny amount of lithium). This continued until the universe had expanded and cooled to below the temperature where fusion would occur.
  (For evidence see – The nuclear abundances of hydrogen and helium)

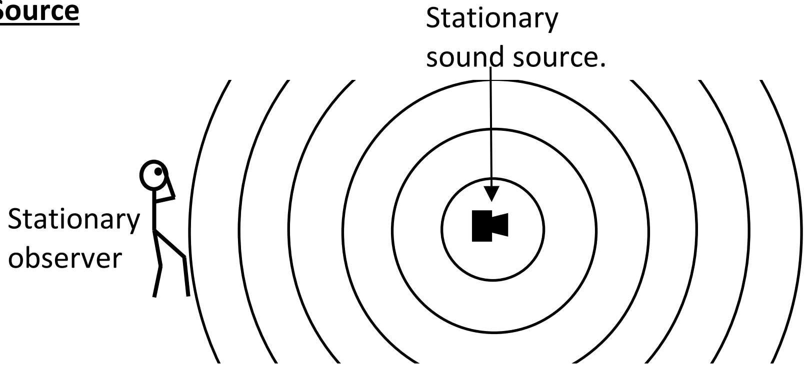
- After approximately 300,000 years the universe that cooled sufficiently that it reached a temperature where atoms could form. Once atoms formed the universe became transparent to radiation. The radiation emitted by the still hot universe was free to radiate across the universe. Over time the wavelength of this radiation has been increased as the space it travelled through expanded. The wavelength of this radiation now lies in the microware region of the e-m spectrum.
  (For evidence see – Cosmic microwave background radiation)

- Gravity caused the collapse of some of the clouds of hydrogen and helium to form stars and galaxies.

6.2 I know that the Doppler Effect causes shifts in wavelengths of sound and light.

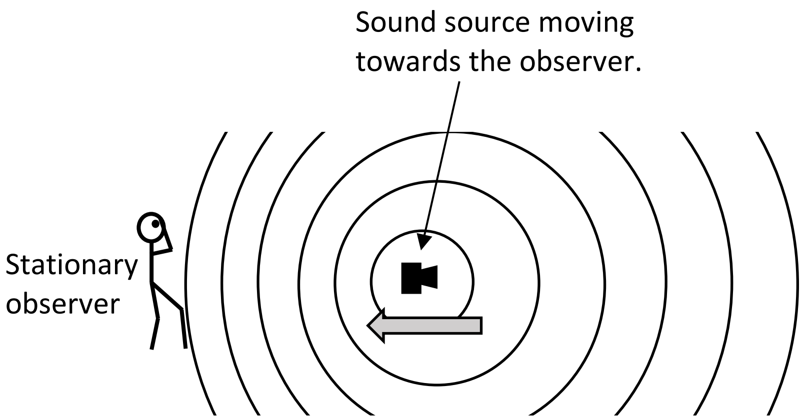
The Doppler Effect is the apparent change in wavelength of a source of waves due to the relative motion of the source.

Stationary Source



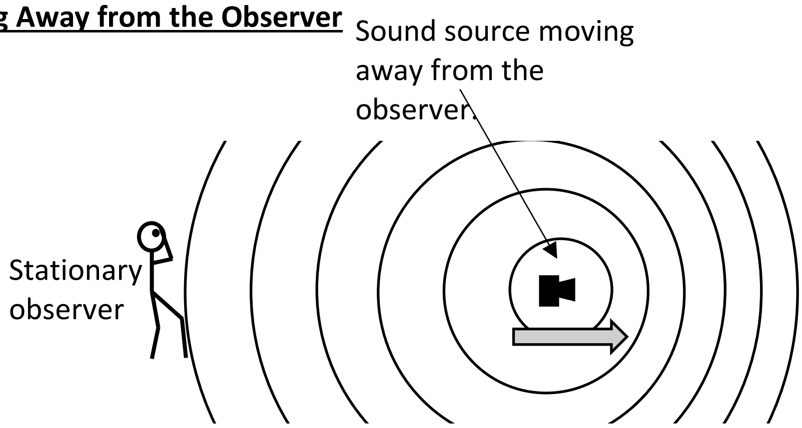
With a stationary source the wavelength and frequency of sound reaching the observer is the same as the wavelength emitted by the source.

Source Moving Towards the Observer



When the sound source is moving towards the observer more wavefronts per second are received by the observer. The wavelength observed is shorter than the wavelength emitted by the source and so the frequency observed will be increased.

<u>Source Moving Away from the Observer</u>



When the sound source is moving away from the observer fewer wavefronts per second are received by the observer. The wavelength observed is longer than the wavelength emitted by the source and so the frequency observed will be decreased.

The Doppler Effect applies to **all** sources of waves e.g. all the electromagnetic spectrum.

6.3 I can use the relationship $f_o = f_s \left(\frac{v}{v \pm v_s}\right)$ to solve problems involving the observed frequency, source frequency, source speed and wave speed.

The frequency of the waves observed from a moving source can be calculated using the relationship below

Frequency of the source when stationary (Hz)

Frequency observed (Hz)

$$f_o = f_s \left(\frac{v}{v \pm v_s}\right)$$

Speed of sound (ms^{-1})

Speed of sound (ms^{-1})

Speed of the sound source (ms^{-1})

- Use + when the source is moving away from the observer.
- Use – when the source is moving towards the observer.

This relationship only applies to sound waves and other waves where v is less than 0.1c. It cannot be used for any of the waves in the electromagnetic spectrum as the relationship does not take into account relativistic effects.

