2022

Higher

Our Dynamic Universe Notes

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Thanks to Natalie for the cut and paste job!

CONTENTS

HOW TO LAY OUT EQUATIONS

REMINDERS

- **I**nformation
- **E**quation
- **S**ubstitution
- **S**olution
- **U**nits
- **U**nderline

Where t is the common factor, the only horizontal equation you need is *s=vt*

VERTICAL	HORIZONTAL
t	***t***
a	***s***
v	***v***
u	
s	

The roots (solution) of a quadratic equation

$$ax^2 + bx + c = 0$$

are given by:

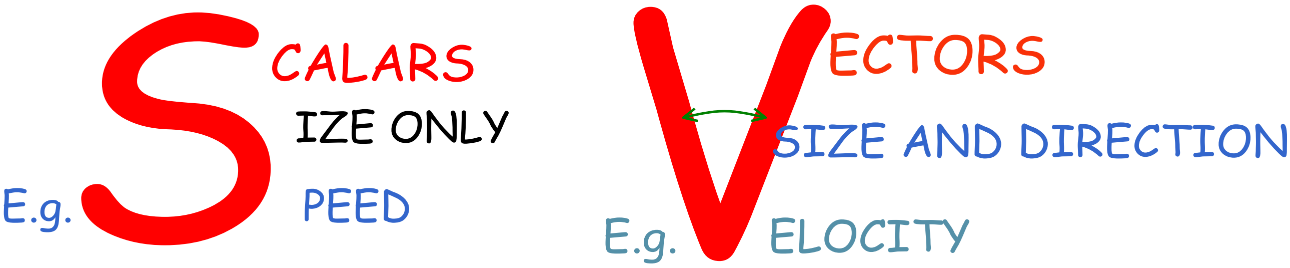
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If you need to find the value of t in the equation $s = ut + \frac{1}{2}at^2$ then the equation for a quadratic can be used as below.

$$t = \frac{-u_v \pm \sqrt{u_v^2 + 2as_v}}{a}$$

CHAPTER 1: BACKGROUND INFORMATION

BACKGROUND INFORMATION



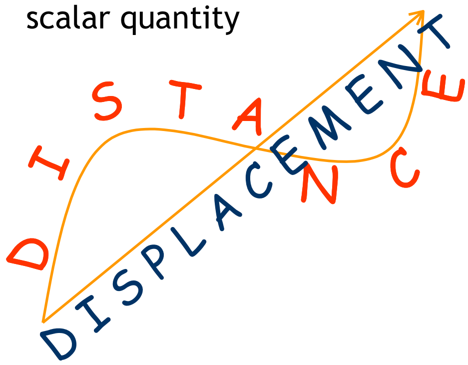
SCALAR QUANTITY (magnitude + unit)	VECTOR QUANTITY (magnitude, direction + unit)
Distance	Displacement
Speed	Velocity
Mass	Acceleration
Energy	Momentum
Power	Impulse
Temperature	Field Strength
Time	Force
Etc.	(including weight, friction, tension , upthrust etc)

Distance = "how far we've travelled"

⇒ symbol *d*

⇒ units metres, *m*

⇒ scalar quantity



Displacement = "how far we've travelled in a straight line (from A to B)" (include your direction)

⇒ symbol *s*

⇒ units, metres, *m*

⇒ Vector quantity

⇒ Must quote the direction

SPEED - an example of a SCALAR QUANTITY

(metres per second) $speed = \dfrac{distance}{time}$ $\dfrac{(metre)}{(second)}$

VELOCITY - an e.g. of a VECTOR QUANTITY **QUOTE DIRECTION**

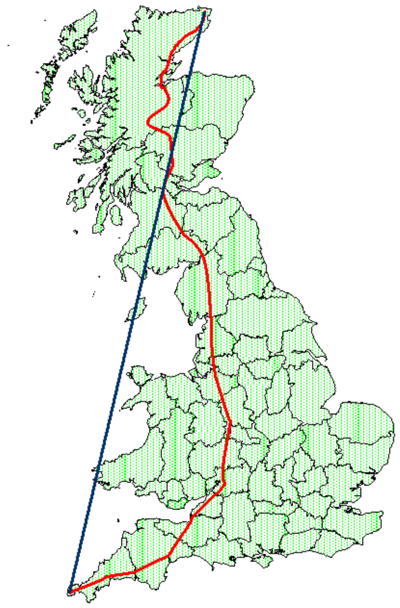
(metres per second) $velocity = \dfrac{displacement}{time}$ $\dfrac{(metre)}{(second)}$

What was the average speed for the journey from John O'Groats to Land's End?

d = 1193 miles

t = 26½ hours

$$speed = \frac{1193}{26 \cdot 5} = 45\ mph$$



What was the average velocity for the journey?

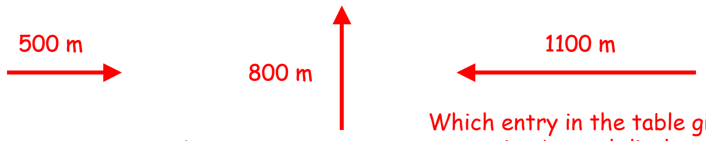
s = 615 miles

t = 26½ hours

$$speed = \frac{615}{26 \cdot 5} = 23 \cdot 2\ mph$$

1992 Higher Paper 1, Q 4

A competitor completes the following sequence of displacements in 10 minutes during part of an orienteering event.

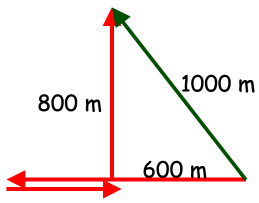


	Total displacement (m)	Average speed (m s^{-1})	Average velocity (m s^{-1})
A	↖ 1000	1.7	↖ 4.0
B	↘ 1000	1.7	↘ 1.7
C	↖ 1000	4.0	↖ 1.7
D	↖ 2400	4.0	↖ 4.0
E	↖ 1000	10	↖ 10

Which entry in the table gives the competitor's total displacement, average speed and average velocity for this part of the event?

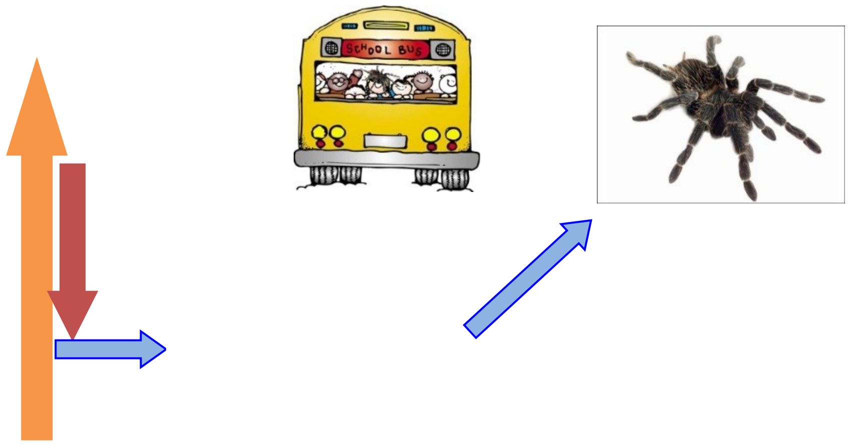
$$\text{average speed} = \frac{\text{distance}}{\text{time}} = \frac{2400}{10 \times 60} = 4.0s$$

$$\text{velocity} = \frac{\text{distance}}{\text{time}} = \frac{1000}{600} = 1.7\,m\,s^{-1}$$



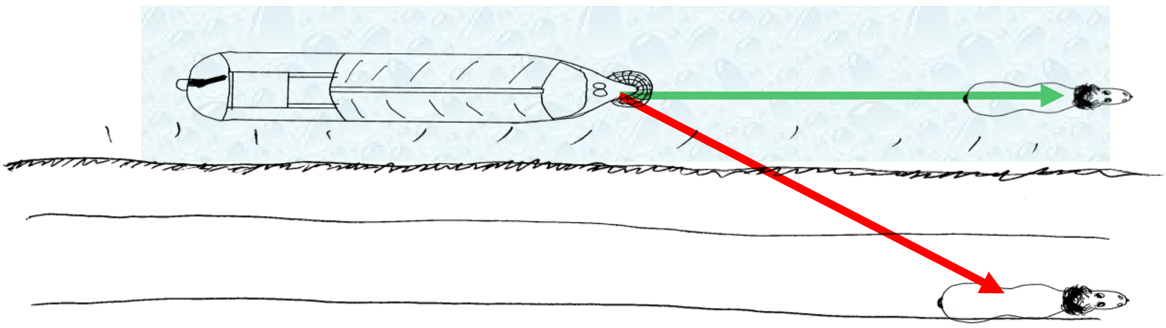
RESOLVING VECTORS

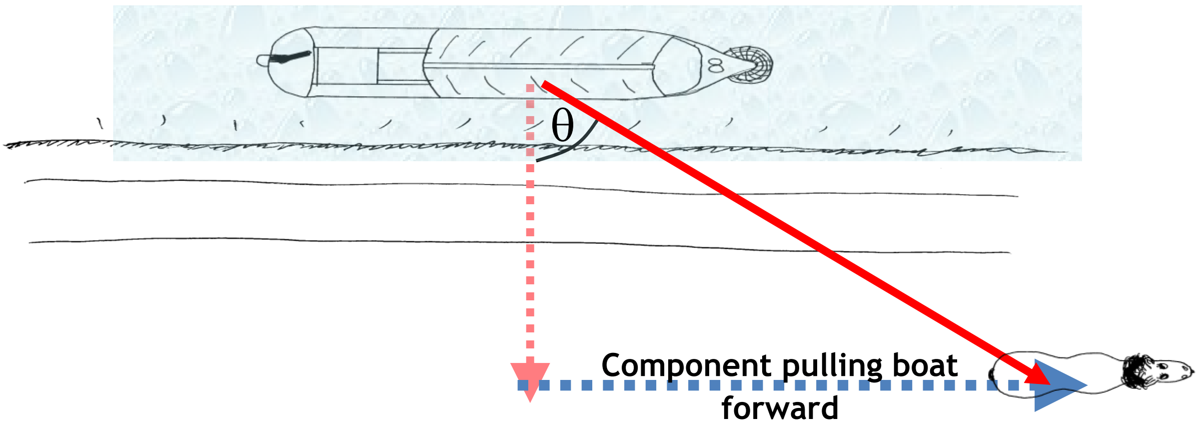
When you find the resultant of a number of different vectors you are finding the effect of all the vectors acting on an object. For example if a spider is walking across your head as you are walking down a bus which is travelling north, the spider's displacement is not just the direct path across your head but also depends on your direction down the bus as well as the direction of the bus.



Sometimes however it is useful to break vectors down into how they affect the object say horizontally or vertically, forward or sideways.

For example, consider a horse pulling a barge along a canal. We are only interested in the ability of the horse to pull the boat through the water. For maximum forward motion the horse should be pulling directly in front of the boat. The problem with this is that not many horses walk on water and are consigned to the towpath. When this happens part of the work done by the horse pulls the boat towards the bank. We can resolve the force from the horse, transmitted through the rope into the part that pulls the boat forward and the part that pulls the boat towards the bank.



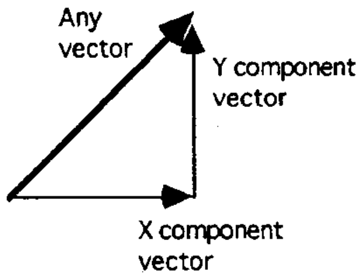


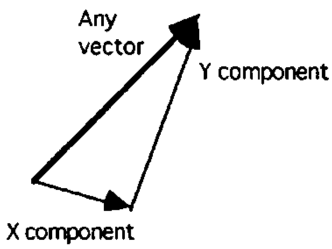
Weetabix Question: Explain why the boat doesn't crash into the bank.

BREAKING DOWN VECTOR QUANTITIES INTO COMPONENT PARTS.

Any vector can be made by adding two components which for convenience we put at right angles to each other.We call them rectangular components, X along and Y up.

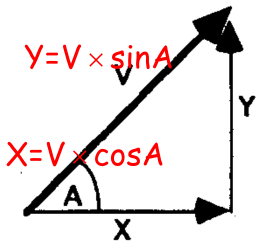
The X and Y vectors are the rectangular components of the main vector.





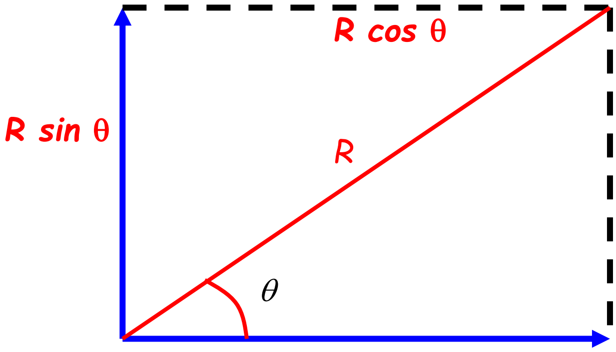
We have an infinite choice of rectangular components so we can choose the pair which suits us. Usually we take components as horizontal and vertical, but sometimes it is more convenient to take these as parallel to a slope and into the slope.

All components are calculated like this:



$Y = V \times \sin A$ ....(1)

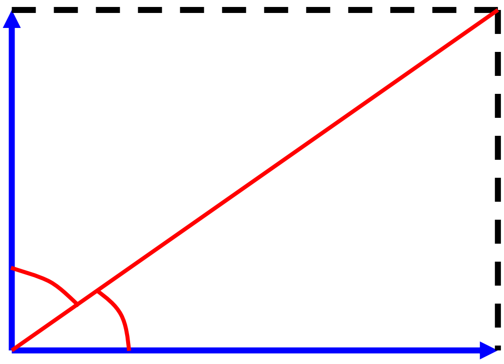
$X = V \times \cos A$ ....(2)



Adjacent X ***R cos θ***

$$R^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x}$$



opposite θ
y

adjacent θ X

$$\alpha = 90 - \theta$$

PYTHAGORAS

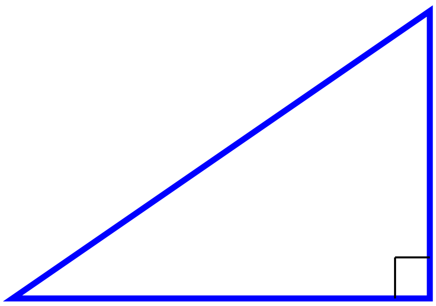
$$c^2 = a^2 + b^2$$

$$\sin\theta_1 = \frac{b}{c} \qquad \cos\theta_1 = \frac{a}{c}$$

$$\sin\theta_2 = \frac{a}{c} \qquad \cos\theta_2 = \frac{b}{c}$$

$$\therefore \sin\theta_1 \equiv \cos\theta_2$$

$$\tan\theta_1 = \frac{b}{a} \qquad \tan\theta_2 = \frac{a}{b}$$



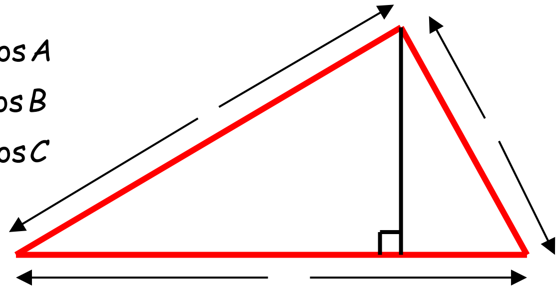
COSINE RULE

The cosine rule for a triangle states that:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

OR $\quad b^2 = c^2 + a^2 - 2ca\cos B$

OR $\quad c^2 = a^2 + b^2 - 2ab\cos C$



To prove these formula consider the following triangle, ABC:

Drop a line from C to form a perpendicular with AB at F.

$CF = b\sin A \qquad$ and $\qquad AF = b\cos A$

so $\quad BF = AB - AF = c - b\cos A$

Using Pythagoras' theorem in the triangle BFC:

$$\begin{aligned} BC^2 &= BF^2 + CF^2 \\ \text{or} \quad a^2 &= (c - b\cos A)^2 + b^2\sin^2 A \\ &= c^2 - 2bc\cos A + b^2\left(\sin^2 A + \cos^2 A\right) \\ &= b^2 + c^2 - 2bc\cos A \end{aligned}$$

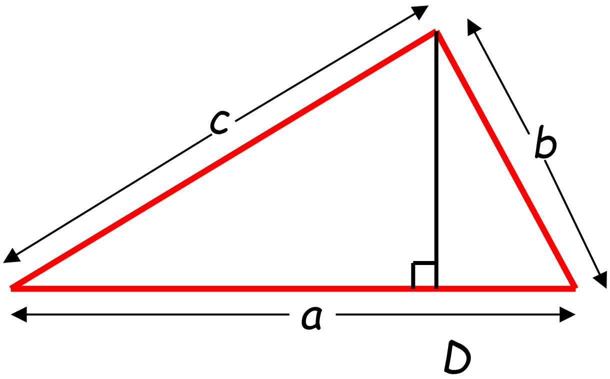
SINE RULE

The sine rule for a triangle states that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To prove these formula consider the following triangle, ABC:

Drop a line from C to form a perpendicular with BC at D.



$$AD = c\sin B = b\sin C$$

$$\therefore \frac{b}{\sin B} = \frac{c}{\sin C}$$

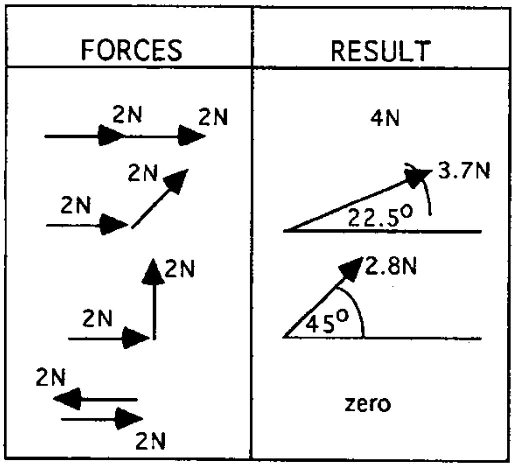
RESOLVING VECTORS

RESULTANT FORCES

The resultant of a number of forces is the single force which has the same effect as the several forces actually acting on the object.

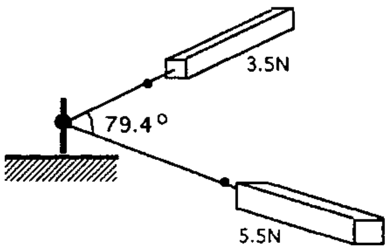
To find the resultant of several vectors we add them. The direction of vector quantities makes their addition more difficult.

e.g. Two forces, each of 2 N, can be added to give any result between zero and 4 N.



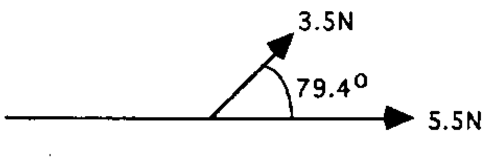
ADDING VECTOR QUANTITIES.

A force of 3.5 N pulls at an angle of 79.4° to a force of 5.5 N as shown.

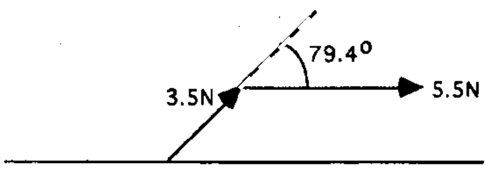


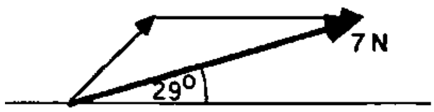
What single force are these equal to?

VECTOR DIAGRAM:



This is the same as and is equivalent to





which is 7 N at an angle of 29° to the direction of the 5.5 N force.

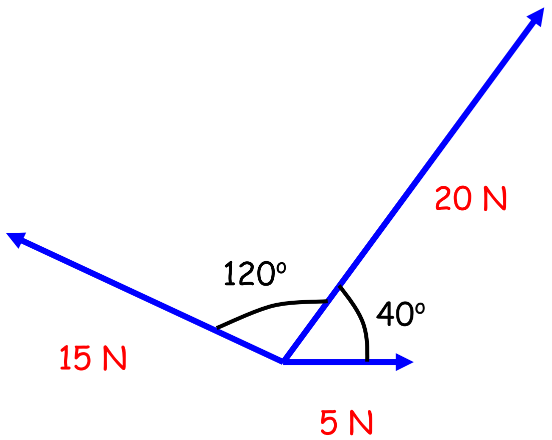
RESOLVING VECTORS BY SCALE DIAGRAMS

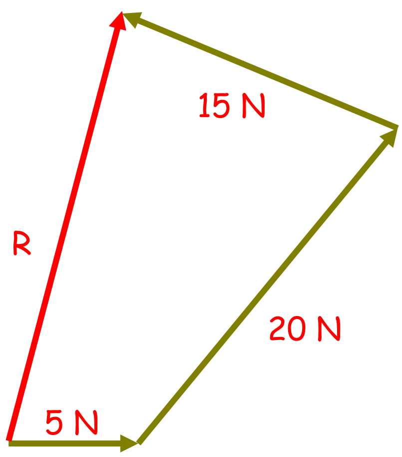
1. Always draw a small plan to show all vectors. Include the direction.
2. Choose an adequate scale (as large as possible but which fits on the page). Use an easy conversion.
3. Use a sharp pencil and draw the first vector (any one to start).
4. Put an arrow on the end and ensure you have clearly indicated the start.
5. On the head of the first vector draw the next vector. Ensure the angles are measured correctly and accurately.
6. Keep adding vectors until all are drawn.
7. CHECK!
8. Draw a line from the START of the FIRST to the HEAD of the FINAL vector. This is the RESULTANT.
9. Find the angle and measure or calculate it. DON'T MISS THIS OFF!

EXAMPLE OF RESOLVING VECTORS

We've met scale diagrams before, especially with forces. Remember: HEAD TO TAIL RULE or COMPONENTS. Don't forget that FORCES DOWN A SLOPE ARE A SPECIAL CASE which we'll deal with in the Forces section.

RESOLVING BY SCALE DIAGRAM





Measure: 19 N at 71° N of E.

RESOLVING BY COMPONENTS

Example 1

Horizontally (L-R +ve)

$$
\begin{aligned}
F_H &= 5 + 20\cos 40 - 15\cos 20 \\
&= 5 + 15.3 - 14.1 \\
&= 6.2\,N
\end{aligned}
$$

Vertically (Upwards +ve)

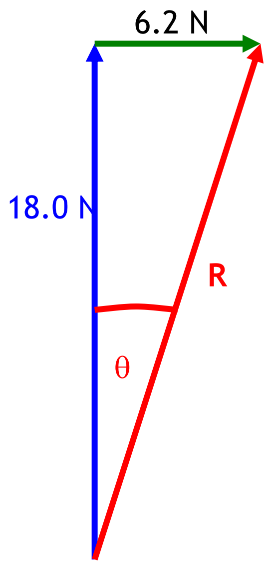
$$
\begin{aligned}
F_V &= 20\sin 40 + 15\sin 20 \\
&= 12.9 + 5.1 \\
&= 18.0\,N
\end{aligned}
$$

$F_v^2 = 6.2^2 + 18^2 = 38.4 + 324 = 362.4$

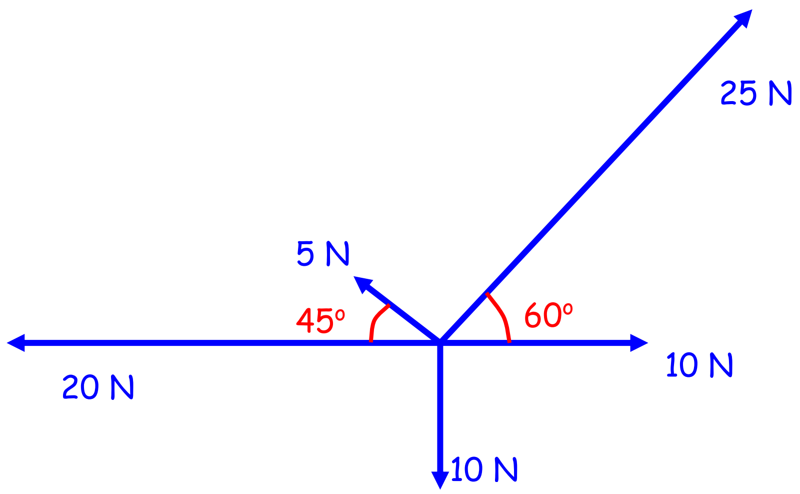
$F_v^2 = \sqrt{362.4} = 19.0N$

$tan\theta = \dfrac{6.2}{18} = 0.344$

<u>θ=19°</u>



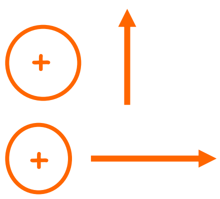
Example 2



1. **Resolve vectors horizontally:**
2. **Choose a direction to be positive.**
3. **Find components to each vector.**

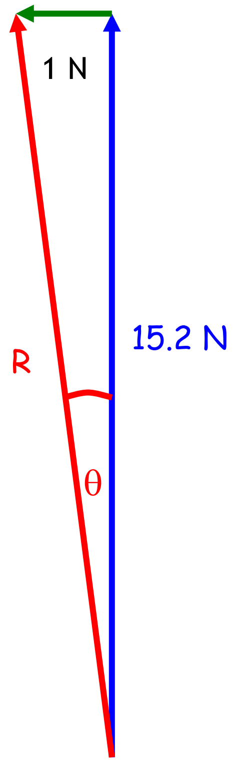
$\mathbf{Horizonal\ vector}\ \boldsymbol{F_h} = \mathbf{10 + 25cos60 - 5cos45 - 20 + 0 = -1N}$

4. **Resolve vectors vertically:**
5. **Choose a direction to be positive.**
6. **Find components to each vector.**



$$\mathbf{Vertical\ vector}\ \boldsymbol{F_v} = \mathbf{0 + 25sin60 + 5sin45 - 10 + 0 = 15.2N}$$

7. Find the resultant vector by Pythagoras:



Pythagoras states that the square of the long side of a right-angled triangle is the sum of the squares of the other two sides.

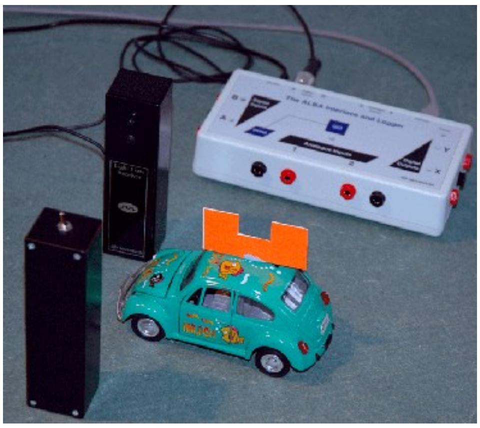
$$R^2 = a^2 + b^2$$
$$R^2 = 15.2^2 + 1^2$$
$$R^2 = 231.04 + 1$$
$$R = \sqrt{232}$$
$$R = 15.2N$$

Find θ:

$$\tan\theta = \frac{b}{a}$$
$$\tan\theta = \frac{1}{15.2} = 0.07$$
$$\theta = 3.9°$$

ACCELERATION- RATE OF CHANGE OF VELOCITY

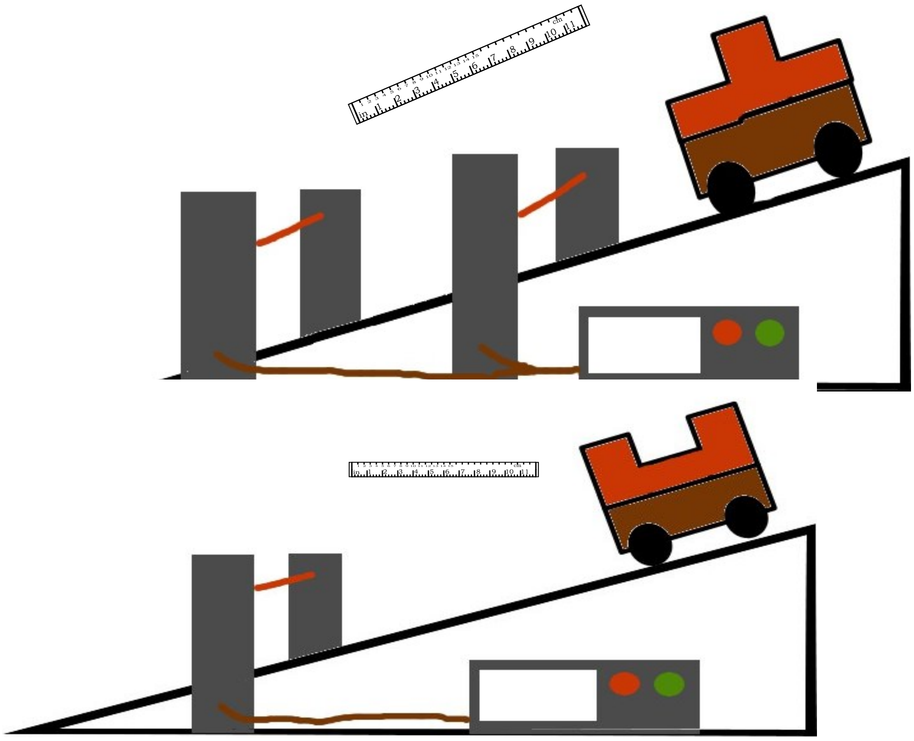
Acceleration is the rate of change of velocity. Acceleration is a vector quantity and is measured in metres per second squared, (m s^{-2}).



$$(ms^{-2})\ \text{acceleration} = \frac{\text{change in velocity}}{\text{time for the change}} \left(\frac{ms^{-1}}{s}\right)$$

$$a = \frac{dv}{dt} = \frac{v-u}{t}$$

Acceleration can be measured with a double mask and one set of light gates/ light bridges or a single mask and two sets of light gates/bridges.



Measurements	Calculations
t_1 time to pass first light gate	$u = \frac{l}{t_1}$
t_2 time to pass second light gate	$v = \frac{l}{t_2}$
t_3 time between light gate	$a = \frac{v-u}{t_3}$
length of mask	l

PRACTICAL 1: ANGLE OF SLOPE AND ACCELERATION

Using equipment of your choice investigate the effect of the angle of a slope on acceleration. Look back at these notes to be sure that you know what relationship you should be looking for.

Produce a full write up including an estimation of uncertainty in the experiment. A sheet is available to help you with your write up.Do not complete if you've already done this in the introduction booklet.

CHAPTER 2: EQUATIONS OF MOTION

SUMMARY OF CONTENT EQUATIONS OF MOTION

No		CONTENT
3.		**Equations of Motion**
	eq	$d = \bar{v}t,\ s = \overline{vt}\,;\ \ s = \frac{1}{2}(u+v)t;$ $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$
	a)	I can use the equations of motion to find distance, displacement, speed, velocity, and acceleration for objects with constant acceleration in a straight line.
	b)	I can interpret and draw motion-time graphs for motion with constant acceleration in a straight line, including graphs for bouncing objects and objects thrown vertically upwards.
	c)	I know the interrelationship of displacement-time, velocity-time and acceleration- time graphs.
	d)	I can calculate distance, displacement, speed, velocity, and acceleration from appropriate graphs (graphs restricted to constant acceleration in one dimension, inclusive of change of direction).
	e)	***I can give a description of an experiment to measure the acceleration of an object down a slope***

THE EQUATIONS OF MOTION

There are “lots” of equations of motion that you need to know for this course and some that it is helpful to know. For all equations you are expected to know and define the terms.

s = displacement, u = initial velocity, v = final velocity a = acceleration, t = time

$_h$ = horizontal, $_v$ = vertical

Vital equations	Helpful Equations
$v = u + at$ $v = \frac{s}{t}$ $v^2 = u^2 + 2as$ $s = ut + \frac{1}{2}at^2$ $\bar{v} = \frac{(v+u)}{2}$ (for a vehicle with constant acceleration)	$s = vt - \frac{1}{2}at^2$ $v = u + at$ $v^2 = u^2 + 2as$ $s = ut + \frac{1}{2}at^2$

PROVING THE FIRST EQUATION OF MOTION

$$v = u + at$$

This equation comes from the definition of acceleration

"acceleration is the rate of change of velocity"

$$a = \frac{dv}{dt} = \frac{\text{change in velocity}}{\text{time for the change}}$$

$$a = \frac{v-u}{t_v - t_u}$$

$a = \dfrac{v-u}{t}$ where t is the time for the change of velocity

rearrange

$\underline{v = u + at}$

PROVING THE SECOND EQUATION OF MOTION

$$s = ut + \tfrac{1}{2}at^2$$

This equation comes from the fact that displacement is the area under a velocity time graph.

displacement = area under a velocity time graph

$s = (b \times h) + (\tfrac{1}{2}b \times h)$

$s = ut + \tfrac{1}{2}t \times (v - u)$

but

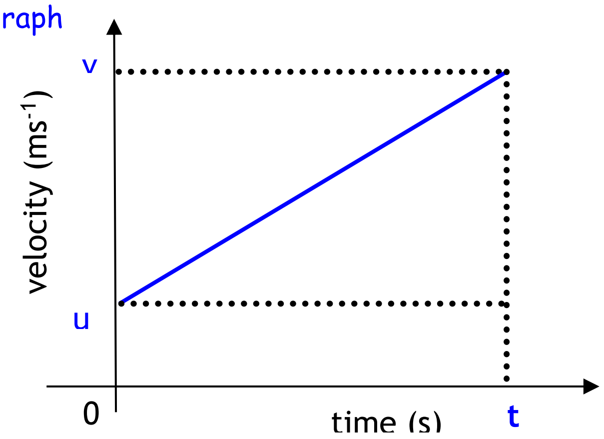
$a = \dfrac{v-u}{t}$ or $(v-u) = at$

so substitute

$s = ut + \tfrac{1}{2}t \times (at)$

rearrange

$\underline{s = ut + \tfrac{1}{2}at^2}$



PROVING THE THIRD EQUATION OF MOTION

There are many different ways of proving this equation and I have looked through them all. I found this one easiest to tackle but if you like another method then use that but do check it out with me first to make sure that it is viable!

start with the equation closest to what we want to prove!

$v = u + at$

square both sides

$v^2 = (u + at)^2$

expand the brackets (here is the first problem!)

$v^2 = (u + at)(u + at)$

$v^2 = u^2 + uat + uat + a^2t^2$

(make sure you've squared both a and t in the final term)

collect the terms

$v^2 = u^2 + 2uat + a^2t^2$

it is beginning to look close to what we want!

(here is the second problem!

substituting x1 changes nothing but makes life a little easier)

Substitute $\frac{2}{2} = 1$

$v^2 = u^2 + 2uat + \frac{2}{2}a^2t^2$

Take out the common factor, 2a

$v^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$

BUT (and here is the good bit!)

$s = ut + \frac{1}{2}at^2$

Replace

$\underline{v^2 = u^2 + 2as}$

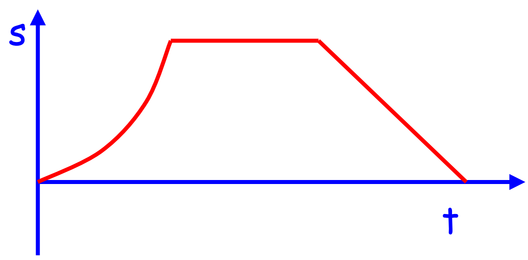
MOTION GRAPHS

Type of graph	Gradient	Area under graph
Displacement - time	Velocity	-----
Velocity - time	Acceleration	Distance travelled (displacement)
Acceleration - time	-----	Change in velocity (v-u)

A displacement - time graph is describing how the displacement of an object is changing with time. We know that:

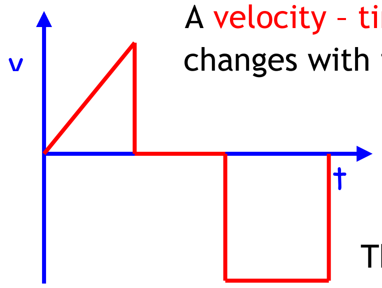
$v = \frac{ds}{dt}$ $\quad i.e. \quad \frac{\text{change in displacement}}{\text{time taken to change}}$

$v = \frac{s_2 - s_1}{t_2 - t_1}$



Therefore velocity must be equal to the gradient of a displacement - time graph.

$$\text{gradient} = \frac{\text{rise}}{\text{run}} \left(\frac{\text{displacement}}{\text{time}}\right)$$



A velocity - time graph describes how the velocity of an object changes with time. We know that

$$a = \frac{dv}{dt} = \frac{\text{change in velocity}}{\text{time to change}}$$

Therefore acceleration must be equal to the gradient of a velocity - time graph

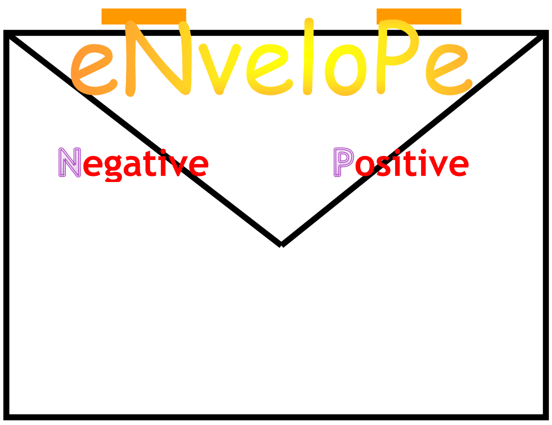
$$\text{gradient} = \frac{\text{rise}}{\text{run}} \left(\frac{\text{velocity}}{\text{time}}\right)$$

The area under a velocity - time graph = base × height

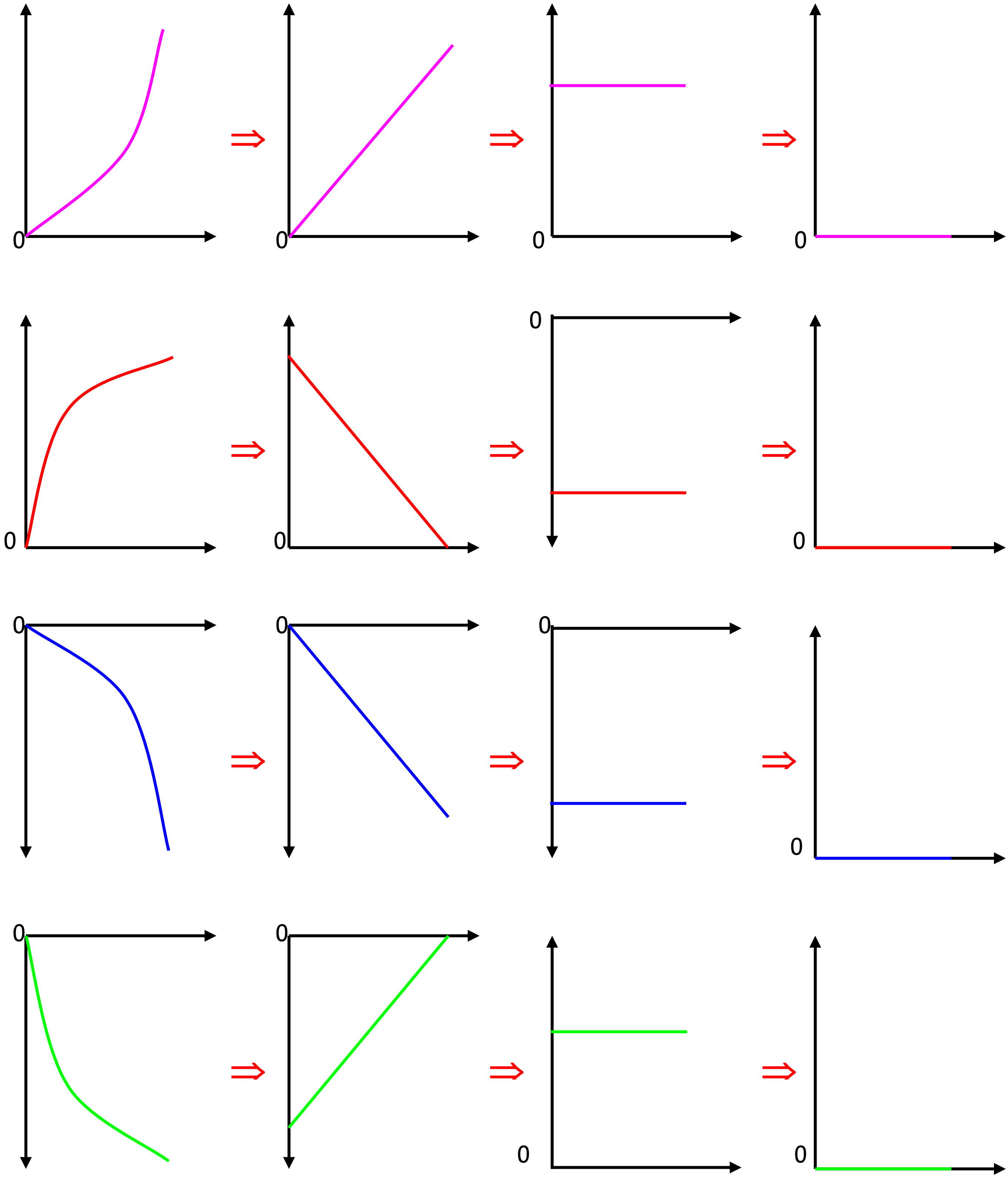
= time × velocity

must therefore be equal to displacement, $s = v \times t$.