Example

The siren on a fire engine travelling towards a burning house at $30ms^{-1}$, emits a sound of frequency 1350Hz. The house owner hears a fire engine approaching his burning house. What frequency would the house owner hear the fire engine siren? Take the speed of sound to be $340ms^{-1}$.

Solution

$f_0 = ?$

$f_s = 1350\text{Hz}$

$v = 340\text{ms}^{-1}$

$v_s = 30\text{ms}^{-1}$

$$f_o = f_s\left(\frac{v}{v - v_s}\right)$$

$$f_o = 1350\left(\frac{340}{340 - 30}\right)$$

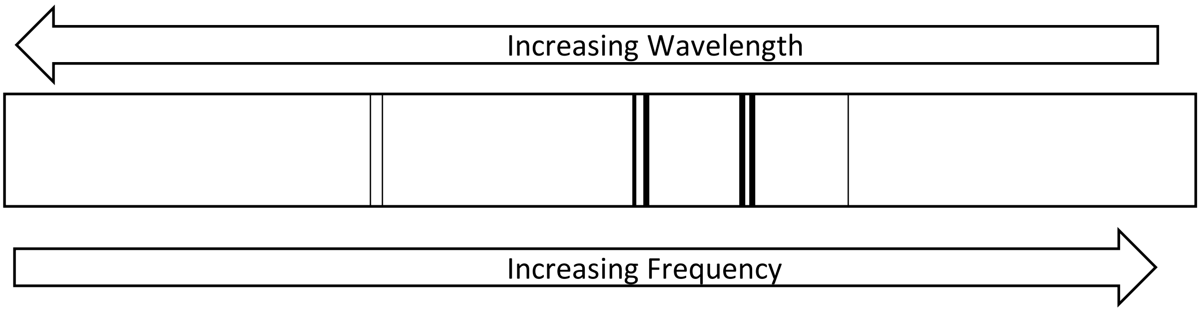
$$f_o = 1481\text{Hz}$$

Question book pages 48 to 51 questions 1 to 18.
Doppler Effect Homework

6.4 I can define the terms redshift for the light from object moving away and blueshift for the light from an object moving closer.

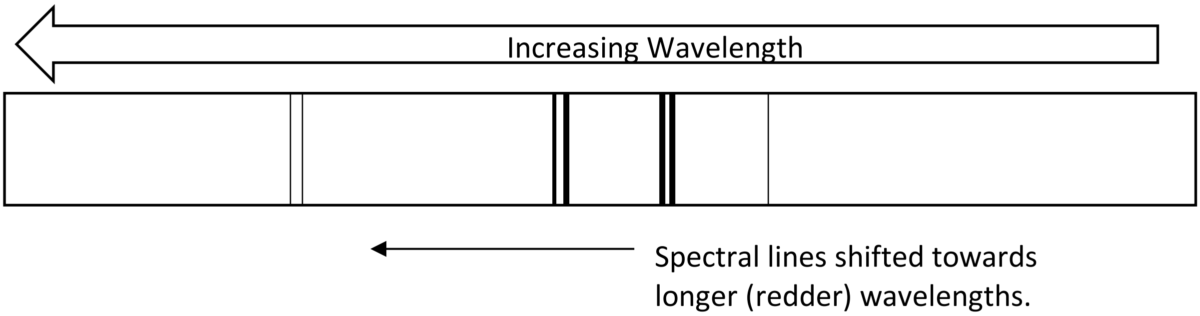
When examining light sources we are usually looking at the light emitted by stars or galaxies. As studied in the Particles and Waves unit, the light emitted by stars and galaxies usually arrive as either emission or absorption spectra.
As light is a wave the Doppler Effect will change the frequency and wavelength of light observed compared to the frequency emitted by the source. This can be observed by examining the lines in the spectrum of stars and galaxies and comparing the observed lines to reference lines obtained from a light source on Earth.

<u>Spectra as seen on Earth</u>



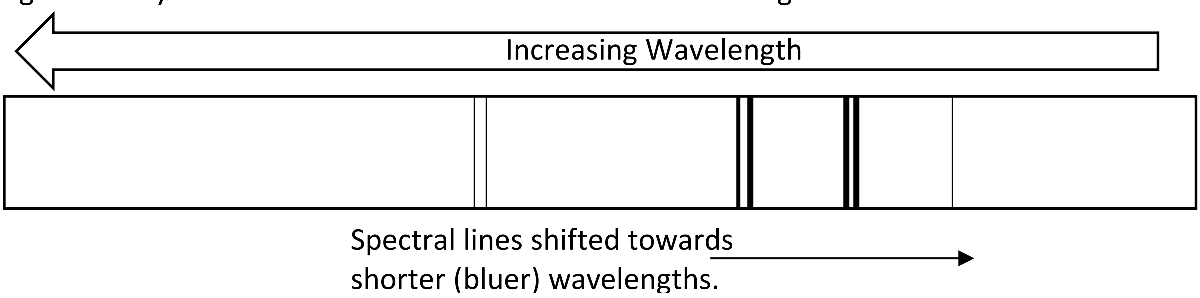
<u>Redshifted Spectra</u>

Redshift occurs when the source of light is moving away from the observer. The frequency of light seen by an observer will be decreased and the wavelength increased.



<u>Blueshifted Spectra</u>

Blueshift occurs when the source of light is moving towards the observer. The frequency of light seen by an observer will be increased and the wavelength decreased.



6.5 I know that the redshift, *z*, of a galaxy is given by $z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$ and for galaxies moving less than 0.1c the redshift is given by $z = \frac{v}{c}$.