GRADIENTS OF GRAPHS

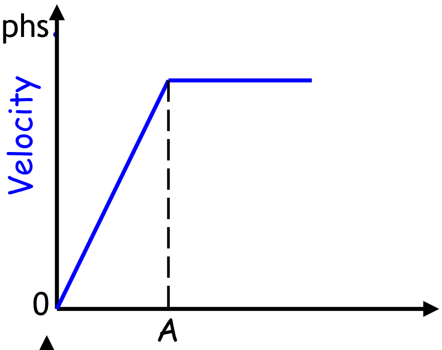


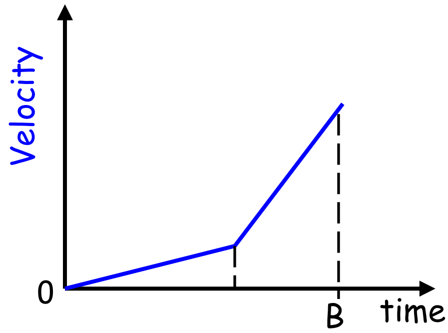
On the next page you'll see a set of graphs. Each graph shows a kinematic quantity plotted against time. You can decide on the quantity. The arrows indicate that the graph on the right is a graph of the gradient of the previous graph. For example if the second graph on the second line represents displacement against time the graph to the right of this would be the corresponding velocity-time and the last one on the second row is the corresponding acceleration-time graph. Therefore if you learn the patterns in these graphs it will help you in your revision and should speed up how you manage to answer some questions.

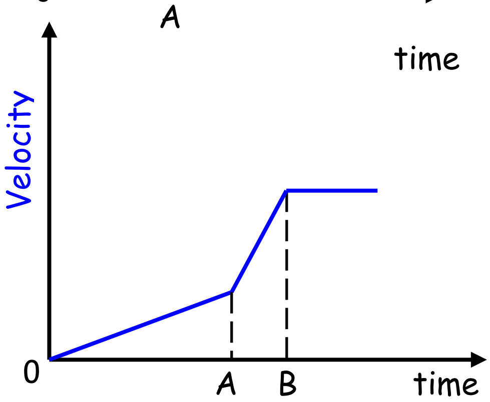


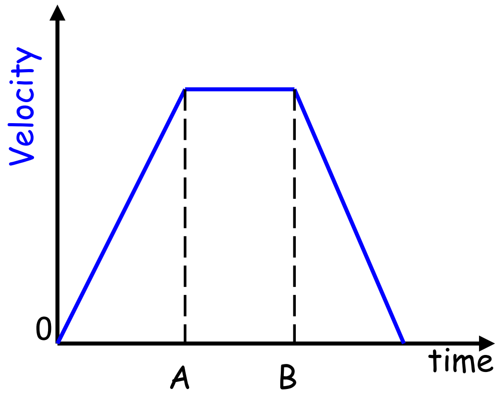
TASK

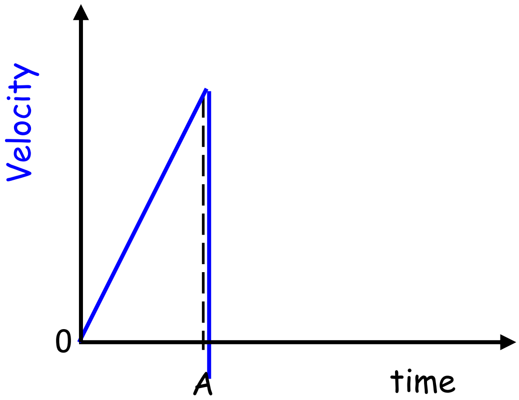
Sketch the acceleration-time graphs for each of the following velocity-time graphs:

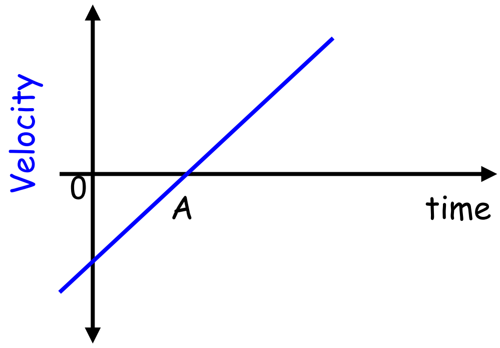












PATTERNS OF RESULTS IN TABLES

Constant velocity results are the most obvious to spot. They are either steadily increasing if they are displacement measurements or they are all about the same if they are velocity measurements.

With constant acceleration, the acceleration figures are constant while the velocity figures increase steadily.

Tables of results for constant acceleration that show displacement figures are a little more difficult because they are not always quoted in the same way. If it is a straightforward set of ‘distance from the start’ figures, then they show an increase that is itself increasing.

An object falling under the influence of the force of gravity.

Distance fallen /m	0	5	20	45	80	125	180	245	320	405	500
Time of fall /s	0	1	2	3	4	5	6	7	8	9	10

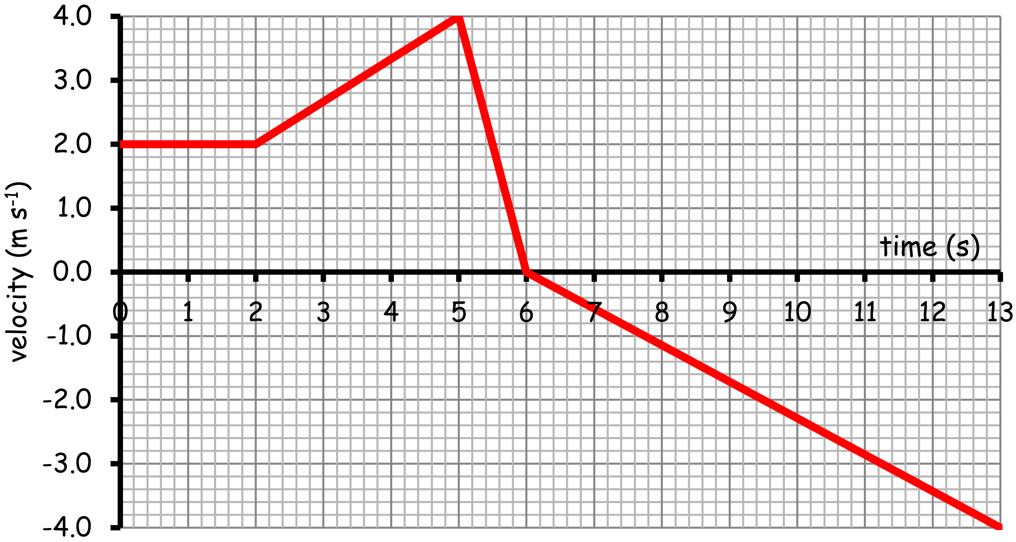
These displacement figures are proportional to t^2. Sometimes the displacement measurements are given as part of a strobe photograph exercise in which case the figures are displacements that occur during each interval of time. In this case steady acceleration shows up as a steady increase in the displacement measurements.

A stroboscopic photograph using a 1.0 Hz flash of the same object as above gives the following information:

Displacement between Exposures /m	5	15	25	35	45	55	65	75	85	95
Time from start /s	1	2	3	4	5	6	7	8	9	10

Find the displacement from the area under the velocity-time graph and the acceleration from the gradient of the velocity-time graph.

The figure below represents the velocity of a particle moving in a straight line.

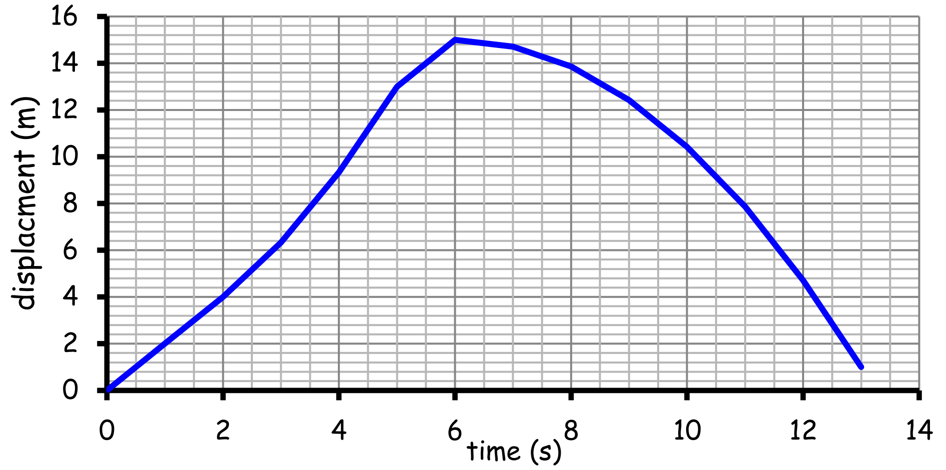


Firstly, describe the graph in words to yourself (roughly).

- Constant velocity / zero acceleration for 2 seconds.
- Constant acceleration from 2 to 5 seconds.
- Deceleration to rest in 1 second.

- Acceleration in opposite direction for 7 seconds.

Below are the corresponding displacement-time and acceleration-time graphs.



ESSENTIAL PRACTICALS

PRACTICAL 1: ACCELERATION

In this exercise, you are asked to use your understanding of the definition of acceleration to guide your selection and use of the apparatus.

The apparatus consists of a ramp, a trolley with a mask attached for operating photo-diode switches, and two photo-diode switches each connected to an electronic timer. In addition, you can use a hand-held stopwatch to find the time taken by the trolley to pass between the two photo-diode switches.

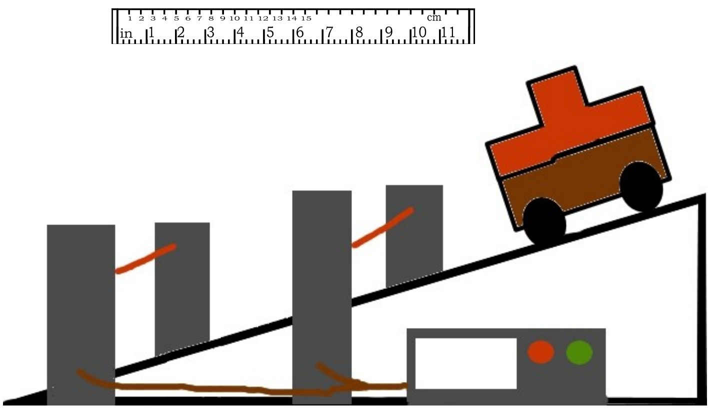
Use the apparatus to find the acceleration of the trolley as it rolls down a moderately sloped ramp.

Provide a written account of your method including how you arrived at your value for the acceleration.

PRACTICAL 2: ACCELERATION

Aim: To measure the acceleration of a trolley moving down a slope using a computer.

Apparatus: 1 ramp, 1 trolley and single mask, 2 light gate, computer and interface, 1 power supply

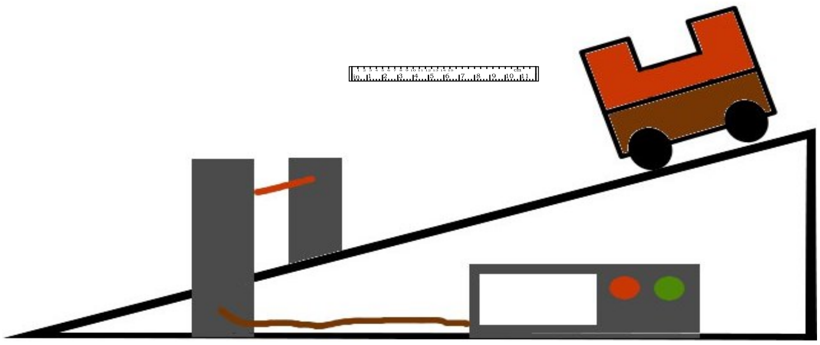


Instructions:

- Set up the apparatus as shown in the diagram.
- After selecting the acceleration programme, allow the trolley to run down the track.
- Note the value of the acceleration.
- Repeat 5 times. Calculate the mean acceleration and random uncertainty.
- Explain, in detail, how the mask arrangement allows the computation of the acceleration.

PRACTICAL 3 ACCELERATION

Aim: To measure the acceleration of a trolley moving down a slope using a computer.



Apparatus: 1 ramp, 1 trolley and double mask, 1 light gate, computer and interface, 1 power supply

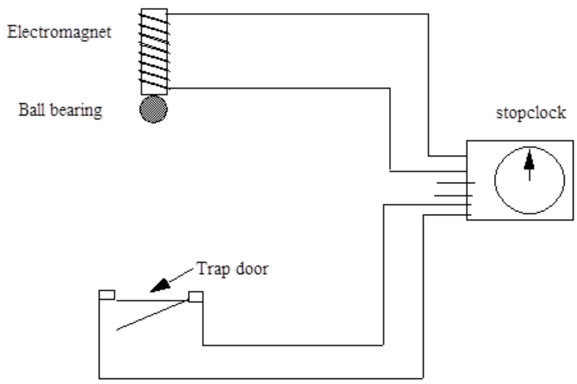
Instructions:

- For 5 different angles of slope find the corresponding acceleration.
- Using an appropriate format to find the relationship between the angle of slope and the acceleration.

PRACTICAL 4: EQUATIONS OF MOTION

Aim: To calculate the acceleration due to gravity using

Apparatus: electromagnet, trap door, stop clock, metre stick, ball bearing.



Instructions:

- Measure the distance between the bottom of the ball bearing and the trap door.
- Allow the ball to fall several times and find the average time it takes to reach the trap door.
- Show that $s = ut + \frac{1}{2}at^2$ the equation in this case reduces to $h = \frac{1}{2}at^2$,

  where *h* is the height dropped and *g* is the acceleration due to gravity.
- Use the equation to estimate the acceleration due to gravity.
- Estimate the uncertainty in your answer.
- How could the method be improved.

PRACTICAL 5 USING $v^2=u^2+2as$

- Hold the card above the light gate and next to the ruler so that its height above the gate may be measured carefully.
- Release the card so that it cuts through the light beam; a velocity measurement should appear in the table on the screen.
- Repeat this measurement from the same height several times; enter the height value in the height column of the table in the computer program.
- Repeat this procedure for a new starting height 2 cm above the first.
- Collect a series of measurements, each time increasing the height by 2 cm.

Analysis

- Depending upon the software, the results may be displayed on a bar chart as the experiment proceeds. Note the relative increase in values of velocity as greater heights are chosen.
- The relationship between velocity and height fallen is more precisely investigated by plotting a XY graph of these two quantities. (Y axis: velocity; X axis: height fallen.)
- Use a curve matching tool to identify the algebraic form of the relationship. This is usually of the form 'velocity is proportional to the square root of height'.
- Use the program to calculate a new column of data representing the square of the velocity. Plot this against height on a new graph. A straight line is the usual result, showing that the velocity squared is proportional to the height fallen.

CHAPTER 3: FORCES,ENERGY AND POWER

SUMMARY OF FORCES, ENERGY & POWER

4.		Forces, energy and power
	eq	$W = mg$ $F = ma$ Ew or $W = Fd$ $Ep = mgh$ $Ek = ½\, mv^2$ $E = Pt$
	a)	I can use vector addition and appropriate relationships to solve problems involving balanced and unbalanced forces, mass, acceleration and gravitational field strength.
	b)	I can identify and explain the effects of friction on moving objects. I do not need to use reference to static and dynamic friction.
	c)	I can identify and explain terminal velocity, in terms of forces
	d)	I can interpret and produce velocity-time graphs for a falling object when air resistance is taken into account.
	e)	I can analyse motion using Newton's first and second laws.
	f)	I can use free body diagrams and appropriate relationships to solve problems involving friction and tension.
	g)	I can resolve a vector into two perpendicular components.
	h)	I can resolve the weight of an object on a slope into a component acting parallel (down the slope) and a component acting normal to the slope.
	i)	I can use the principle of conservation of energy and appropriate relationships to solve problems involving work done, potential energy, kinetic energy and power.

NEWTONS THREE LAWS OF MOTION

From previous courses it is assumed that you learned Newton's Three Laws of Motion, a corner stone of Physics.

NEWTON'S FIRST LAW:

A body will remain at rest or move at steady speed in a straight line unless acted upon by an unbalanced force.

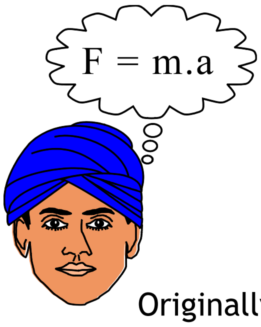
Or

Unless an unbalanced force acts on an object the object will move at constant velocity (which means constant speed in a straight line without quoted direction)

Or

A body will remain at rest or travel at constant velocity, unless acted upon by an unbalanced force.

NEWTON'S SECOND LAW



We normally write as a formula:

$$F_R = ma$$

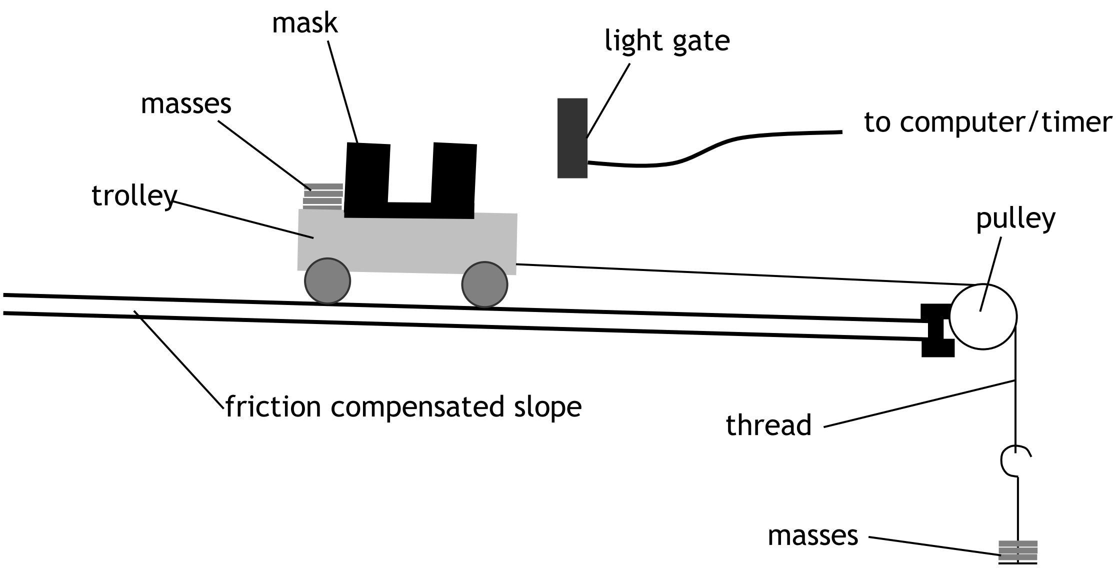
$$\text{Resultant Force} = \text{mass} \times \text{gravitational field strength}$$
$$(\text{Newtons}) = (\text{Kilogram}) \times (\text{Newtons per kilogram})$$

Originally written as **The rate of change of momentum equals force**

$$\frac{\Delta mv}{t} = F$$

$$F = \frac{m(v-u)}{t}$$

Where $a = \frac{(v-u)}{t}$



The apparatus shown above can be used to investigate the relationship between unbalanced force, mass and acceleration.

To find a relationship between three variables then two experiments must be carried out keeping one of the variables constant in each.

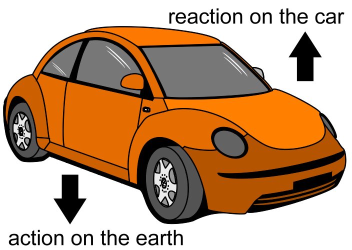
It is found that:

$a \propto F$ when m is constant

$a \propto \frac{1}{m}$ when F is constant

Hence: $a \propto \frac{F}{m}$

NEWTON'S THIRD LAW:



To every action there is an equal and opposite reaction.
(the action and reaction act on different bodies)

For every action there is an equal but opposite reaction.

or

If A exerts a force on B, B exerts an equal but opposite force on A

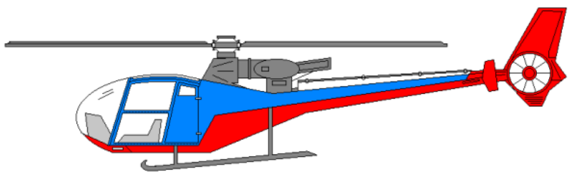
BALANCED FORCES VERSUS NEWTON PAIRS

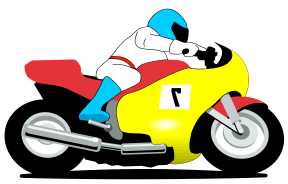
If the ship is travelling at constant velocity then:-

Weight is equal to the buoyancy force or upthrust, but in the opposite direction. Engine force is equal to the drag forces but in the opposite direction. These are the balanced forces and are directly caused by each other. Balanced forces are two separate forces acting on one object. A Newton Pair is a pair of forces acting on the two interacting objects. The size of the force on the first object equals the size of the force on the second object.