The relationship below is the definition of the redshift of a galaxy. This applies to galaxies moving at any speed.

Wavelength observed from
the moving galaxy (nm)

Redshift (no units) $z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$ Wavelength as measured from a stationary source (nm)

If the galaxy is moving at non-relativistic speeds i.e. $< 0.1c$ the following relationship can be used.

Speed of the galaxy (ms^{-1})

Redshift (no units) $z = \frac{v}{c}$

Speed light (ms^{-1})

Example

One of the Sodium D lines as measured on earth occurs a 568.8205nm. In a spectrum produced from a distant galaxy the same line occurs at 597.2615nm.

a. Calculate the redshift of the galaxy.
b. Find the speed of the galaxy.

Solution

a.

$\lambda_{observed} = 597.2615\text{nm}$

$\lambda_{rest} = 568.8205\text{nm}$

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

$$z = \frac{597.2615 - 568.8205}{568.8205}$$

$$z = 0.04999996$$

b. $z = 0.04999996$

$c = 3.0 \times 10^8 \text{ms}^{-1}$

$$z = \frac{v}{c} \quad \Rightarrow \quad v = zc$$

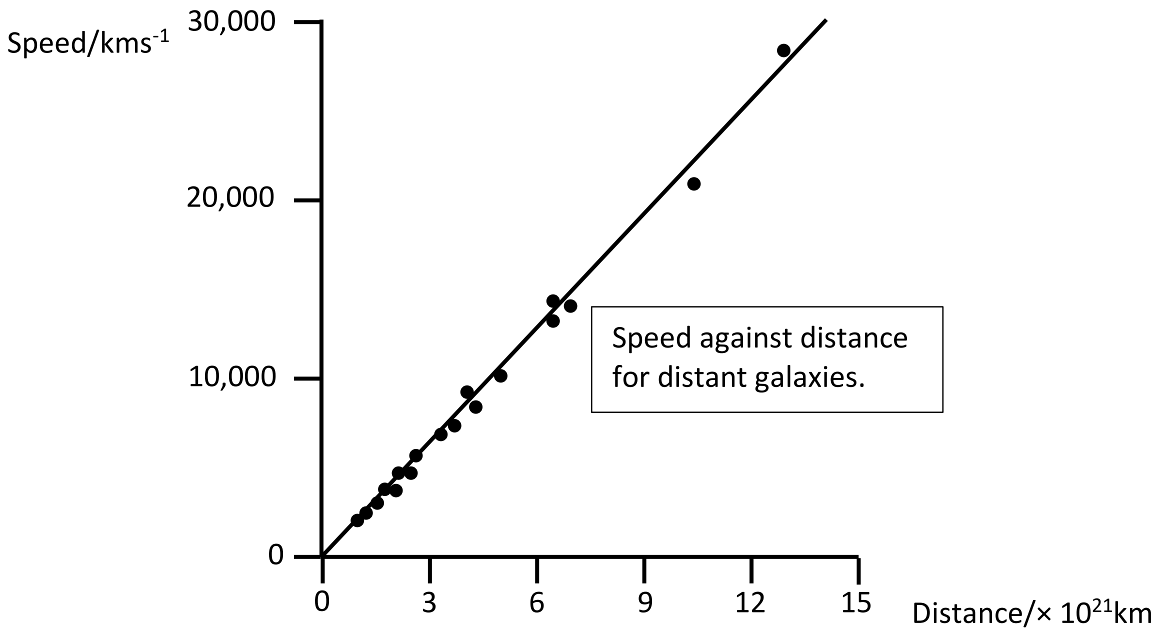
$$v = 0.04999996 \times 3.0 \times 10^8$$

$$v = 1.5 \times 10^7 \text{ms}^{-1}$$

Question book pages 51 to 52 questions 19 to 20.

6.6 I can use Hubble's Law $v = H_o d$ to solve problems involving the Hubble constant, the recession velocity of a galaxy and its distance from us.

In 1929 when Edwin Hubble plotted a graph of speed against distance for distant galaxies. The graph came out as a straight line similar to the one shown below.



From this graph Hubble's Law can be derived. The constant H_o is obtained from the gradient of the graph.

Hubble's Constant $= 2.3 \times 10^{-18}\text{s}^{-1}$

Speed of the distant galaxy (ms^{-1})

$$v = H_o d$$

Distance to the distant galaxy (m)

Example

By examining the spectra of a distant galaxy it is determined that it is moving at $1.7 \times 10^{7}\text{ms}^{-1}$. Use Hubble's Law to find the distance to this galaxy.

Solution

$v = 1.7 \times 10^{7}\text{ms}^{-1}$
$H_o = 2.3 \times 10^{-18}\text{s}^{-1}$
$d = ?$

$$v = H_o d$$

$$1.7 \times 10^{7}\text{ms}^{-1} = 2.3 \times 10^{-18} \times d$$

$$d = 7.4 \times 10^{24}\text{m}$$

Question book page 53 question 6.

Example
When a spectrum of light from a distant galaxy is analysed and it is found that a line which occurs at 590.0nm in the galaxies spectrum would be at 568.8nm when measured on Earth. Find the distance to the galaxy.

Solution
<u>Step 1 Find the redshift of the galaxy.</u>

$\lambda_{observed} = 590.0\text{m}$

$\lambda_{rest} = 568.8\text{nm}$

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

$$z = \frac{590.0 - 568.8}{568.8}$$

$$z = 0.0372$$

<u>Step 2 Find the speed of the galaxy</u>

$z = 0.0370$

$c = 3.0 \times 10^{8}\text{ms}^{-1}$

$$z = \frac{v}{c} \quad \Longrightarrow \quad v = zc$$

$$v = 0.0372 \times 3.0 \times 10^{8}$$

$$v = 1.12 \times 10^{7}\text{ms}^{-1}$$

<u>Step 3 Use Hubble's Law to find the distance</u>

$H_o = 2.3 \times 10^{-18}\text{s}^{-1}$

$$v = H_o d$$

$$1.12 \times 10^{7} = 2.3 \times 10^{-18} \times d$$

$$d = 4.9 \times 10^{24}\text{m}$$

Question book pages 53 to 55 questions 3 to 13.
Redshift and Hubble's Law Homework

6.7 I know that Hubble's law allows us to estimate the age of the Universe.

Hubble's Law allows us to find an approximate age of the universe if it is assumed that the universe has been expanding at a constant speed.

The kinematic relationship for constant speed $v = \frac{d}{t}$ can be rearranged to give

$$t = \frac{d}{v}$$

Hubble's Law $v = H_o d$ can also be rearranged to give

$$H_0 = \frac{v}{d}$$

By examining these two relationships it can be seen that

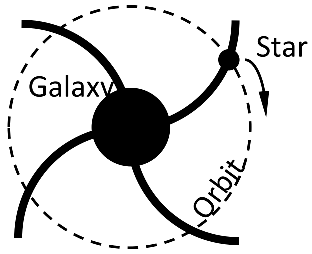
$$t = \frac{1}{H_o}$$

$$t = \frac{1}{2.3 \times 10^{-18}} = 4.3 \times 10^{17}\text{s} \approx 15 \text{ Billion Years}$$

The current best estimate of the age of the universe is 13.8 Billion Years. The difference between these two values due to the expansion rate of the universe not being varying as it expanded.

6.8 I know that the mass of a galaxy can be estimated from the orbital speed of its stars.

In galaxies the stars orbit around the centre of the galaxy. The mass of a galaxy within the orbit of a star can be found from the universal law of gravitation and orbital speed of the star.

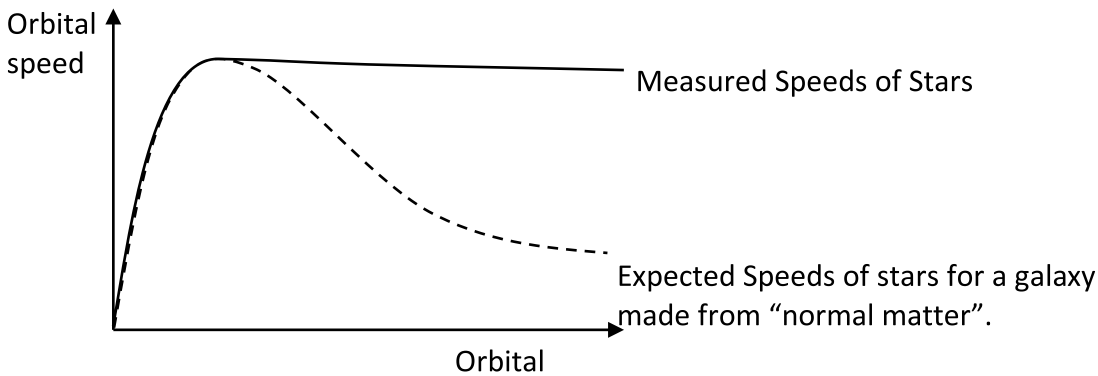


6.9 I know that evidence supporting the existence of dark matter comes from estimations of the mass of galaxies.

The mass of a galaxy can be estimated in two ways

- From orbital speed of stars.
- Observation of the "normal matter" in the galaxy, which consists adding up all the masses of stars together with all the dust and gas.

When a graph of orbital speed against orbital radius is plotted for stars in galaxies, the graph below is obtained.



The solid line indicates the measured speeds of stars and the dashed line indicates the speeds expected if the galaxy was made from normal matter. The difference between the two graphs shows that most of the mass of the galaxy is made from invisible matter. This unknown form of matter is called **dark matter.**

6.10 I know that evidence supporting the existence of dark energy comes from the accelerating rate of expansion of the Universe.

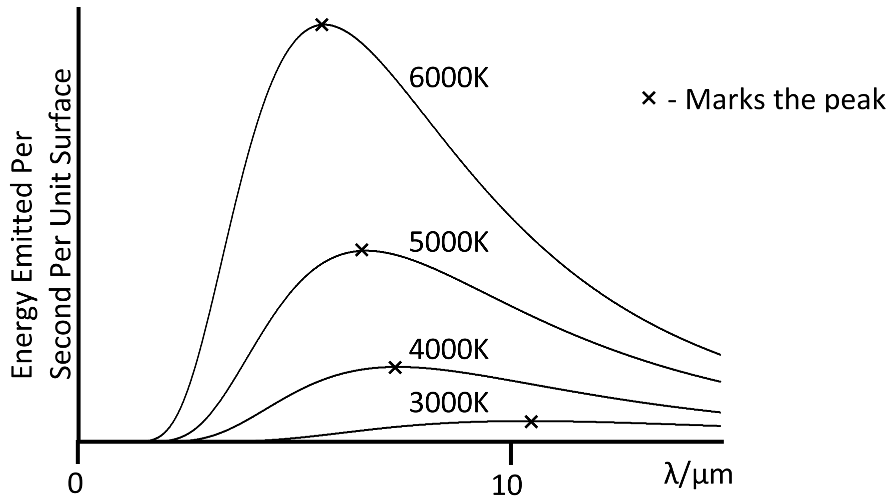
There is a gravitational force of attraction between all the galaxies in the universe. It is expected that this attractive force would cause the expansion of the universe to slow down. Observations of distant supernovae indicate that the expansion of the universe is speeding up. The reason for unexpected increase in the expansion rate is not known. The cause of the speeding up of the expansion rate is called **dark energy**.