TASK: For each of these 4 balanced forces state the Newton Pairs.

If the helicopter is hovering at constant height the engine force is equal in size but opposite in direction to the weight. These are balanced forces. **TASK: State the Newton Pairs for each of these Forces.**





What forces are acting on the bike?

F_W (weight) $= F_r$ (reaction force from ground)

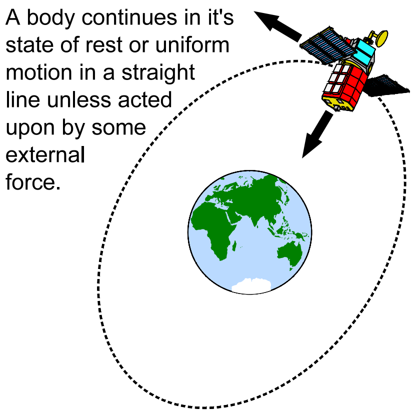
F_F (frictional) = F_E (engine)

TASK:

Which of these equal forces are Newton Pairs and which are balanced forces?

So how do we know that there is a resultant force on the satellite?

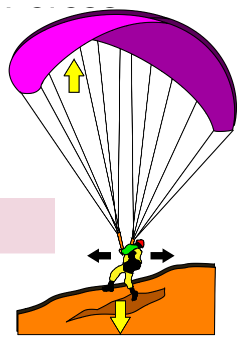
Name the balanced forces on the paraglider.



THE NEWTON

One Newton is equal to the force which causes an acceleration of one metre per second squared when applied to a mass of one kilogram.

RESULTANT FORCES

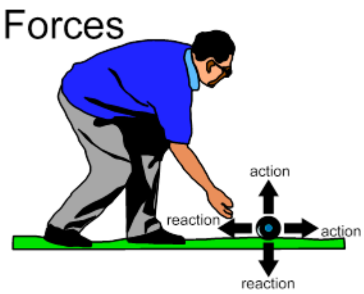


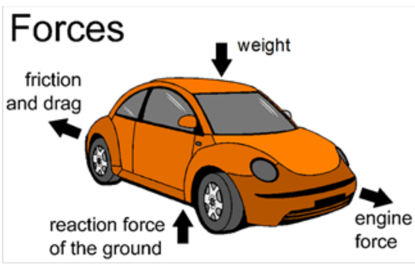
The resultant of a number of forces is the single force which has the same effect as the several forces actually acting on the object.

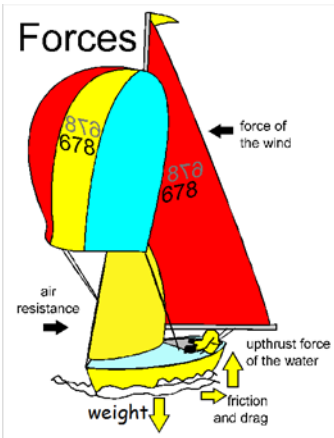
FREE BODY DIAGRAMS

Free body diagrams are diagrams that show all the forces (or sometimes components of forces) acting on a mass. If the size of the forces are known these can be added, if not they can be written in under the names of the forces.

E.g.







TASK: Draw a freebody diagram of:

a) you standing on the floor or sitting on a chair
b) Mostly Harmless (Mrs H's canal boat) going at 4 mph along the Forth and Clyde Canal.

VECTORS AND TENSION

WHAT IS A VECTOR?

A vector is a quantity where magnitude and direction are both important.

WHY ARE VECTORS IMPORTANT?

Calculations can be positive or negative e.g. equations of motion/momentum.

Sometimes we want to use components of vectors e.g. in projectile motion we only use the vertical or horizontal components in our calculations.

WHICH QUANTITIES ARE VECTOR QUANTITIES?

Force including **Tension, Weight, Friction,** etc.

Acceleration	**Displacement**	**Field strength**
Velocity	**Impulse**	**Momentum**

WHEN DO WE QUOTE DIRECTION FOR VECTOR QUANTITIES?

In all equations of motion questions direction should be noted, also in momentum calculations. If in doubt, add a direction, even if it is an arrow or just a left or right, up or down.

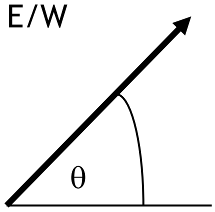
CASES TO BE AWARE OF:

In equation of motion questions you always use components if they are at an angle, but it may ask for the final velocity which is a vector quantity. Therefore you must recombine horizontal and vertical components.

A SPECIAL CASE OF RESOLVING VECTORS: FORCES DOWN A SLOPE.

We can take COMPONENTS of any vector quantity to help us with our calculations

Usually these are horizontal and vertical or N/S and E/W



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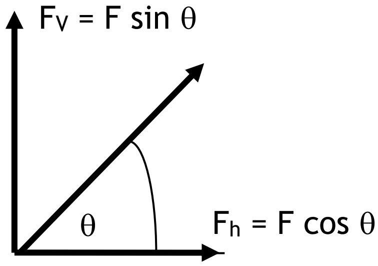
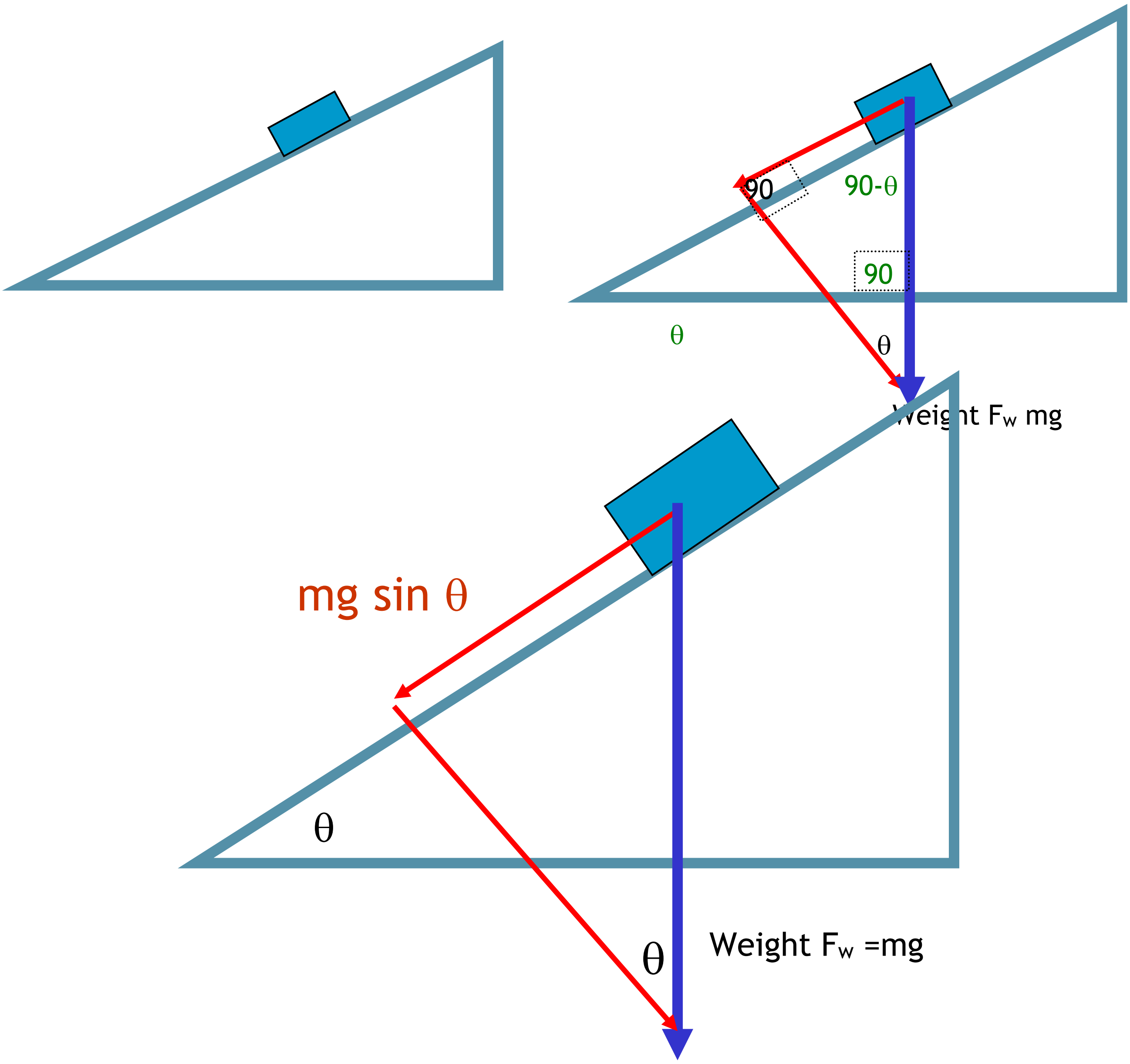
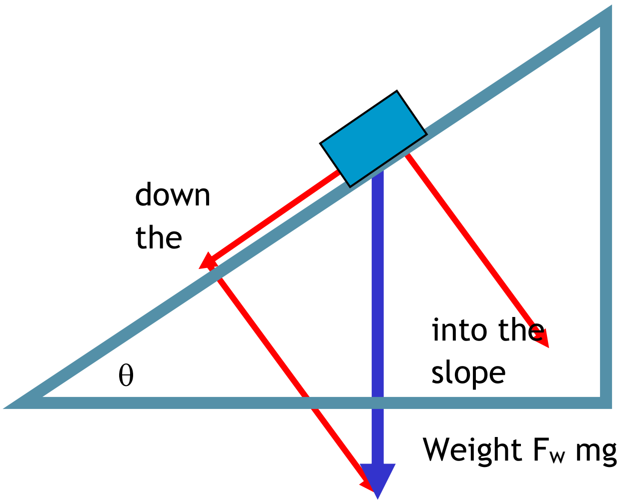


When objects are on an inclined plane (a slope!) we do not always want to separate the force into horizontal and vertical components but we want to separate then into components acting down the slope and at right angles to the slope.



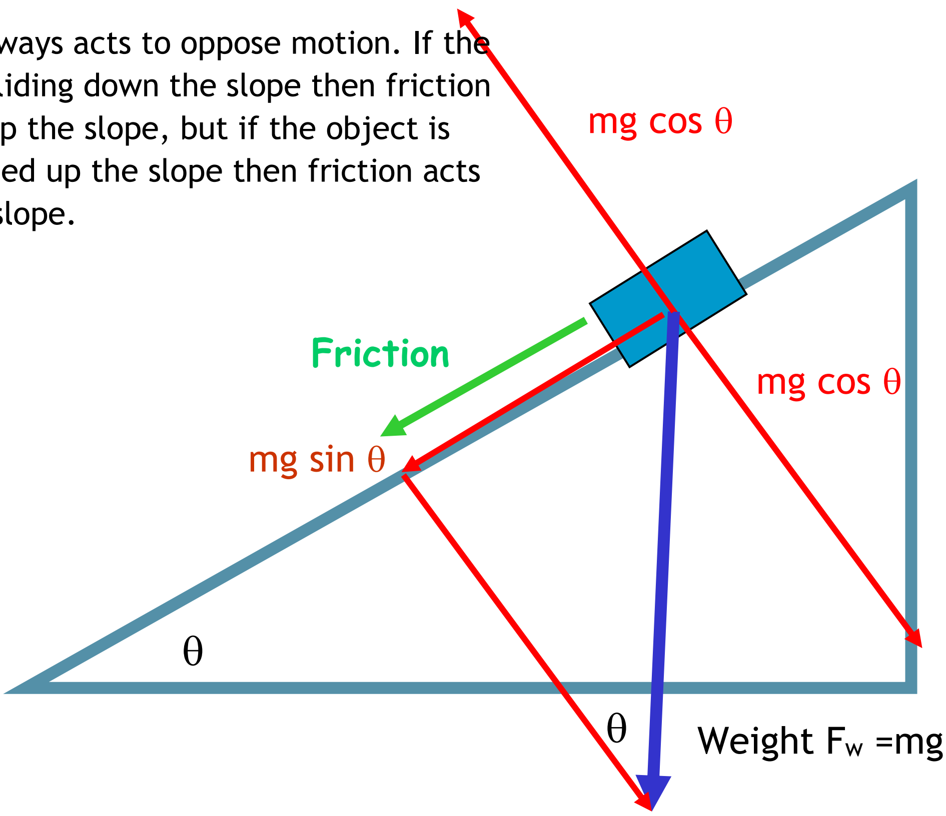
So the important component that gives the block the motion down the slope is **mg sin θ**

The component mg cos θ pushes into the slope. This force is balanced by an equal but opposite force from the slope and is called the reaction forceThis leavethe component parallel to the slope as the part responsible for causing the acceleration down the slope, but friction also plays a part in the accelerationaccording to Newton's second law.



FORCES WHEN AN OBJECT MOVES UP THE SLOPE

Friction always acts to oppose motion. If the object is sliding down the slope then friction must act up the slope, but if the object is being pushed up the slope then friction acts down the slope.



FORCES WHEN AN OBJECT MOVES DOWN THE SLOPE

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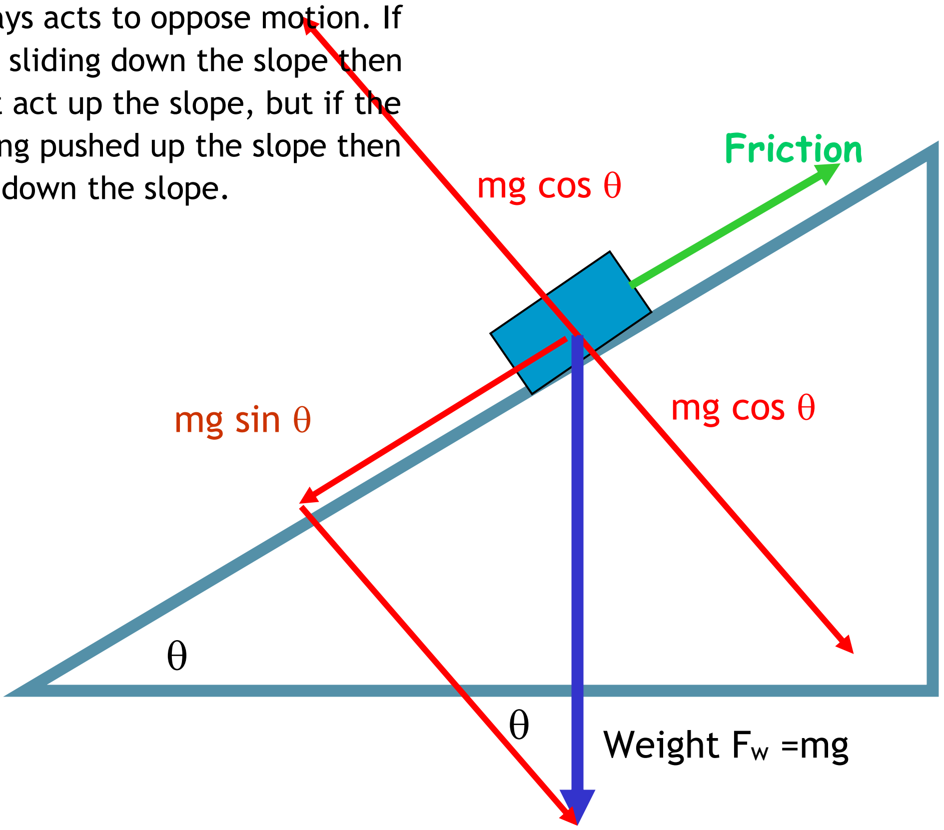
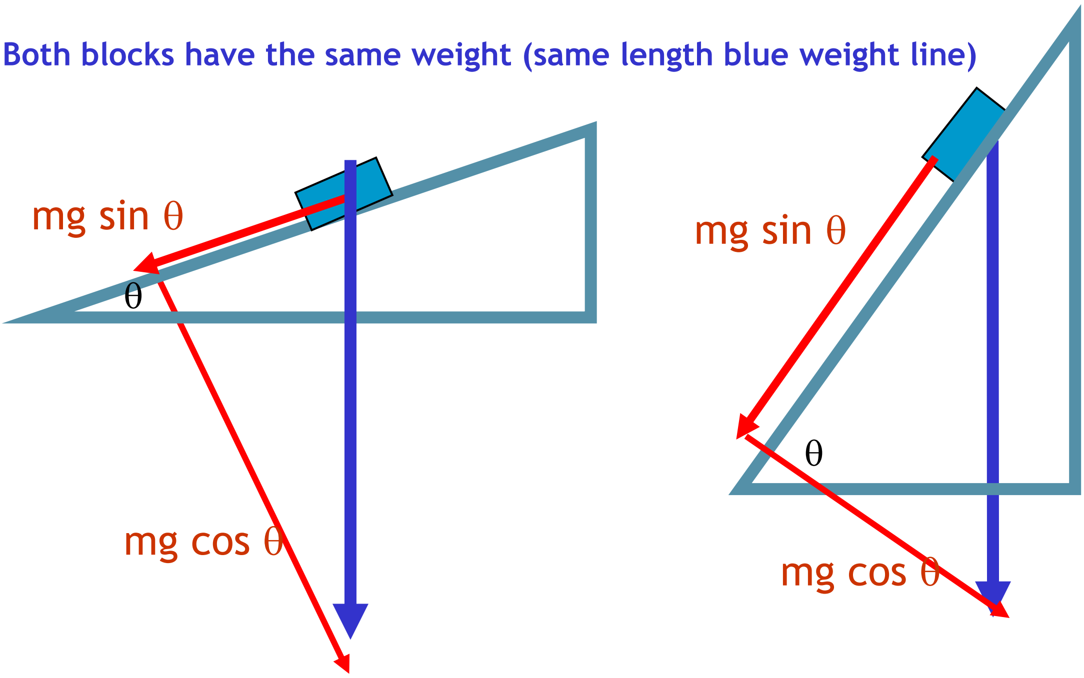
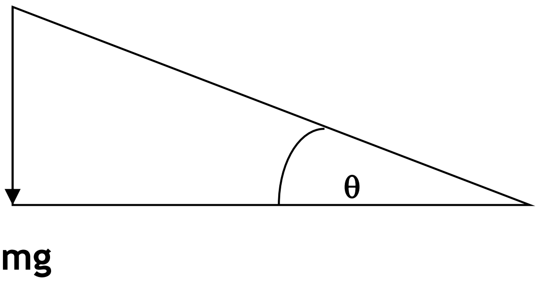


Notice as the slope gets steeper the component of weight down the slope increases, so there is more chance of the object moving!



A common mistake that students often make is that they think mg or weight is the vertical component and draw the diagram as below. They then try to calculate the force down the slope and find the hypotenuse. This gives them a value greater than the weight. THIS IS NOT POSSIBLE. Remember the weight is the hypotenuses of your diagram and you are taking components. The problem with this is that the component of the weight down the slope would be greater than the weight. To avoid this always draw the weight line longer than the slope (see previous diagrams)



wrong method so don't you do it

$$\sin\theta = \frac{opp}{hyp}$$

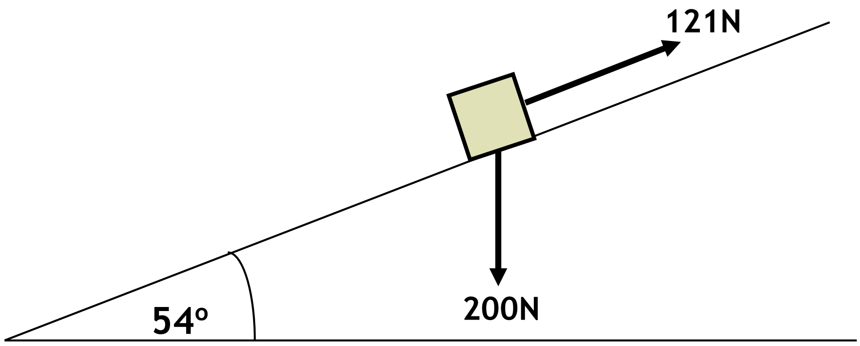
$$\sin\theta = \frac{mg}{hyp}$$

$$hyp = \frac{mg}{\sin\theta}$$ wrong answer!

The object will accelerate if there is an unbalanced force on an object. This comes from the **resultant** of all the forces. In most of the cases the acceleration is equal to the component of weight down the slope less the friction which slows the object down.

Example 1

a) Determine the mass of the block shown?
b) Determine the resultant force on the block?
c) Determine the acceleration of the block shown?