6.11 I know that the temperature of stellar objects is related to the distribution of emitted radiation over a wide range of wavelengths.

6.12 I know that the peak wavelength of the radiation emitted is shorter for hotter objects than for cooler objects.

6.13 I am aware of the qualitative relationship between radiation emitted per unit surface area per unit time and the temperature of a star.

The graph below shows Planck's Law which shows the relationship between the power per unit area of a heated object at each of the wavelengths emitted. The graph does this for several different temperatures.
For stars, the graph shows the power emitted per unit area (Energy per second per metre squared) at each wavelength so it is independent of the size of the star. Each line on the graph shows a different surface temperature. So the graph can be applied to any size or temperature of star.



Examining this graph, you can see that

- At all temperatures a wide range of wavelengths are produced.
- Higher temperatures produce more shorter wavelength radiation.
- Peak wavelength decreases for higher temperatures.
- The higher the temperatures there is a greater energy emitted per second per unit area.

6.14 I am aware of evidence supporting the big bang theory.

There are several pieces of evidence which supports the big bang theory.

Cosmic Microwave background radiation
Microwave radiation comes from all points in the sky. This radiation is the remains of radiation emitted early in the life of the universe when it was very hot. As the universe expanded, the wavelength of this radiation was increased as it passed through the expanding space. It now has a peak wavelength indicating a temperature of 2.7K.

The abundance of the elements hydrogen and helium
The Big Bang theory predicts the ratio of hydrogen to helium in the universe to close agreement with the measured values.

Redshift of galaxies
All galaxies in the universe, except those in the local group, show redshift. This indicates that all galaxies are moving away from each other as predicted by the Big Bang theory.

Olbers' paradox
Olbers asked why the sky was dark at night. If the universe is infinite in extent and of infinite age then any line of sight in the sky should land on the surface of a star. This should make the night sky as bright as the daytime sky. Even if dust or gas blocked the view, this material would over a long period of time heat up to the same temperature as the surface of the stars. So as the sky is dark at night the universe cannot be static and of infinite age. Olbers' paradox is not direct evidence for the Big Bang theory but indicates that the universe is of finite age.

Question book pages 56 to 57 question 1.
Big Bang Theory Homework

Significant Figures

Success Criteria

7.1 I understand how to find the correct number of significant figures in data

7.2 I understand how to handle significant figures in calculations

7.1 I understand how to find the correct number of significant figures in measured data.

Significant figures show how precisely measurements were taken in an experiment.
Two measurements of distance 12m (2 significant figures) and 12.1m (3 significant figures) shows that the second measurement was more precise than the first.

SIGNIFICANT FIGURES RULES

<u>Rule 1 Every non-zero digit is significant.</u>
145m has 3 significant figures.
1.2kg has 2 significant figures.

<u>Rule 2 Zeros in between non-zero digits are all significant.</u>
4008km has 4 significant figures.
1.0006g has 5 significant figures.

<u>Rule 3 Zeroes on the left of the **first** non-zero digit are **not** significant.</u>
0.12s has 2 significant figures.
0.0003 has 1 significant figure.
0.4006 has 4 significant figures.
These zeros only show the position of the decimal point.

<u>Rule 4 Zeroes to the right of a non-zero digit after the decimal point are significant.</u>
0.04g has 1 significant figure.
0.040g has 2 significant figures.
41.0g has 3 significant figures.

<u>Rule 5 Zeros to the right of the last non-zero digit before the decimal point may be significant.</u>
500 could have 1, 2 or 3 significant figures. This depends of how the measurement was taken. We will assume that all the zeros are significant if it is in the data given in a question (unless you know otherwise).
If a decimal point is included then the zeros are all significant.
500.0 has 4 significant figures.

7.2 I understand how to handle significant figures in calculations

Calculations with Significant Figures.
Your answer should have the same number of significant figures as the lowest number of significant figures in the data used.
Example Calculation
Calculate the mass of water heated.

$\mathrm{E = 17490J \quad (5\ s.f.)}$

$\mathrm{m = ?}$

$\mathrm{\Delta T = 3.2°C \ (2\ s.f.)}$

$\mathrm{c = 4180Jkg^{-1}°C^{-1}\ (4\ s.f)}$

$$\mathrm{m = \frac{E}{c\Delta T}}$$

$$\mathrm{m = \frac{17490}{4180 \times 3.2}}$$

$$\mathrm{m = 1.307565789kg}$$

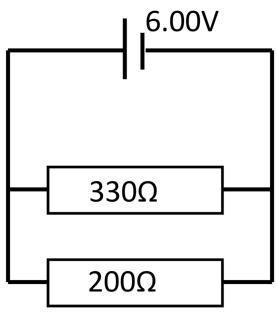
$$\mathrm{m = 1.3kg\ (2\ s.f.)\ as\ \Delta T\ is\ to\ 2\ s.f.}$$

Intermediate Calculations
Intermediate answers must carry **at least** one extra significant figure
Example
In the circuit shown calculate the total power dissipated in the circuit.

$$\mathrm{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}}$$

$$\mathrm{\frac{1}{R_T} = \frac{1}{330} + \frac{1}{200}}$$

$$\mathrm{R_T = 124.5283\Omega}$$



The data is to 3 significant figures so you should keep at least 4 significant figures when used in the next step.

Correct Calculation with 4 s.f.

$$\mathrm{P = \frac{V^2}{R}}$$

$$\mathrm{P = \frac{6^2}{124.5}}$$

$$\mathrm{P = 0.28909W}$$

$$\mathrm{P = 0.289W\ to\ 3\ s.f.}$$

Incorrect Calculation with 3 s.f

$$\mathrm{P = \frac{V^2}{R}}$$

$$\mathrm{P = \frac{6^2}{125}}$$

$$\mathrm{P = 0.288W}$$

$$\mathrm{P = 0.288W\ to\ 3\ s.f.}$$

Notice that if you don't carry extra significant figures you can get the wrong answer.
Significant Figures Problem Sheet

Uncertainties

Success Criteria

8.1 I can state that measurement of any physical quantity is liable to uncertainty.

8.2 I can express uncertainties in absolute or percentage form.

8.3 Express the numerical result of an experiment in the form: final value ± uncertainty.

8.4 I can distinguish between random uncertainties and systematic errors.

8.5 I can calculate the arithmetic mean of a value of a reading using repeated measurements and explain why this should be done in experiments.

8.6 I can calculate the random uncertainty in the mean value of a set of measurements.

8.7 I can identify and provide a minimum numerical value for various sources of measurement uncertainty.

8.8 I can combine uncertainties from different measurements to give a single value for uncertainty in a final result.

8.1 I can state that measurement of any physical quantity is liable to uncertainty.

No measurement is exact. There is always a limitation to the precision and accuracy of measuring instruments. This is means that the measurement of any quantity should be accompanied by an uncertainty which indicates the range of values within which the measurement could lie. Uncertainties are not exact quantities they are only approximations and so are usually only given to one significant figure.
Uncertainties are sometimes called **errors**.

8.2 I can express uncertainties in absolute or percentage form.

Uncertainties can be given as an **absolute uncertainty**
eg. $(20 \pm 1)\text{mm}$

This measurement likely to be between 19mm and 21mm.

Or as a **percentage uncertainty**

$$\%\text{uncertainty} = \frac{\text{absolute uncertainty}}{\text{measurement reading}} \times 100$$

$$\%\text{uncertainty} = \frac{1}{20} \times 100 = 5\%$$

$$= 20\text{mm} \pm 5\%$$

Uncertainties, **in the final answer**, are usually expressed to a single significant figure.

8.3 Express the numerical result of an experiment in the form: final value ± uncertainty

Percentage uncertainties are needed to compare uncertainties in different quantities but give final answers as an absolute uncertainty **unless asked otherwise**.

<u>Example</u>
A final answer is $t = 4.8\text{s} \pm 8\%$

The final answer should be given as an absolute uncertainty

$$\frac{8}{100} \times 4.8 = 0.384\text{s}$$

The uncertainty is rounded to one significant figure giving

$$t = (4.8 \pm 0.4)\text{s}$$

Uncertainties Question 1 and 3

8.4 I can distinguish between random uncertainties and systematic errors.

Random uncertainty occurs when repeated measurements give different readings. This is due to small changes in the setup between runs, temperature changes, reaction time etc. It is assumed that these small changes are equally likely to make the reading too big as they are to make it too small. More accurate measurement can be obtained by taking the arithmetic mean of repeated readings to reduce the effects of these small changes.

Systematic errors
These are due to faults in apparatus, procedures or design of an experiment.
Systematic errors affect all the readings in the same way, making them all too large or all too small.
Taking and arithmetic mean from repeated readings **does not** reduce systematic errors as all the readings will be offset from their true values making them all either be too big or too small.
Systematic errors can be difficult to find and remove from experiments.

8.5 I can calculate the value of a reading using repeated measurements by finding an arithmetic mean and explain why this should be done in experiments.

The arithmetic mean can be calculated by

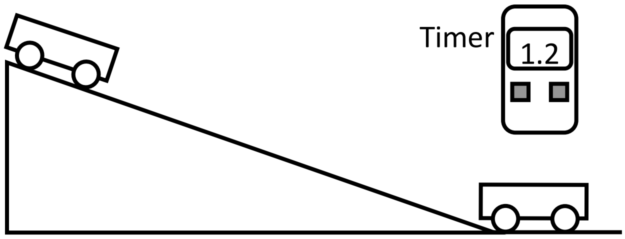
$$\text{Arithmetic Mean} = \frac{\text{Sum of the readings}}{\text{Number of readings}}$$

This relationship is **NOT** given in the relationship sheet.

In a more mathematical form, for n readings of x

$$\bar{x} = \frac{1}{n}\sum_{i=0}^{n} x_i$$

Example
A trolley is rolled down a slope and the following times obtained
Times 1.21s, 1.25s, 1.31s, 1.22s, 1.22s
Calculate the arithmetic mean of these results.



Solution
The line over the t indicating an arithmetic mean is frequently dropped in these calculations

$$t = \frac{\text{Sum of the readings}}{\text{number of readings}}$$

$$t = \frac{1.21 + 1.25 + 1.31 + 1.22 + 1.22}{5}$$

$$t = 1.24\text{s}$$

Finding values in an experiment by taking repeated readings and finding an arithmetic mean reduces the random uncertainty in these readings. The greater the number of readings the smaller the random uncertainty.

8.6 I can calculate the random uncertainty in the mean value of a set of measurements.

The random uncertainty in a measurement can be calculate using the relationship

$$\text{Random uncertainty} = \frac{\text{max. value} - \text{min. value}}{\text{number of values}}$$

This relationship gives the absolute uncertainty in the measurement.