$F_w = mg \qquad 200 = m \times 9.8$

$\underline{\underline{m = 20.4kg}}$

$F_w \sin\theta = \text{component down the slope}$

$F_{un} = F_w \sin\theta - F$

$F_{un} = 200\sin 54 - 121$

$\underline{\underline{F_{un} = 40.8N}}$

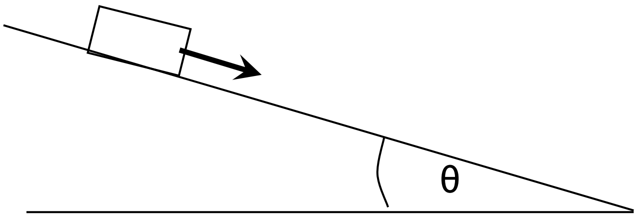
Notice $F_w \cos\theta$ is balanced from a reaction force from the slope

$F_{un} = ma$

$40.8 = 20.4 \times a$

$\underline{\underline{a = 2ms^{-2}}}$

MOTION ON A SLOPE REVISION TEST



If an object is on a slope then it may move down the slope. Depending on the forces acting on the object it may be travelling at a constant velocity or it may be accelerating.

1. Describe the forces acting on the object when:

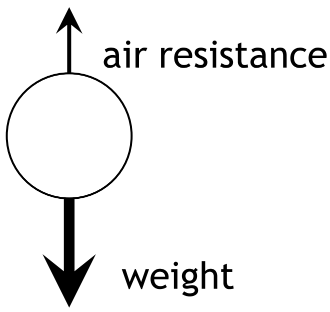
(a) it is moving with constant velocity.

(b) it is accelerating down the slope.

2. Explain what you think will happen to the acceleration if the angle is increased

For further notes on motion down a slope refer to the Powerpoint presentation on the network.

FREEFALL



When an object is allowed to fall towards the Earth it will accelerate because of the force due to gravity acting on it. This will not be the only force acting on it though. There will be an upwards force due to air resistance.

Air resistance increases with speed.

If an object is allowed to fall through a large enough distance then the force due to air resistance may increase to become the same magnitude as the force due to gravity.

When this situation occurs the forces acting on the object will be balanced. This means the object will fall with constant velocity or terminal velocity.

PARACHUTES- AN EXAMPLE OF TERMINAL VELOCITY

The instant the parachutist leaves the plane

- ✓ Her vertical speed is zero
- ✓ Air resistance is zero
- ✓ Weight is the unbalanced force on the parachutist
- ✓ The parachutist accelerates downwards according to F=ma
- ✓ The parachutist accelerates at 9.8 m s^{-2}

As the parachutist falls her speed increases

- ✓ AIR RESISTANCE/ DRAG increases
- ✓ WEIGHT remains constant
- ✓ There is still an unbalanced force on the parachutists but this is less than before.
- ✓ The parachutist accelerates downwards but the acceleration is **less than** $9.8ms^{-2}$
- ✓ The parachutist ***does not slow down but speeds up slower!***

Finally AIR RESISTANCE equals WEIGHT

1:http://www.bbc.co.uk/schools/gcsebitesize/science/add_gateway_pre_2011/forces/fallingrev1.sht

- ✓ The forces on the parachutist are now BALANCED (overall effect ZERO acceleration)
- ✓ The parachutist travels at CONSTANT SPEED (acceleration is zero)
- ✓ The parachutist travels at **TERMINAL VELOCITY**

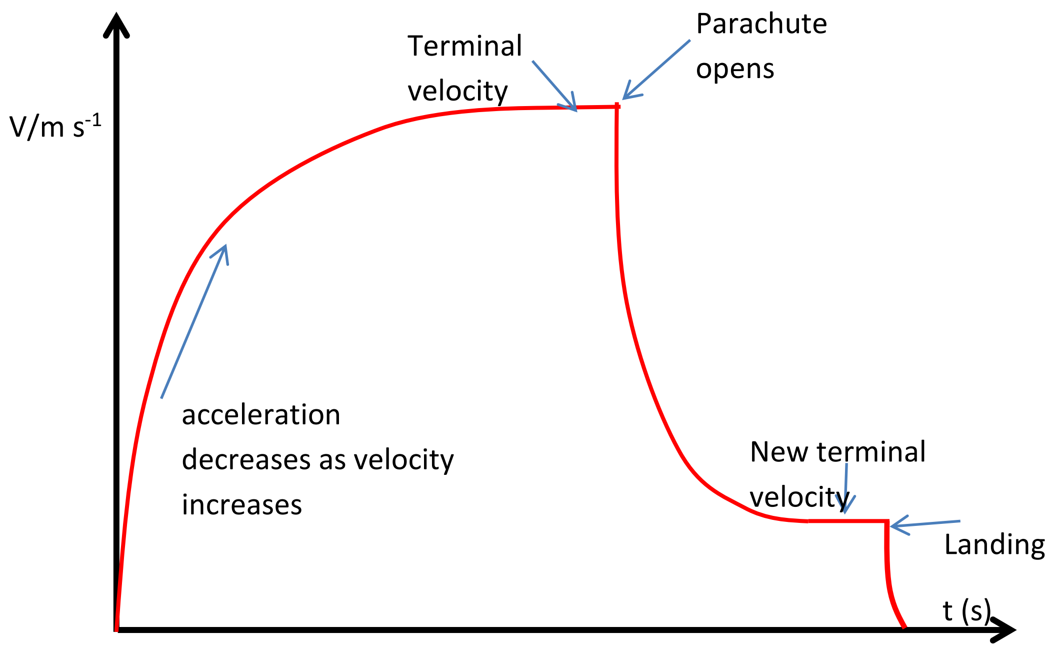
The parachute opens

- ✓ AIR RESISTANCE/ DRAG INCREASES
- ✓ WEIGHT remains constant
- ✓ AIR RESISTANCE>>WEIGHT
- ✓ There is an unbalanced force on the parachutist upwards
- ✓ The parachutist decelerates (slows down very quickly)

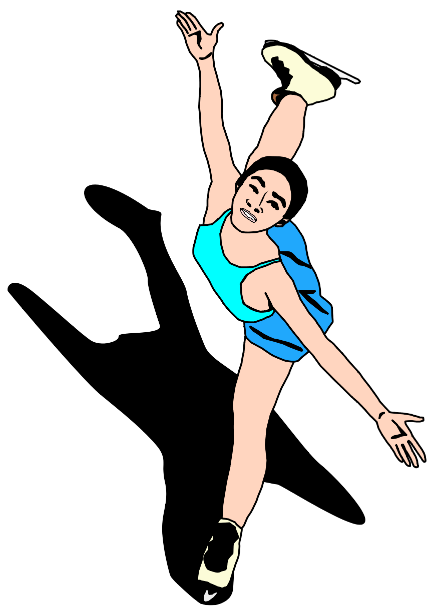
Is it true that you go upwards when the parachute is open? That is what you see on the TV. The Camera operator hasn't opened her parachute so begins to fall faster than the parachutist she is filming. To keep filming the parachutist, the camera operator, has to point her camera upwards, giving the illusion that the parachutist has shot upwards.

As the parachutist decelerates

- ✓ AIR RESISTANCE DECREASES
- ✓ Air resistance decreases until it equals the weight.
- ✓ The forces on the parachutist are now balanced.
- ✓ The parachutist travels at CONSTANT SPEED (acceleration is zero)
- ✓ The parachutist travels at a NEW TERMINAL VELOCITY
- ✓ Obviously <u>less than</u> before but still enough to break a leg on impact.



FRICTION



In winter sports, we need friction to be as low as possible so that we can achieve high speeds.

Ice skaters actually move on a layer of water, and don't skate on ice at all. When ice is subjected to high pressure it melts.

The narrow blades of the skates create a very high pressure and thus the skaters glide along on a layer of water they've just melted. The water refreezes as soon as they've moved on.

This is called "regelation" (sounds like something that happens to a football team, but it's spelt differently!)

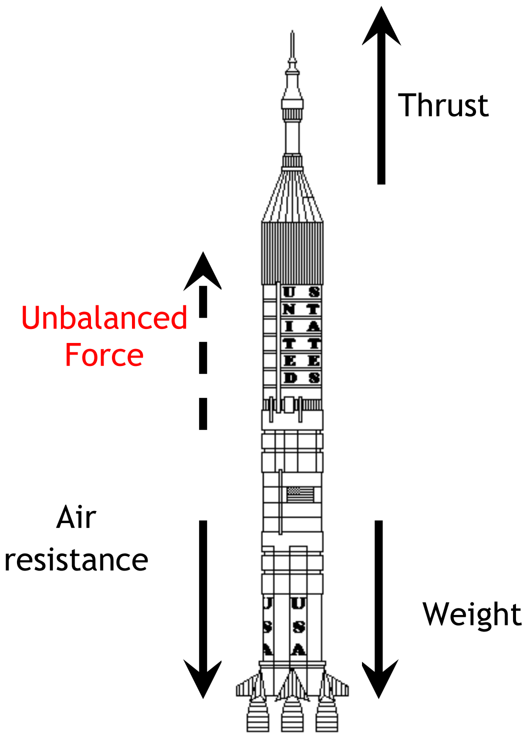
ROCKETS

Whenever you consider rockets, always start by drawing a freebody diagram.

The motion of a rocket once it starts to lift off is complex. We need to consider all the forces acting on the rocket.

At the instant of lift off: in order for the rocket to lift off, the thrust must be greater than the weight. This produces an unbalanced force which makes the rocket accelerate upwards.

unbalanced Force = Thrust- weight



$$a = \frac{F_u}{m}$$

where F_U = Thrust – weight, easy to calculate if we know the thrust and the mass of the rocket.

So far so good, but we've only moved a couple of centimetres at this point.

As the lift off continues: when the rocket begins moving, **the air resistance will increase as the speed increases.** This acts against motion so the **unbalanced force will be reduced.** But wait a minute, **as the rocket uses up fuel its mass will decrease, reducing the weight increasing the unbalanced force.**

Not only that as the rocket gets further from the Earth the gravitational field strength will reduce making the weight smaller again.

As altitude increases, the air will get thinner, thus reducing the air resistance.

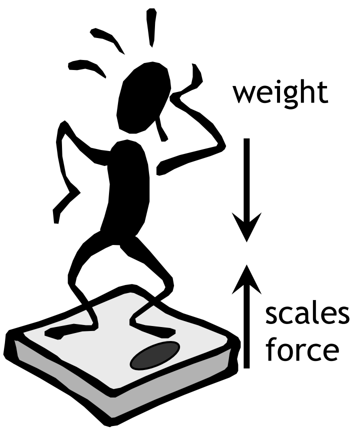
All of this is based on the assumption that the thrust remains constant. If **thrust** changes then this becomes an even bigger headache.

How do all of these different factors act together to affect the acceleration of the rocket?

Essentially the net effect is that the acceleration increases as the rocket rises.

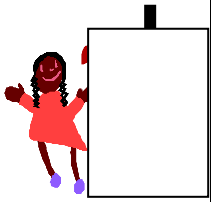
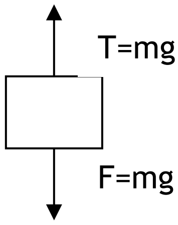
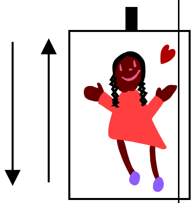
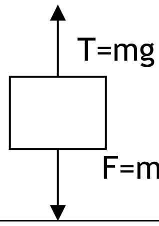
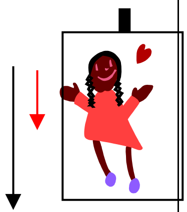
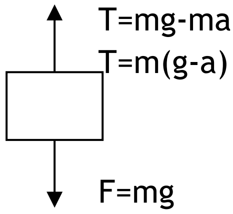
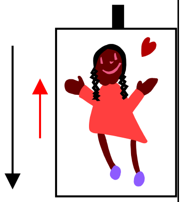
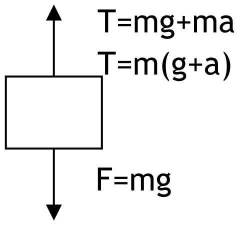
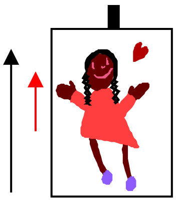
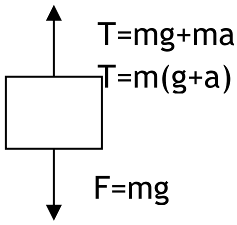
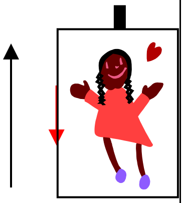
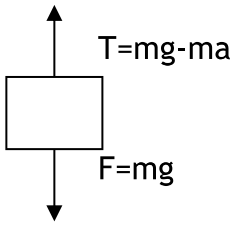
https://www.youtube.com/watch?v=wbSwFU6tY1c

THE PHYSICS OF LIFTS



Have you noticed that when you are in a lift you experience a strange feeling when the lift starts to move and as it begins to slow to a stop. However when the lift is in the middle of its journey you cannot tell if you are moving at all.

This is because at the start and end of the journey you will experience an acceleration and consequently an unbalanced force. This unbalanced force is what you 'feel'

Direction	Lift	Notes	Free body diagram
Stationary		waiting together - stationary The forces are balanced	
Moving at constant speed up or down.		Moving at constant velocity up or down the forces are balanced	
Accelerating downwards.		unbalanced forces downwards, less tension in the lift cable as some of weight used for acceleration	
decelerating downwards.		unbalanced force upwards. The cable is having to slow the lift down and support its weight	
accelerating upwards		The cable must support the weight & provide an accelerating/unbalanced Force	
decelerating upwards.		Part of the weight can be used as a decelerating force	

When you stand on a set of scales the scales is actually measuring the **upwards** (reaction) force. This is the force the scales exert on you. Now this is fine when you are in your bathroom trying to find your weight.

Normally, you and your bathroom scales will be stationary and so your weight will be equal to the upwards force [Newton Pair].

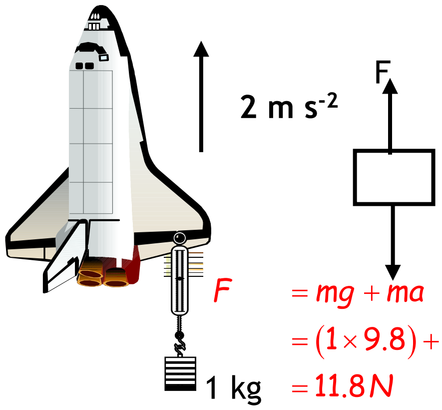
When you weigh yourself when you are accelerating the reading on the scales will **not** be equal to your weight. The reading will give you an indication of the unbalanced force acting on you, which could then be used to calculate an acceleration. This unbalanced force could be acting up or down depending on the motion of the lift.

Accelerometers which can use this principle are found in more and more electronic devices. Game consol handsets, phones where you can move between functions by shaking the handset, laptops that know when they're falling and protect themselves before they hit the ground, the list goes on and on.

The forces are similar for an object accelerating upwards and decelerating downwards.

The forces are similar for an object accelerating downwards or decelerating upwards.

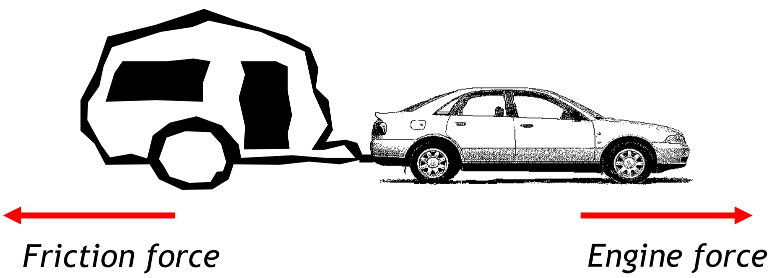
A mass of 1 kg hangs from a spring balance which is suspended from a rocket.



$$\begin{aligned} F &= mg + ma \\ &= (1 \times 9.8) + (2 \times 1) \\ &= 11.8\,N \end{aligned}$$

State the reading on the balance if the rocket accelerates upwards at 2 m s^{-2}.

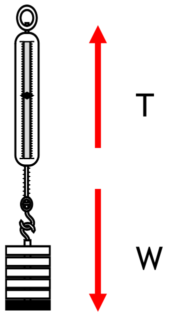
WHAT IS TENSION?

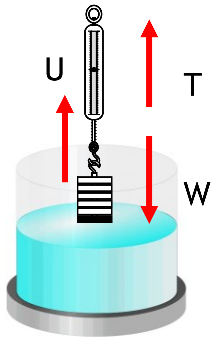


Tension is a force, measured in Newtons.

Think of it as the "balancing force". It acts in the direction to balance out forces to:

- **prevent movement**
- **to couple.**





Examples of the way tension acts.

As soon as other forces start to act on the object the size and/or the direction of the tension alters.

At the start, W=T.

As you submerge the mass the reading on the spring balance decreases as the upthrust, U, increases.

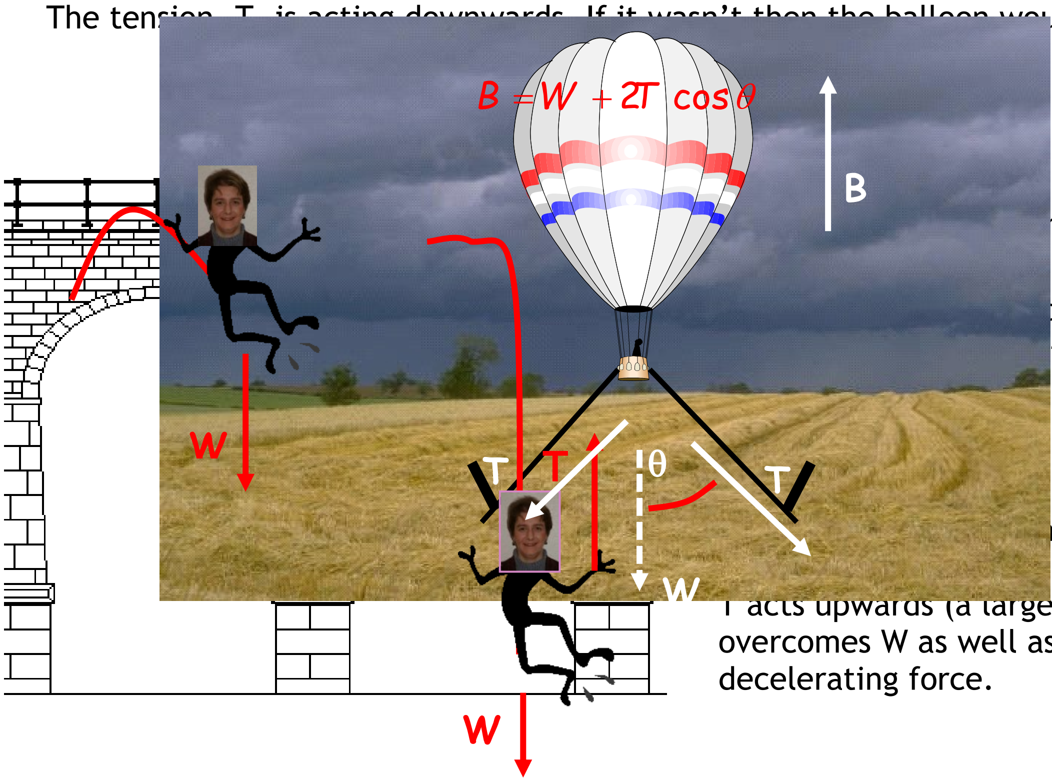
Once the mass is totally immersed the upthrust remains constant so the reading on the spring balance will remain constant.

$$W = T + U$$

$$T = W - U$$

TENSION AS A BALANCING FORCE

The tension, T, is acting downwards. If it wasn't then the balloon would rise.



r leaps off she is

ds. The tension in

is zero Newtons.

the elastic provides

l as providing the

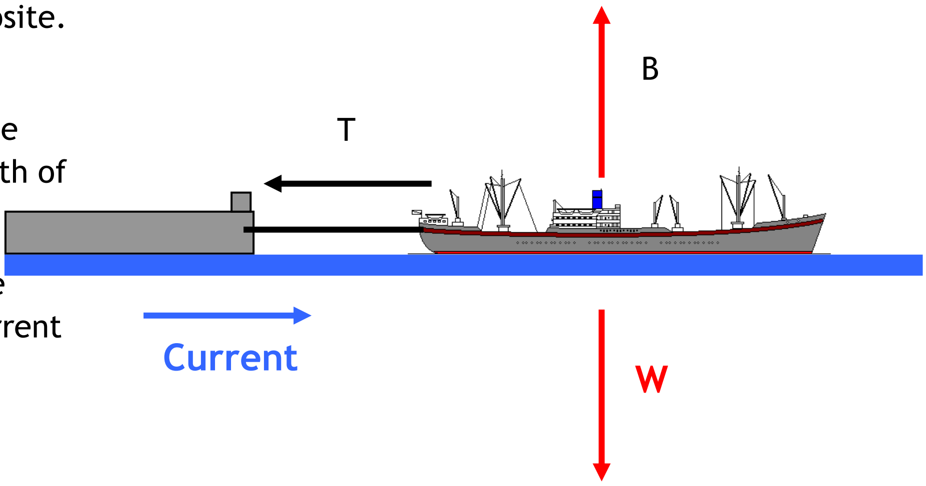
t.

T acts upwards (a large force) and overcomes W as well as providing the decelerating force.

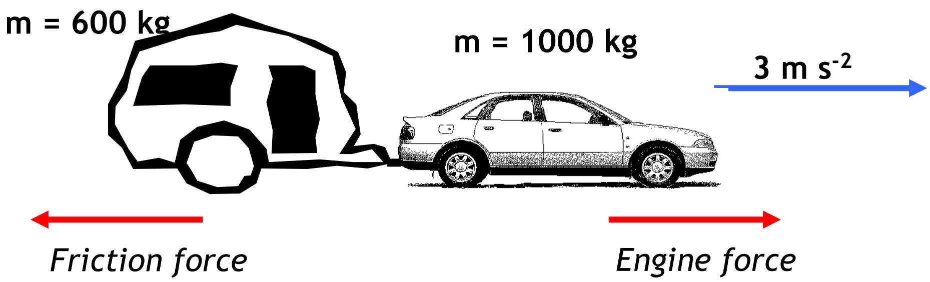
In the boat example opposite.

The tension, T, in the rope will depend on the strength of the current.

You will often have to use components when the current



The tension in the towbar, T, is the size of the force required to accelerate the caravan with the same value as the car. However, you must remember to add the frictional force. Assuming the frictional force, F_V, of the caravan is 400 N then the tension is given by:



Assuming the frictional force, F_C, of the car is also 400 N, the total force provided

$$T = ma + F_v$$
$$T = (600 \times 3) + 400$$
$$T = 2200\,N$$

$$\begin{aligned} Force &= (m_1 + m_2)a + F_v + F_c \\ &= (1000 + 600)3 + (400 + 400) \\ &= 4800 + 800 \\ &= 5600\,N \end{aligned}$$

by the car is that needed to accelerate the car + caravan is:

If the vehicle is travelling at constant speed then since the forces are balanced:

$$T = F_v = 400\,N$$

NEWTON'S SECOND LAW: ENERGY AND POWER

Newton's Second Law *F=m a*

Gravitational Potential Energy= mass × gravitational field strength × height

$$E_p = mgh$$

Kinetic Energy= ½ × mass × velocity squared

$$E_k = \tfrac{1}{2} mv^2$$

$$work\ done = Force\ \times distance$$

$$\boldsymbol{E_w = F\ \times d}$$

LAW OF CONSERVATION OF ENERGY

***Energy cannot be created or destroyed, it can only be changed from one form to another (or from one object to another)*.**

Example

(a) Thomas drops a football of mass 0.2 kg from a height of 2.25 m. Calculate the velocity of the ball at the instant before it hits the ground. Ignore air resistance

Before the ball is dropped, it possesses only gravitational potential energy.

At the instant before the ball hits the ground, all the gravitational potential energy has been converted to kinetic energy. So, provided that there are no energy losses,

$$\Delta\ Ep\ lost = \Delta\ Ek\ gained$$

$$\Delta\ mgh = \Delta\ ½\ mv^2$$

$$gh = ½v^2$$

('m' appears on both sides of equation, so can be cancelled out).

$$9.8 \times 2.25 = 0.5\ v^2$$

$$22.1 = 0.5\ v^2$$

$$v^2 = 22.1/0.5 = 44.2$$

$$\underline{\underline{v = \sqrt{44.2} = 6.6\ ms^{-1}}}$$

Another important concept it power. Power is the equivalent to the energy transformed or dissipated per second.

$$P = \frac{E}{t} = \frac{E_w}{t} = \frac{Fs}{t} = Fv$$

The equation above is found by combining some of the equations below

$$E_p = mgh, \quad E_k = \frac{1}{2} mv^2, \quad E_w = Fs,$$

You have probably not met that Power can equal force × velocity but you should be able to see that this is equivalent. Other used of power and energy are given below. We have yet to meet I as equal to irradiance, but it will turn up in the next unit.

$$Power = \frac{\text{Energy}}{Time}$$

$$P = IV, \quad = \quad I^2R, \quad = \frac{V^2}{R}$$

Where the *I* in the last equation is **irradiance** and **not current, so don't get confused!**

$$P = IA$$

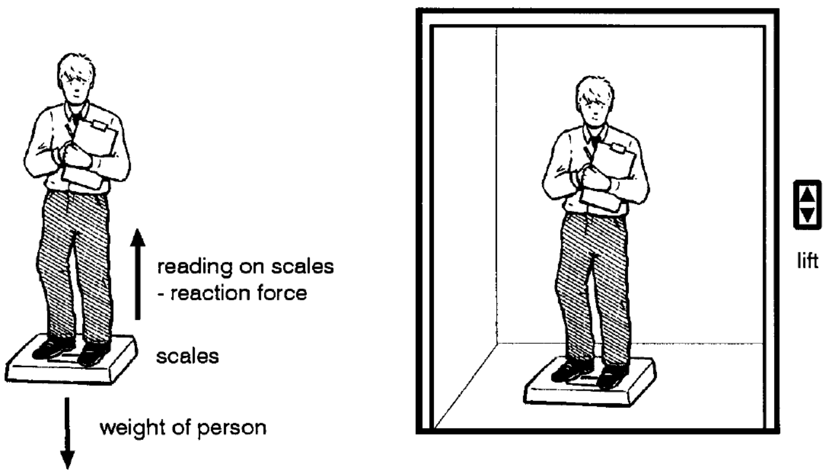
$$Power = Irradiance \times area$$

NEWTON'S SECOND LAW ENERGY AND POWER/ PRACTICALS

PRACTICAL 1 TO CALCULATE THE ACCELERATION OF A LIFT.

Apparatus: 1 set of bathroom scales.

Instructions:



- Use the scales to determine your weight and your mass.
- Stand on the scales inside of the lift and note the reading on the scales while the lift is stationary.
- Press the button to send the lift downwards.

Note a) The minimum reading on the scales as the lift accelerates downwards.

b) The reading on the scales as the lift travels at constant speed.

c) The maximum reading on the scales as the lift decelerates upwards.

- Repeat the experiment for the upward journey of the lift.

- Use your results to calculate the acceleration of the lift at the three stages of each journey.

EXTENSION:

Draw free body diagrams of the forces with the relative sizes for the three stages of each journey.

CHAPTER 4: COLLISIONS AND EXPLOSIONS

SUMMARY OF COLLISIONS AND EXPLOSIONS

5.		Collisions and explosions
	eq	$p = mv$ $\quad$ $Ft = mv - mu$ $\quad$ $E_k = \frac{1}{2}mv^2$
	a)	I can use the principle of conservation of momentum and an appropriate relationship to solve problems involving the momentum, mass and velocity of objects interacting in one dimension.
	b)	I can explain the role in kinetic energy in determining whether a collision is described as elastic and inelastic collisions or in explosions.
	c)	I can use appropriate relationships to solve problems involving the total kinetic energy of systems of interacting objects.
	d)	I can use Newton's third law to explain the motion of objects involved in interactions.
	e)	I can draw and interpret force-time graphs involving interacting objects.
	f)	I know that the impulse of a force is equal to the area under a force-time graph and is equal to the change in momentum of an object involved in the interaction.
	g)	I can use data from a force-time graph to solve problems involving the impulse of a force, the average force and its duration.
	h)	I can use appropriate relationships to solve problems involving mass, change in velocity, average force and duration of the force for an object involved in an interaction.

COLLISIONS AND EXPLOSIONS

INERTIA

Inertia is the tendency of a body to remain at rest, or if moving, to continue its motion in a straight line.

(NB the inertia is given by its mass)

Click on this link and watch the example

https://www.youtube.com/watch?v=zWeKRMh3kT8

http://www.stevespanglerscience.com/experiment/trick-with-tablecloth

MOMENTUM

The **momentum** of a body is defined as **the product of mass and its velocity**.

It has the symbol p and units $kgms^{-1}$.

Momentum is a vector quantity.

REVISION OF NEWTON'S LAWS OF MOTION

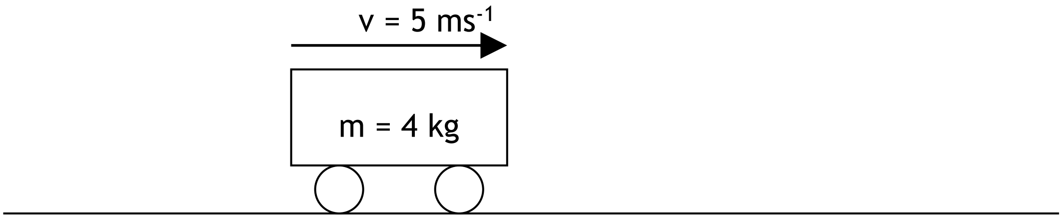
First Law: unless a resultant force acts on a body its velocity will not change.

Second Law: The rate of change of momentum of a body is proportional to the resultant force that acts.

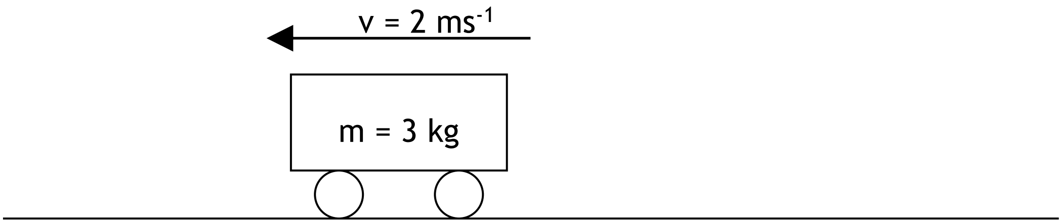
Third Law: In any interaction between two objects the force exerted by object A on object B is equal in size but opposite in direction to the force exerted by object B on A.

For each of the example find the total momentum of the system. Remember that momentum is a vector quantity.

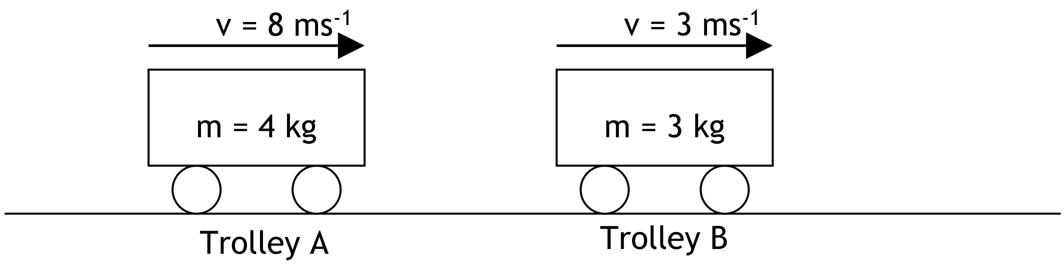
Example 1



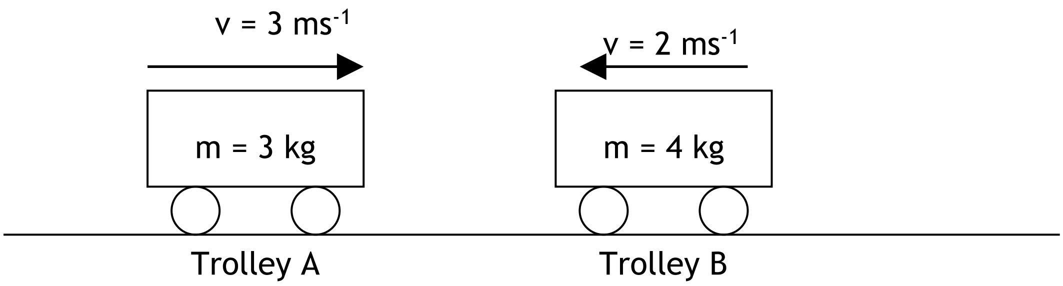
Example 2



xample 3



Example 4



Answers

Example 1 = 20kgms^{-1} to the right,

Example 2 = 6kgms^{-1} to the left,

Example 3 = 41kgms^{-1} to the right

Example 4 = 1kgms^{-1} to the right

MOMENTUM AND COLLISIONS

Any object that has mass and moves has momentum. When objects collide momentum can be transferred to the objects with which they collide.

TASK

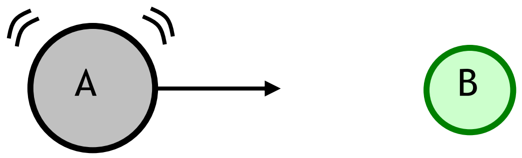
List 10 situations in which momentum is transferred during collisions.

Experiments show that

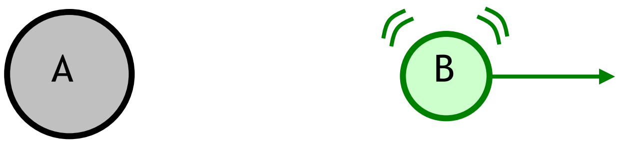
During any collision, in the absence of external forces the total momentum before the collision is equal to the total momentum after the collision.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

BEFORE:



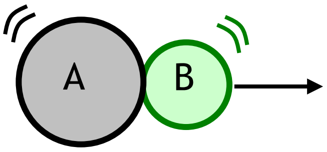
AFTER:



Pass on ALL momentum.

Share momentum. A has passed on some of its momentum.

Collide and reverse direction. Black undergoes a very large change in momentum.



Stick together.

View the video and think about the conservation of momentum

http://www.youtube.com/watch?v=fRY7vSarkio

If you think of situations where objects collide you should notice that you have 2 types of collision.

- when objects stick together
- when the objects bounce off each other after the collision.

Loosely we call the first type of collision *INELASTIC*, and the second type of collision *ELASTIC*. In both types of collision MOMENTUM IS CONSERVED as is TOTAL ENERGY.

For Higher we need a tighter definition for the two types of collision.

An inelastic collision is defined where the total kinetic energy after the collision is reduced, ie kinetic energy is lost in the collision. NB the total energy remains the same but it is converted to other forms mainly as heat and sound.

An elastic collision is one where no kinetic energy is lost. (In reality there are very few examples of perfectly elastic collisions)- Although it should be recognised that particles collide elastically.

SUMMARY

Type of Collision	***Momentum***	***Kinetic Energy***	***Total Energy***
Elastic	**Conserved**	**Conserved**	**Conserved**
Inelastic	**Conserved**	**Reduced**	**Conserved**
Explosions	**Conserved** **zero at start and finish**	**here Ep is converted to E_k so E_k increases**	**Conserved**

During ALL collisions, remember that TOTAL energy is CONSERVED (as well as momentum). We are just looking at ΔE_k for defining inelastic and elastic collisions.

EXAMPLE MOMENTUM PROBLEM

A 2 kg trolley moving to the **right** at **10 ms^{-1}** collides with a **10 kg** trolley which is also moving to the **right** at **1 ms^{-1}**. Immediately after the collision, the **2 kg** trolley rebounds to the **left** at **5 ms^{-1}**.

(a) Calculate the **velocity** of the **10 kg** trolley immediately **after** the collision.

(b) Show that the collision is **elastic**.

Total momentum = (2 kg x 10 ms^{-1}) + (10 kg x 1 ms^{-1})
= 20 + 10
= 30 kg ms^{-1}

Total momentum = (2 kg x -5 ms^{-1})+ (10kg x v)
= -10 + 10v kg ms^{-1}

Total momentum just before collision = Total momentum just after collision
30 = (-10 + 10v)
10 v = 30 - (-10)
10v = 40
v = 40/10 = <u>4 ms^{-1} (ie., 4 ms^{-1} to the right)</u>

Total kinetic energy before collision
= (1/2 x 2 x 10^2) + (1/2 x 10 x 1^2)
= 100 + 5
= 105 J

Total kinetic energy after collision
= (1/2 x 2 x 5^2) + (1/2 x 10 x 4^2)
= 25 + 80
= 105 J

Total kinetic energy just before collision = Total kinetic energy just after collision
SO, <u>COLLISION IS ELASTIC.</u>

Note

You should set out all your momentum problems like this - This makes it easier for you (and anybody marking your work) to see exactly what you are doing.

- *Always include a sketch to show the masses of the colliding objects and their velocities just before and just after the collision.*
- *Take plenty of space on your page - Some people take a new page for every problem.*
- *Take care with your calculations and be careful with directions. Remember:*

DIRECTION IS VITAL!

Task: You try this….

Two objects collide. Object 1 has a mass of 0.2kg and object 2 has a mass of 1kg. The smaller object has a starting velocity of 10 ms^{-1} and the larger object has an initial velocity of 1 ms^{-1}. After the collision object 1 rebounds with a velocity of 5 ms^{-1} Determine the velocity of object 2?

Is the collision elastic or inelastic? YOU MUST JUSTIFY YOUR ANSWER. To justify your answer you must find out if Ek is lost. If Ek is lost the collision is inelastic.
(Answer 4 ms^{-1}, elastic)

I. ELASTIC COLLISIONS

In an **elastic collision**:

- ✓ the 2 colliding objects **bounce apart** after the collision.
- ✓ **momentum is conserved**. (The total momentum just before the collision = the total momentum just after the collision.)
- ✓ **total energy is conserved**
- ✓ **kinetic energy is conserved**. (The total kinetic energy just before the collision = the total kinetic energy just after the collision.)

II. INELASTIC COLLISIONS

In an **inelastic collision**:

- ✓ the 2 colliding objects **stick together** after the collision (but only one sign of an inelastic collision, not definitive!)
- ✓ **momentum is conserved**. (The total momentum just before the collision = the total momentum just after the collision.)
- ✓ **total energy is conserved**
- ✓ **kinetic energy decreases**. (The total kinetic energy just after the collision is less than the total kinetic energy just before the collision.) Some **kinetic energy** is changed into **sound**, **heat** and **energy of deformation** (which changes the shape of the objects) during the collision.

When objects collide they exert a force on each other according to Newton's Third Law.

$$F_a = -F_b$$

$$F_a = m_a a_a$$

$$F_a = \frac{m(v-u)}{t}$$

$$F_a t_a = m_a (v_a - u_a)$$

This quantity *Ft* is called the *IMPULSE* and it is equal to the *CHANGE IN MOMENTUM* of the object.

Impulse = change in momentum

(N s) = (kg m s^{-1})

REARRANGING MOMENTUM EQUATIONS

In ALL linear collisions and explosions MOMENTUM is CONSERVED.

Momentum Before = Momentum after

$$\Delta p_1 = \Delta p_2$$

Momentum before is equal momentum of object 1 and momentum of object 2 prior to the collision. Momentum after is equal momentum of object 1 and momentum of object 2 after the collision.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Also Δp of body 1 is the same as Δp of body 2

(ie the impulse received by each body in a collision is equal)

Impulse on 1 = impulse on 2

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

Let's see how these equations come about:-

Let's start with Newton's Third Law of Motion. This says that the force acting on object B is equal in size but opposite in direction to that on object A. Newton's Third Law is that same as saying that Momentum is conserved. (NB This only applies to different bodies and not to explosions, which are simply balanced forces).

$$-F_1 = F_2$$

(for ease I have used 1 & 2 instead of A & B for my bodies!)

but Newton's Second Law says :

$$F = ma$$

$$\therefore -m_1a_1 = m_2a_2$$

but acceleration is equal to $a = \frac{v-u}{t}$ so substitute

$$-\frac{m_1(v_1-u_1)}{t_1}=\frac{m_2(v_2-u_2)}{t_2}$$

During the collision the time body 1 is in contact with body 2 must be the same so that *t* cancels out, $-m_1(v_1-u_1)=m_2(v_2-u_2)$

This tells us that the change in momentum of body 1 is equal to the change in momentum of body 2.

Expand the brackets. $-m_1v_1+m_1u_1=m_2v_2-m_2u_2$

Change in p_1 = change in p_2

but $-m_1v_1+m_1u_1=m_1v_1-m_1u_1$

$\therefore m_1u_1-m_1v_1=m_2v_2-m_2u_2$

rearrange again $m_1u_1+m_2u_2=m_1v_1+m_2v_2$

momentum before = momentum after **QED!**

SUMMARY OF MOMENTUM & COLLISIONS

Force is measured in Newtons.

Momentum is measured in kilograms metres per second.

Impulse is measured in Newton seconds and is given by the area under a Force -time graph.

Change in momentum is measured in kilograms metres per second. It is equal to the impulse.

During any collision, in the absence of external forces total momentum before a collision is equal to the total momentum after the collision. HOWEVER, remember that the momentum of each individual object is likely to change, one will lose momentum, and one will gain it!