<u>Example</u>
Using the trolley rolling down a slope data from section 8.5 calculate the random uncertainty in the time and express value as an absolute uncertainty.

<u>Solution</u>
Times 1.21s, 1.25s, 1.31s, 1.22s, 1.22s and $\bar{t} = 1.24\text{s}$

$\text{max. value} = 1.31\text{s}$
$\text{min value} = 1.21\text{s}$
$\text{number of values} = 5$

$$\text{Random uncertainty} = \frac{1.31 - 1.21}{5}$$
$$\text{Random uncertainty} = 0.02\text{s}$$

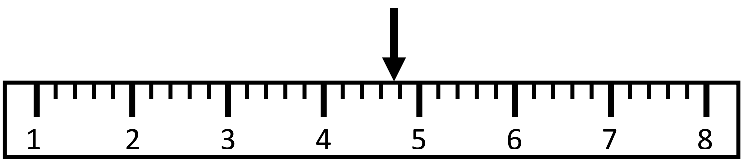
Expressing this as a value with absolute uncertainty gives

$$t = (1.24 \pm 0.02)\text{s}$$

8.7 I can identify and provide a minimum numerical value for various sources of measurement uncertainty.

Scale reading uncertainty
The scale-reading uncertainty is a measure of how precisely an instrument scale can be read. This provides an estimate of the **minimum** scale-reading uncertainty incurred when using an analogue display and a digital display. Frequently, when doing experiments, estimates of the scale reading uncertainty is larger than the minimum. In an investigation you must provide a realistic estimate of uncertainty not just the minimum value.

The minimum scale reading uncertainty for an analogue scale is taken as ± half the smallest scale division.

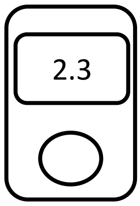


<u>Example</u>
The reading on the ruler scale, Reading $= (4.7 \pm 0.1)\text{mm}$
For very widely spaced scales a reasonable estimate should be made.

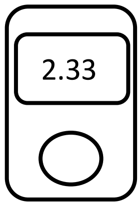
For a digital scale, the minimum scale reading uncertainty is taken as ± 1 in the least significant digit.

Example

The reading on the voltmeter scales



Voltmeter 1



Voltmeter 2

Reading on voltmeter 1 $= (2.3 \pm 0.1)\text{V}$

Reading on voltmeter 2 $= (2.33 \pm 0.01)\text{V}$

Other reading errors can occur - reading the wrong scale!

Example

Rulers can come with millimetre, half-millimetre, inch and centimetre scales, frequently on the same ruler. It is important to read the correct scale.

Uncertainties Questions 2, 4, 5, 6

8.8 I can combine uncertainties from different measurements to give a single value for uncertainty in a final result.

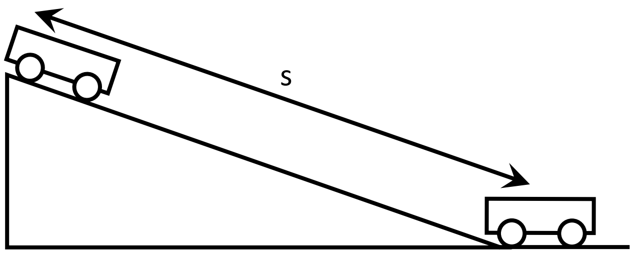
When an experiment is performed with several values being in measured it is necessary to combine their uncertainties to give an uncertainty in the final value.

The procedure to be followed is

- Calculate the percentage uncertainties in all the quantities measured
- Select the largest percentage uncertainty
- Calculate the absolute uncertainty in the final value using the largest percentage uncertainty
- State the final answer in as the final value ± absolute uncertainty.

Example

A trolley is run down a slope. Using the data provided calculate the average speed of the trolley.



Data

$t = 1.21\text{s}, 1.25\text{s}, 1.31\text{s}, 1.22\text{s}, 1.22\text{s}$

$s = 0.850\text{m}$ as measured with a ruler of interval 1mm.

Solution

$$\bar{t} = \frac{1.21 + 1.25 + 1.31 + 1.22 + 1.22}{5} = 1.24\text{s}$$

$$\text{Random uncertainty in time} = \frac{1.31 - 1.21}{5} = 0.02\text{s}$$

$$\text{\%uncertainty in time} = \frac{0.02}{1.24} \times 100 = 2\% \quad \text{to one significant figure}$$

Note

When a random uncertainty is measured it is not normally necessary to include a reading uncertainty with this quantity as it is usually smaller than the random uncertainty.

Distance, s, is measured with a metre stick as 0.850m.

The minimum reading uncertainty of a metre stick is half the interval of the scale, ±0.5mm

$$\%\text{uncertainty in distance, s} = \frac{0.0005}{0.850} \times 100 = 0.06\%$$

The percentage uncertainty in the time is 2% and in the distance 0.06%. The 2% value is larger so will be used to find the uncertainty in the final value.

$$\bar{v} = \frac{s}{t} = \frac{0.850}{1.24} = 0.685\text{ms}^{-1}$$

$$\text{Absolute uncertainty in speed} = \frac{2}{100} \times 0.685 = 0.01\text{ms}^{-1}$$

The final answer is

$$\bar{v} = (0.685 \pm 0.01)\text{ms}^{-1}$$

Note
This calculated minimum uncertainty is unrealistically small. A more realistic uncertainty in the measured distance would need to be estimated when doing an actual investigation.