MOMENTUM & IMPULSE EQUATION SUMMARY

F=ma

$-F_1= F_2$

p=mv

$$a=\frac{v-u}{t}$$

momentum before = momentum after

$$m_1u_1 + m_2u_2 = m_2v_2 + m_1v_1$$

$$m_1u_1 + m_2u_2 = (m_2 + m_1)v \quad \text{if they stick together}$$

Change in p_1 = change in p_2

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$\Delta p_1 = \Delta p_2$$

$$\Delta p = mv - mu$$

$$\Delta p = m(v - u)$$

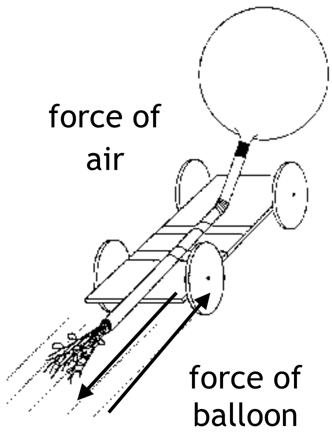
Change in momentum = impulse

$$Ft = mv - mu$$

$$\Delta p = Ft$$

EXPLOSIONS AND NEWTON'S LAWS

Far from being some abstract physics law from a text book the law of conservation of momentum has direct consequences in our lives. If you have ever been in a plane or on a jet ski then the motion of these vehicles depends on this law.

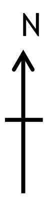


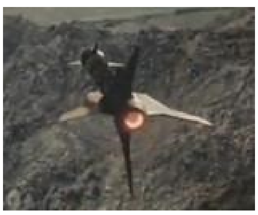
The propulsion system works by the engine expelling some form of exhaust at high speed in one direction. The conservation of momentum means that something else must move in the opposite direction to conserve momentum. In the cases above that thing is the engine, which is attached to the vehicle causing it to move.

This is essentially what is meant by Newton's third law: every action[force] has an equal and opposite reaction[force].

In a simple explosion two objects start together at rest then move off in opposite directions. Momentum must still be conserved, as the total momentum before is zero, the total momentum after must also be zero.

Example: An early Stark Jericho missile is launched vertically and when it reaches its maximum height it explodes into two individual warheads.





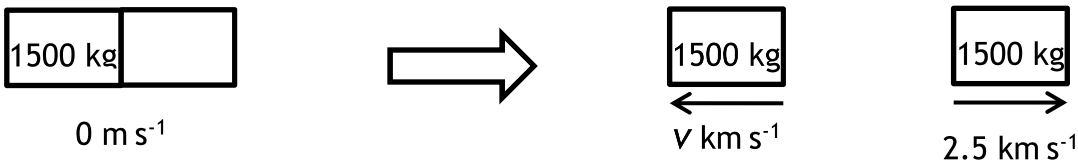
www.Iron-man-armor.blogspot.com

www.marvalcinematicuniverse.wikia .com

Both warheads have a mass of 1500 kg and one moves off horizontally, with a velocity of 2.5 km s^{-1} (Mach 9) at a bearing of 090°.

Calculate the velocity of the other warhead.

Solution:



$$0 = m_1v_1 + m_2v_2$$

$$0 = 1500 \times v_1 + 1500 \times 2.5 \times 10^3$$

$$1500 \times v_1 = -1500 \times 2.5 \text{x} 10^3$$

$$v_1 = -\frac{3.75 \times 10^6}{1500}$$

$$v_1 = \underline{-2.5 \times 10^3 \text{ m s}^{-1}}$$

The negative sign in the answer indicates the direction of v_1 is opposite to that of v_2, i.e. 270° rather than 090°.

<u>Second warhead is travelling at 2.5 km s^{-1} on a bearing of 270°.</u>

COMPARING EXPLOSIONS AND COLLISION

Explosion	Collision
p_{total} before = p_{total} after	p_{total} before = p_{total} after
$0 = m_1v_1 + m_2v_2$	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
$m_1v_1 = -m_2v_2$	$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$ $-m_1 (v_1 - u_1) = m_2 (v_2 - u_2)$
$\frac{m_1v_1}{t} = \frac{-m_2v_2}{t}$	$\frac{-m_1(v_1-u_1)}{t} = \frac{m_2(v_2-u_2)}{t}$
$m_1a_1 = -m_2a_2$	$-m_1a_1 = m_2a_2$
$F_1 = -F_2$	$-F_1 = F_2$

FORCE-TIME GRAPHS

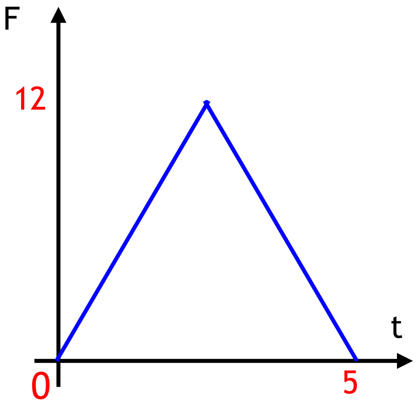
For force-time graphs:

area under graph

= Ft

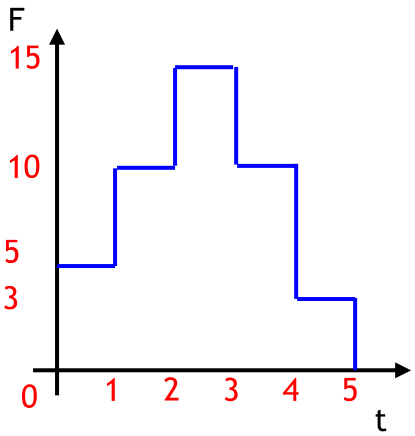
= Δp (change in momentum)

= impulse



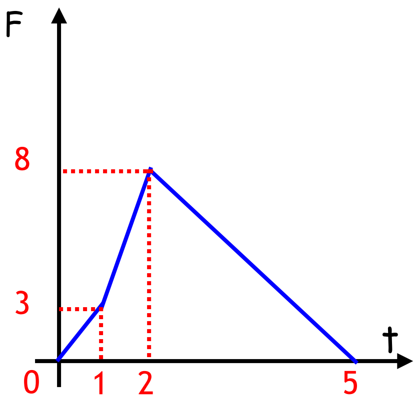
Impulse = area under the graph

= ½ x 12 x 5 = 30 N s.



Impulse = area under the graph

$$= (1 \times 5) + (1 \times 10) + (1 \times 15) + (1 \times 10) + (1 \times 3)$$
$$= 43 N s$$



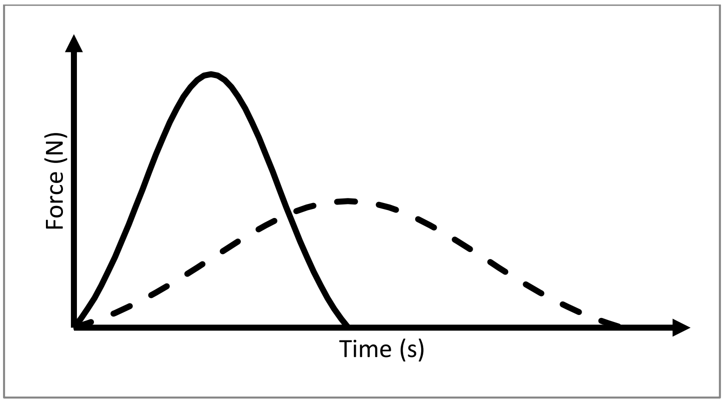
Impulse = area under the graph

$$= \tfrac{1}{2}bh + bh + \tfrac{1}{2}bh + \tfrac{1}{2}bh$$
$$= (\tfrac{1}{2} \times 1 \times 3) + (1 \times 3) + (\tfrac{1}{2} \times 1 \times 5) + (\tfrac{1}{2} \times 3 \times 8)$$
$$= 19 N s$$

Impulse = area under the graph

$$= \overline{F}t + \overline{F}t + \overline{F}t$$
$$= (1\tfrac{1}{2} \times 1) + (5\tfrac{1}{2} \times 1) + (4 \times 3)$$
$$= 19 N s$$

A more realistic example of a Ft graph is the one shown below (represented by the solid line), which could be a graph of a collision between two cars that bounce off each other. At Higher you would not be expected to calculate the impulse from this type of graph as it is considered too difficult. However, you could be asked what this graph might look like if safety features were in place. Obviously this would have the same area under the graph as the same momentum change must occur, but safety features are designed to decrease the force on the occupants, resulting in an increased time of contact.

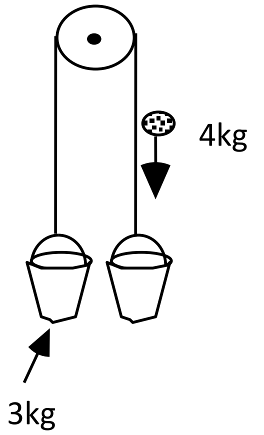


WORKED EXAMPLES

Two difficult worked examples of momentum and collisions and explosions. Try to work out how you would start each question before you look at the solutions.

Worked Example 1

Calculate the velocity of the bucket and the putty after the collision.



A 4kg lump of putty is dropped from 2m into a bucket as shown.

Find the velocity the putty hits the bucket:

$v = ?$

$u = 0ms^{-1}$

$s = 2m$

$m = 4kg$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + (2 \times 9.8 \times 2)$

$v = 6.3ms^{-1}$

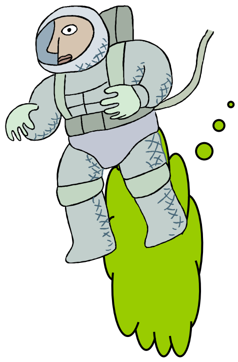
This is the INITIAL velocity of the putty during the collision. Notice too that as the buckets are joined it is the combined mass of the buckets that is being accelerated.

momentum before = momentum after

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$(4 \times 6.3) + (6 \times 0) = (10)v$$
$$25.2 + 0 = 10v$$
$$v = \frac{25.2}{10} = 2.5\,m\,s^{-1}$$

This is the initial velocity during the collision so this can be used to find the final velocity of the bucket.

Worked Example 2



An astronaut of mass 100 kg (including his equipment) is at rest with respect to his spaceship which is 100 m away. He has 1600 g of oxygen left in his tank which he uses to breathe at 0.8 g per second and which he can release in bursts of 50g at a time with a velocity of 100 m s^{-1} in order to propel himself towards his ship.

(a) What velocity does the astronaut have towards his ship if he releases 50 g of oxygen?

(b) How long does it take him to reach his ship at this speed?

Momentum before = momentum after = 0 kg m s^{-1}

Find the momentum of the gas $= mv$

$= 0.05 \times 100$

$= 5.0\,kg\,m\,s^{-1}$

$\therefore$ Momentum of astronaut $= -5.0\,kg\,m\,s^{-1} = mv$

$$-5.0 = (100 - 0.05)v$$
$$-5.0 = 99.95v$$
$$v = \frac{-5.0}{99.95} = -0.05\,m\,s^{-1}$$

Assuming he travels at a constant speed $\bar{v} = \frac{s}{t}$ and directly towards the spacecraft:

$$0.05 = \frac{100}{t}$$
$$t = \frac{100}{0.05} = 2000\,s$$

CHAPTER 5: GRAVITATION

SUMMARY OF CONTENT EQUATIONS OF MOTION

5.		Gravitation
	eq	$d = \bar{v}t,\ s = \overline{vt}\ ;\quad s = \frac{1}{2}(u + v)t;$ $v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad F = \frac{Gm_1m_2}{r^2}$
	a)	I can use the equation $F = \frac{Gm_1m_2}{r^2}$
	b)	***I can give a description of an experiment*** **to measure the acceleration of a falling object.**
	c)	I know that the horizontal motion and the vertical motion of a projectile are independent of each other.
	d)	I know that satellites are in free fall around a planet/star.
	e)	I can resolve the initial velocity of a projectile into horizontal and vertical components and their use in calculations.
	f)	I can use resolution of vectors, vector addition, and appropriate relationships to solve problems involving projectiles.
	g)	I can use Newton's Law of Universal Gravitation to solve problems involving force, masses and their separation.

ASTRONOMICAL DATA

This data is required to answer the tutorial questions.

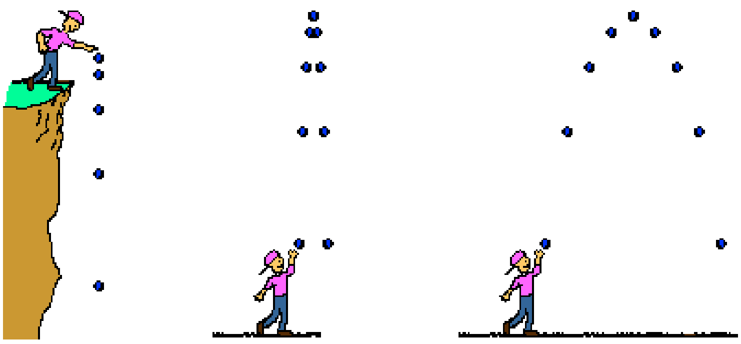
Mass of Earth	M_E	$6{\cdot}0 \times 10^{24}$ kg
Radius of Earth	R_E	$6{\cdot}4 \times 10^{6}$ m
Mass of Moon	M_M	$7{\cdot}3 \times 10^{6}$ kg
Radius of Moon	R_M	$1{\cdot}7 \times 10^{6}$ m
Mean radius of Moon orbit		$3{\cdot}84 \times 10^{8}$ m
Mass of Jupiter	M_J	$1{\cdot}0 \times 10^{27}$ kg
Radius of Jupiter	R_J	$7{\cdot}15 \times 10^{7}$ m
Mean radius of Jupiter orbit		$7{\cdot}8 \times 10^{11}$ m
Universal constant of gravitation	G	$6{\cdot}67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}

GRAVITATIONAL FIELD STRENGTH ON OTHER PLANETS

Solar System object	g (Nkg^{-1})
Mercury	3.7
Venus	8.8
Earth	9.8
(Moon)	1.6
Mars	3.8
Jupiter	26.4
Saturn	11.5
Uranus	11.7
Neptune	11.8

PROJECTILES

Generally projectiles have both horizontal and vertical components of motion. As there is only a single force, the force of gravity, acting in a single direction, only one of the components is being acted upon by the force. The two components are not undergoing the same kind of motion and must be treated separately.



This force creates an acceleration equal to "g" in the vertical direction, which on Earth has a value of 9.8ms^{-2}

PROJECTILES FIRED VERTICALLY

To summarise, for a vertical projectile on Earth:

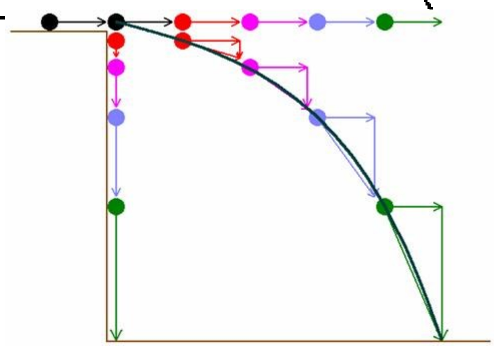
Direction of motion	Forces	Velocity	Acceleration
Horizontal	Air resistance negligible so no forces in the horizontal direction.	Constant (in this case 0 ms^{-1})	None
Vertical	Air resistance negligible so only force of gravity acting in the vertical direction.	Changing with time	Constant or uniform acceleration of - 9.8 $m\ s^{-2}$

PROJECTILES FIRED HORIZONTALLY

The second projectile situation to consider is the horizontal projectile.

In projectile motion we ignore air resistance, or any force other than the force of gravity, in our calculations, but you could be asked the effect of air resistance in a supplementary question.

Analysis of this projectile shows the two different components of motion.



Horizontally: there are no forces acting on the cannonball and therefore the **horizontal velocity is constant.**

Vertically: The force due to gravity is constant in the vertical plane and so the projectile **undergoes constant vertical acceleration**.

The combination of these two motions causes the curved path of a projectile.

Example: The cannonball is projected horizontally from the cliff with a velocity of 100 ms^{-1}. The cliff is 20 m high.

Determine: (a) the vertical speed of the cannonball, just before it hits the water;

(b) if the cannonball will hit a ship that is 200 m from the base of the cliff.

Solution:

Horizontal	Vertical
s = ?	s = 20 m
v= 100 m s^{-1}	u = 0
t = ?	v= ?
	a = 9·8 m s^{-2}
	t= ?

$$v^2 = u^2 + 2as$$
$$v^2 = 0 + (2 \times 9{\cdot}8 \times 20)$$
$$v^2 = 392$$
$$\underline{v = 19.8 ms^{-1}}$$

(b)

$$v = u + at$$
$$19.8 = 0 + 9.8t$$
$$t = \frac{19.8}{9.8} = \underline{2.02s}$$

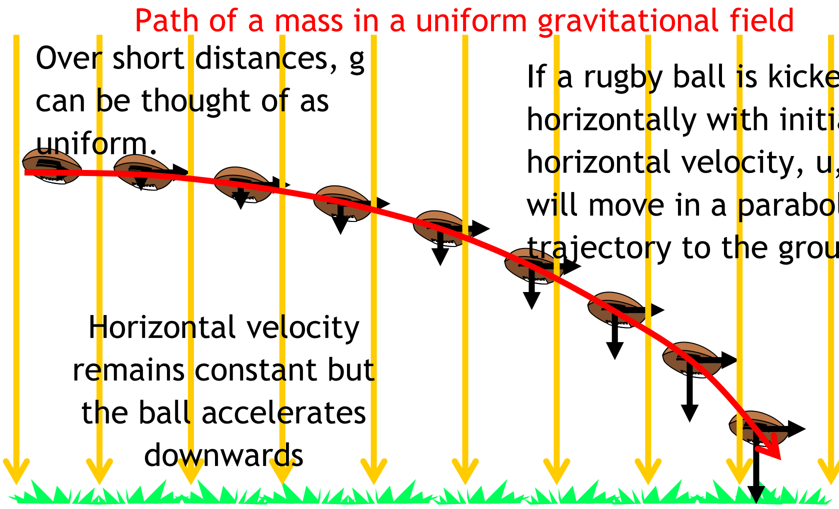
$$s_h = \bar{v}t$$
$$s_h = 100 \times 2.02 = \underline{202m}$$

The cannonball will hit the ship.