Uncertainties Questions – Complete the remaining questions

Quantities, Units and Multiplication Factors

Quantity	**Quantity Symbol**	**Unit**	**Unit Abbreviation**
Acceleration	a	Metres per second squared	ms^{-2}
Acceleration due to gravity	g	Metres per second squared	ms^{-2}
Displacement	s	Metres	m
Distance	d	m	m
Force	F	Newton	N
Gravitational field strength	g	Newtons per kilogram	Nkg^{-1}
Impulse	J (not often used)	Newton second	Ns
mass	m	kilogram	kg
Momentum	p	Kilogram metres per second	kgms^{-1}
Speed	v	Metres per second	ms^{-1}
Time	t	Second	s
Velocity	v	Metres per second	ms^{-1}
Weight	W	Newton	N

Prefix Name	Prefix Symbol	Multiplication Factor
Pico	p	$\times 10^{-12}$
Nano	n	$\times 10^{-9}$
Micro	µ	$\times 10^{-6}$
Milli	m	$\times 10^{-3}$
Kilo	k	$\times 10^{3}$
Mega	M	$\times 10^{6}$
Giga	G	$\times 10^{9}$
Tera	T	$\times 10^{12}$

Relationships required for Physics Higher

$d = \bar{v}t$

$s = \bar{v}t$

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u+v)t$

$F = ma$

$W = mg$

$E_w = Fd$, or $W = Fd$

$E_p = mgh$

$E_k = \frac{1}{2}mv^2$

$P = \frac{E}{t}$

$p = mv$

$Ft = mv - mu$

$F = G\frac{m_1 m_2}{r^2}$

$t' = \frac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$l' = l\sqrt{1-\left(\frac{v}{c}\right)^2}$

$f_o = f_s\left(\frac{v}{v \pm v_s}\right)$

$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$

$z = \frac{v}{c}$

$v = H_0 d$

$W = QV$

$E = mc^2$

$I = \frac{P}{A}$

$I = \frac{k}{d^2}$

$I_1 d_1^2 = I_2 d_2^2$

$E = hf$

$E_k = hf - hf_0$

$v = f\lambda$

$E_2 - E_1 = hf$

$d\sin\theta = m\lambda$

$n = \frac{\sin\theta_1}{\sin\theta_2}$

$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$

$\sin\theta_c = \frac{1}{n}$

$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$

$I_{rms} = \frac{I_{peak}}{\sqrt{2}}$

$T = \frac{1}{f}$

$V = IR$

$P = IV = I^2R = \frac{V^2}{R}$

$R_T = R_1 + R_2 + \ldots$

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$

$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V_S$

$\frac{V_1}{V_2} = \frac{R_1}{R_2}$

$E = V + Ir$

$C = \frac{Q}{V}$

$Q = It$

$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

path difference $= m\lambda$ or $\left(m+\frac{1}{2}\right)\lambda$ where $m = 0, 1, 2\ldots$

random uncertainty $= \frac{\textit{max. value} - \textit{min. value}}{\textit{number of values}}$

or

$\Delta R = \frac{R_{\max} - R_{\min}}{n}$

DATA SHEET

COMMON PHYSICAL QUANTITIES

Quantity	*Symbol*	*Value*	*Quantity*	*Symbol*	*Value*
Speed of light in vacuum	c	$3{\cdot}00 \times 10^{8}$ m s^{-1}	Planck's constant	h	$6{\cdot}63 \times 10^{-34}$ J s
Magnitude of the charge on an electron	e	$1{\cdot}60 \times 10^{-19}$ C	Mass of electron	m_e	$9{\cdot}11 \times 10^{-31}$ kg
Universal Constant of Gravitation	G	$6{\cdot}67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}	Mass of neutron	m_n	$1{\cdot}675 \times 10^{-27}$ kg
Gravitational acceleration on Earth	g	$9{\cdot}8$ m s^{-2}	Mass of proton	m_p	$1{\cdot}673 \times 10^{-27}$ kg
Hubble's constant	H_0	$2{\cdot}3 \times 10^{-18}$ s^{-1}			

REFRACTIVE INDICES

The refractive indices refer to sodium light of wavelength 589 nm and to substances at a temperature of 273 K.

Substance	*Refractive index*	*Substance*	*Refractive index*
Diamond	2·42	Water	1·33
Crown glass	1·50	Air	1·00

SPECTRAL LINES

Element	*Wavelength*/nm	*Colour*	*Element*	*Wavelength*/nm	*Colour*
Hydrogen	656	Red	Cadmium	644	Red
	486	Blue-green		509	Green
	434	Blue-violet		480	Blue
	410	Violet	Lasers		
	397	Ultraviolet	*Element*	*Wavelength*/nm	*Colour*
	389	Ultraviolet	Carbon dioxide	9550 10590	Infrared
Sodium	589	Yellow			
			Helium-neon	633	Red

PROPERTIES OF SELECTED MATERIALS

Substance	*Density*/kg m^{-3}	*Melting Point*/K	*Boiling Point*/K
Aluminium	$2{\cdot}70 \times 10^{3}$	933	2623
Copper	$8{\cdot}96 \times 10^{3}$	1357	2853
Ice	$9{\cdot}20 \times 10^{2}$	273	. . . .
Sea Water	$1{\cdot}02 \times 10^{3}$	264	377
Water	$1{\cdot}00 \times 10^{3}$	273	373
Air	1·29	. . . .	. . . .
Hydrogen	$9{\cdot}0 \times 10^{-2}$	14	20

The gas densities refer to a temperature of 273 K and a pressure of $1{\cdot}01 \times 10^{5}$ Pa.