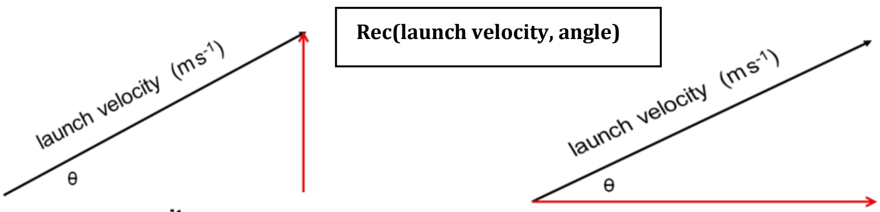
To summarise, for a horizontal projectile on Earth:

Direction of motion	Forces	Velocity	Acceleration
Horizontal	Air resistance negligible so no forces in the horizontal direction.	Constant v	Zero
Vertical	Air resistance negligible so only force of gravity acting in the vertical direction.	Changing with time	Constant or uniform acceleration of $-9.8\ \text{m s}^{-2}$

PROJECTILES AT AN ANGLE



The third and final projectile situation to consider is the projectile at an angle to the horizontal. Projectiles at an angle require us to use our knowledge of splitting vectors into their horizontal and vertical components.



$$\sin\theta = \frac{\text{opposite}}{\text{hyp}}$$

$$\text{opposite} = \text{hyp}\sin\theta$$

$$\text{vertical component} = \text{launch velocity}\sin\theta$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hyp}}$$

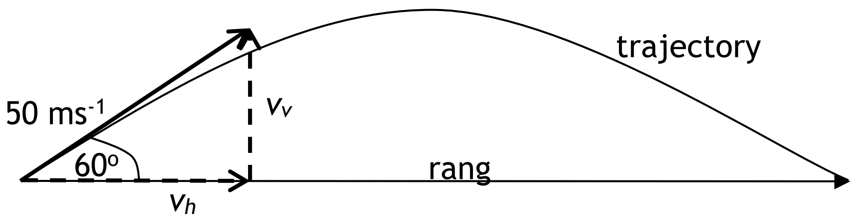
$$\text{adjacent} = \text{hyp}\cos\theta$$

$$\text{horizontal component} = \text{launch velocity}\cos\theta$$

The distance travelled horizontally (the range) is determined by the cosine component of the launch velocity. The time of flight is determined by the sine component of the launch velocity, **providing the angle to the horizontal is given.**

To summarise, for a projectile fired on Earth at an angle to the horizontal:

Direction of motion	Forces	Velocity	Acceleration
Horizontal	Air resistance negligible so no forces in the horizontal direction	Constant $v \cos \theta$	Zero
Vertical	Air resistance negligible so only force of gravity acting in the vertical direction.	Changing with time. Initially $v \sin \theta$	Constant or uniform acceleration of -9.8 m s^{-2}



The velocity at an angle <u>must</u> be split into its vertical and horizontal components before calculating any solution to projectile motion as only the vertical component is acting upon by the force of gravity.

$$v_h = v \cos \theta$$

$$v_v = v \sin \theta$$

For projectiles **fired at an angle** above **a flat horizontal surface** with **no air resistance**:

1. The path of the projectile is symmetrical, about the highest point. This means that:

initial vertical velocity = − final vertical velocity

$$u_v = - v_v$$

2. The time of flight = 2 × the time to highest point (if $s_v = 0$ m)

3. The vertical velocity at the highest point is zero.

4. The overall velocity at the highest point is equal to v_h, the horizontal velocity.

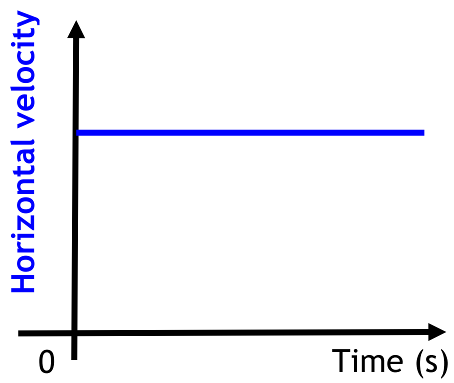
To summarise, for any projectile on Earth:

Direction of motion	Forces	Velocity	Acceleration
Horizontal	Air resistance negligible so no forces in the horizontal	Constant *v cos θ*	Zero
Vertical	Air resistance negligible so only force of gravity acting in the vertical	Changing with time Initially *v* sin *θ*	Constant or uniform acceleration of $-9.8\ m\ s^{-2}$

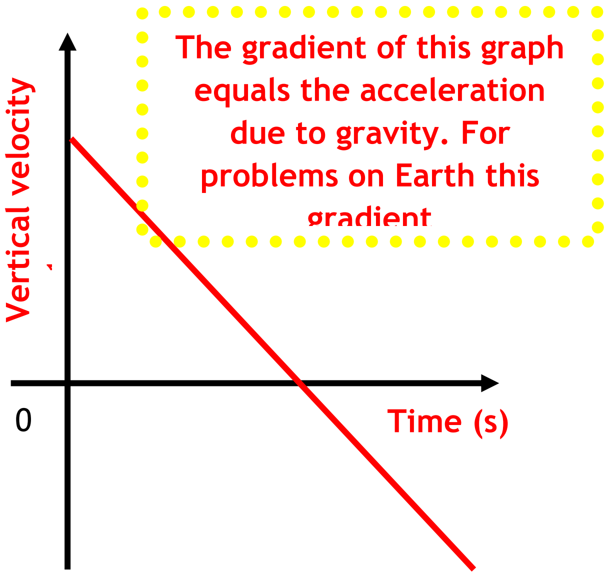
TASK: LAUNCH VELOCITY

Determine experimentally and/or theoretically the angle, which gives the greatest range for a fixed, launch velocity.

NOTES ON PROJECTILE MOTION



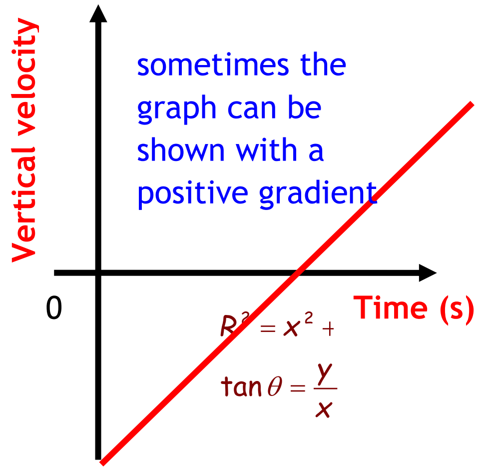
1. Assume air resistance is negligible.
2. Objects travel sideways as well as falling.
3. The motion is symmetrical about the highest point.
4. The flight of a projectile is parabolic.
5. When dealing with projectiles you must deal with the **two motions separately**.
6. HORIZONTALLY, ignoring air resistance, the projectile moves with constant velocity.
7. VERTICALLY the motion is affected by a constant acceleration of $9.8ms^{-2}$ downwards on Earth, the acceleration due to gravity.
8. If we take air resistance into account then the speed tends to decrease in the horizontal direction, however this makes the calculations too difficult so for the higher course you must assume that you're your calculations are occurring in a vacuum!



9. When fired upward the object will decelerate at $9.8ms^{-2}$, reach a maximum height where its vertical velocity is zero. It will then accelerate downwards at $9.8ms^{-2}$.
10. The forces on an object will be the same if an object accelerates downwards as they are if the object decelerates upwards!
11. Don't forget that even when the vertical velocity of the object is zero there is still a force on the object and it is still accelerating downwards at $9.8ms^{-2}$.
12. The velocity of the object changes during the flight.

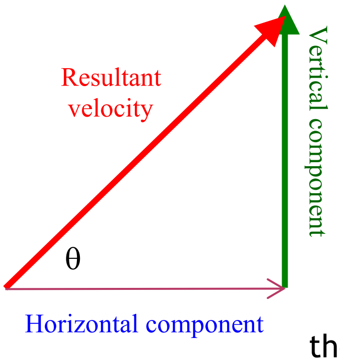
13. Remember that velocity is a vector quantity. We must split the resultant velocity (the velocity that the object is travelling in) into its components, in this case horizontal and vertical. We do this by taking components which has been dealt with elsewhere. *Notice there is only one gradient because there is only one acceleration, but this causes a deceleration in objects moving upwards, away from the Earth and it causes an acceleration in objects moving downwards, towards the centre of the Earth.*

14. We then do <u>not</u> use the resultant again unless we are asked for the resultant at any further point.



15. The horizontal and vertical motions are independent of each other and the only link between the two motions is the TIME!

16. At any point during the flight the horizontal and vertical components are added together to give the resultant velocity of the object.



vertical component = R sin θ
where R is the resultant velocity
horizontal component = R cos θ
where R is the resultant velocity

When working out the projectile problems follow the method below.

a. Draw out a quick sketch of the problem.

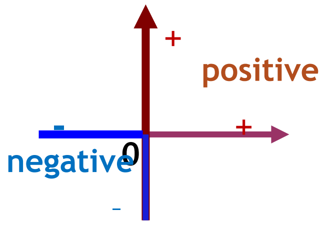
b. Find the horizontal and vertical components of the initial velocity. $u_h = v\cos\theta$ $u_v = v\sin\theta$

c. Draw up a table with the following information and fill in as much as you can.

YOU MUST REMEMBER THAT DIRECTION IS IMPORTANT. You indicate direction by positive and negative signs.

Horizontal motion	Vertical motion
s_H	s_V
$\overline{v_H}$ (usually v cos θ)	u_V (usually v sin θ)
t	v_V
	$a = 9.8ms^{-2}$ ↓
	t
TIME IS THE COMMON FACTOR	
$\boxed{\overline{v_H} = \frac{s_H}{t}}$	$\boxed{\begin{array}{l} s_v = u_v t + \frac{1}{2}at^2 \\ v_v^2 = u_v^2 + 2as_v \\ v_v = u_v + at \\ \overline{v_v} = \frac{v+u}{2} = \frac{s_v}{t} \end{array}}$

You can work by convention i.e. axes, or take each question individually and decide which way you will take as positive. Show this in your summing up of the question. If an object is thrown **upwards** then *u* and *a* must have **opposite** signs.



17. Usually in questions they will ask you to find either the horizontal or vertical quantities but this will involve you working out something that is missing from the horizontal quantities. e.g. Find the horizontal range but you will only be given enough information to find the time from the vertical component. This time will then need to be substituted into the equation for horizontal components.
18. An important thing to remember is that at the top of the throw the **vertical** velocity will be zero. This can be equal to *v* for that part of the journey and *u* for the next part of the journey.This will occur half way through the journey if the **journey is symmetrical** (i.e. it lands at the same height that it was thrown.) These questions are becoming less common as they were considered too easy!
19. Other hints to remember, if the journey is symmetrical then the resultant velocity on landing is the same but in the opposite direction. i.e if an object is projected with an initial velocity of 7·5 ms^{-1} at an angle of 60° to the horizontal, then it will land at a velocity of -7·5 ms^{-1} at an angle of 60° to the horizontal.
20. You can always break journey's into bits if they are too difficult to complete in one go, eg find the time it takes to go to the top of the throw and then find the time it takes to come back down.
21. The vertical displacement for a symmetrical journey is 0 m.
22. Remember you can use the version of the quadratic with it already set up for the equation (s=ut+ ½ at^2) if you feel confident and can remember it (it will not be on the relationship sheet).

    we can rewrite the solution for the quadratic equation as:

    $$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-u_v \pm \sqrt{u_v^2 + 2as_v}}{a}$$

    See the next page on the theory behind this and choose which ever equation suits. Both have things that can catch you out.

23. If you find yourself with a difficult quadratic to solve then you can use two equations instead, ie usually $v^2=u^2+2as$ and then v=u+at.
24. You don't get extra marks for being smart it is being correct that is important and carrying out a method that works!
25. Remember that an object that is thrown with **only a horizontal initial speed** only will land at the same time as an object dropped with zero initial vertical velocity.
26. For all our problems we ignore air resistance. Air resistance and wind would affect the journey but it is too hard to work out. You could be asked what effect this would have on the journey.
27. Sometimes you could be caught out with trying to square root a negative number. You can always try changing the sign convention and then seeing if that works out

better. Remember that the square root of a number has two values, positive and negative. You can't ever end up with a negative time and expect the right answer!

28. If an object hits the ground its velocity on hitting is **not** zero!

USE OF A QUADRATIC WHEN FINDING TIME WITH THE EQUATION OF MOTION

$$s = ut + \frac{1}{2}at^2 \quad \textit{(equation 1)}$$

This can be rewritten as

$$0 = \frac{1}{2}at^2 + ut - s \quad \textit{(equation 1b)}$$

The roots (solution) of a quadratic equation

$$ax^2 + bx + c = 0$$

are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \textit{(equation 2)}$$

Quadratic Equation	Equation of motion
x	t
b	u_v
c	$-s_v$
a	$\frac{a}{2}$ *where a= acceleration*

Substituting equation 1b into equation 2 gives:

$$t = \frac{-u \pm \sqrt{u^2 - \frac{4a}{2} - s_v}}{\frac{2a}{2}} \quad \text{(equation 3)}$$

This can be reduced to:

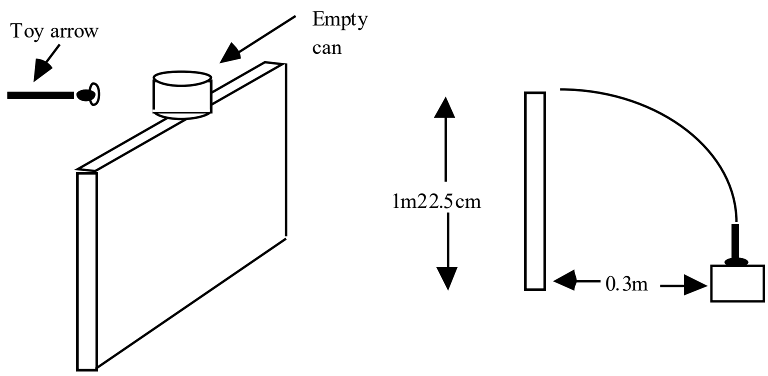
$$t = \frac{-u \pm \sqrt{u^2 + 2as_v}}{a} \quad \text{(equation 4)}$$

So either learn this particular equation (4) for use with equation 1 (but you must always check your directions for each quantity)

Or remember when you substitute equation 1 into equation 2

PROJECTILES: WORKED EXAMPLES

1. Show that the horizontal velocity of the can is 0.6 m s^{-1} immediately after being struck by the toy arrow.



First, consider the vertical velocity in order to find the time.

$$s_v = u_v t + \tfrac{1}{2}at^2$$
$$1.225 = 0 \times t + \left(\tfrac{1}{2} \times 9.8 \times t^2\right)$$
$$t^2 = \frac{2 \times 1.225}{9.8} = 0.25$$
$$t = \sqrt{0.25} = 0.5\,s$$

Horizontally:

$$s_h = u_h t$$
$$0 \cdot 3 = u_h \times 0 \cdot 5,$$
$$u_h = 0 \cdot 6\, ms^{-1}$$

2. Determine the time taken for a stone to reach the ground from the top of a 100m high building if,
   (a) it is thrown vertically downwards at 5 ms^{-1}.
   As u and a are both down, down can be taken as positive or *u,v,a and s* will need to be written as negaive

$$s_v = u_v t + \tfrac{1}{2}at^2$$
$$100 = 5 \times t + \left(\tfrac{1}{2} \times 9.8 \times t^2\right)$$
$$100 = 5t + 4.9t^2 \quad \text{i.e. } 4.9t^2 + 5t - 100 = 0$$

We need to use the solution for a quadratic equation here i.e.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ to solve for t.

$$t = \frac{-5 \pm \sqrt{5^2 - (4 \times 4.9 \times -100)}}{9.8} = \frac{-5 \pm \sqrt{25 + 1960}}{9.8}$$
$$t = \frac{-5 \pm 44.55}{9.8}$$ (a negative value for t is not possible)
$$\therefore t = \frac{-5 + 44.55}{9.8} = 4.04\,s.$$
$$\tfrac{1}{2}at^2 + u_v t - s_v = 0$$

Using

$$\tfrac{1}{2}at^2 + u_v t - s_v = 0$$

We can rewrite the solution for the quadratic equation as:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-u_v \pm \sqrt{u_v^2 + 2as_v}}{a}$$

Trying it on the above problem:

$$t = \frac{-5 \pm \sqrt{5^2 + 2 \times 9.8 \times 100}}{9.8} = \frac{-5 \pm \sqrt{25 + 1960}}{9.8}$$
$$t = 4.04s$$

As you can see this is the same as above.

(a) If this last method is too difficult then try breaking it up into smaller chunks! If you can't do quadratics we need to use one of the other formulae.

$v^2 = u^2 + 2as$ doesn't contain time so that isn't the equation that we finally need but if we use $v = u + at$ we have 2 unknowns. We don't know either v or t so we have to work it out. So first find the final velocity on landing.

s = 100m
u = 5ms^{-1} downwards
v = ?
a = 9.8ms^{-2} downwards
t = ?

$$v^2 = u^2 + 2as$$
$$v^2 = 5^2 + 2 \times 9.8 \times 100$$
$$v^2 = 25 + 1960$$
$$v = \sqrt{1985} = 44.55\ ms^{-1}$$

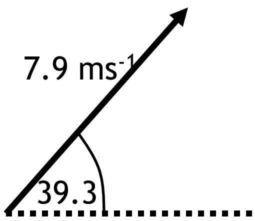
Now put the new velocity into the other equation

$$v = u + at$$
$$44.5 = 5 + 9.8t$$
$$44.55 - 5 = 9.8t$$
$$\frac{39.55}{9.8} = t = 4.04s$$

(b) if it is thrown upwards at an angle of 39.3° to the horizontal at 7.9 ms^{-1}?

(b) Although the stone has both a horizontal **and** vertical component, we only need to consider the VERTICAL one.

The vertical component of the motion, v_v is given by:



$$v_v = 7.9 \sin 39.3$$
$$v_v = 5.00\ m\ s^{-1}$$

[Rec(7.9,39.3)]

Find how long it takes to reach the ground?
s=100m (downwards)
u =5ms^{-1}(upwards)
v=
a=9.8ms^{-2}(downwards)
t=

Find how long it takes for it to reach the top of the flight.
s = ?
u = 5 m s^{-1} upwards
v = 0 m s^{-1}
a = 9.8 m s^{-2} downwards
t =

$$v = u + at$$
$$0 = 5 + -9.8t$$
$$9.8t = 5$$
$$\frac{5}{9.8} = t = 0.510s$$

You now need to know what height this is above the 100m building.

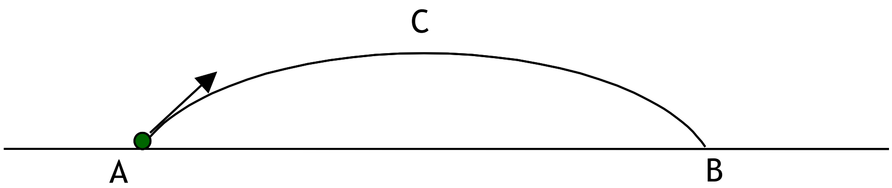
$$v^2 = u^2 + 2as$$
$$0^2 = 5^2 + 2 \times -9.8 \times s$$
$$19.6s = 25$$
$$s = \frac{25}{19.6} = 1.276m$$

So now you need to know the time it takes for the object to fall 1.276 + 100 m to the ground. remember that for this part of the journey the initial starting velocity of the object is zero, as it has reached the highest point.

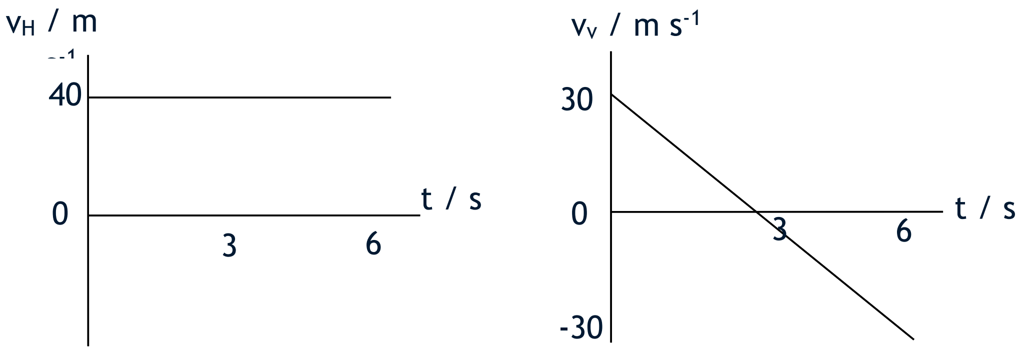
$$s = ut + \tfrac{1}{2}at^2$$
$$101.276 = 0 + \tfrac{1}{2} \times 9.8 + t^2$$
$$\frac{101.276}{4.9} = t^2 = 20.669$$
$$t = \sqrt{20.669} = 4.55s$$

so total time = 4.55 + 0.51= 5.06 s

3. A projectile is fired across level ground taking 6 s to travel from A to B. The highest point reached is C. Air resistance is negligible.



Velocity - time graphs for the flight are shown below

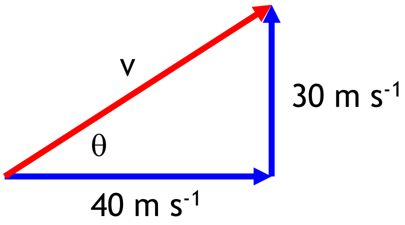


(a) Describe the horizontal and vertical motions of the projectile.
(b) Use a vector diagram to find the speed and angle at which the projectile was fired from point A.
(c) Find the speed at position C. Explain why this is the smallest speed of the projectile.
(d) Calculate the height above the ground of point C.
(e) Find the range AB.

Answer

(a) The horizontal motion is a constant speed (40 m s^{-1}). The vertical motion is a constant acceleration (9.8 m s^{-2}).

(b)



[Pol(40,30)]

$v^2 = 30^2 + 40^2 = 900 + 1600 = 2500$

$v = \sqrt{2500} = 50\,m\,s^{-1}$

$\sin\theta = \frac{30}{50} = 0.6 \quad \theta = 36.9^\circ$

(c) The speed at C is 40 m s^{-1} (the vertical speed is zero).

(d) Consider the vertical motion of the projectile:
s = ?
v = 0 m s^{-1}
u = 30 m s^{-1}
a = -9.8 m s^{-2}
t = 3 s

$s_v = u_v t + \frac{1}{2}at^2 = (30 \times 3) + \left(\frac{1}{2} \times -9.8 \times 3^2\right)$

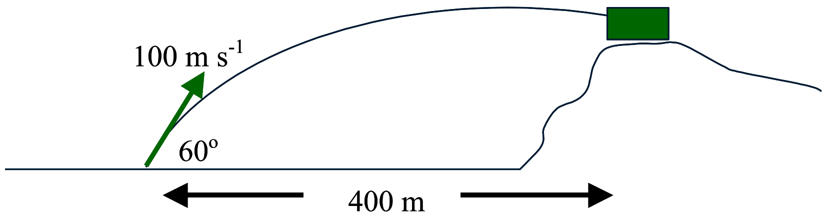
$s_v = 90 - 44.1 = 45.9\,m$

(e) Consider the horizontal motion of the projectile:

$s_H = \overline{v_H} \times t = 40 \times 6$

$s_H = 240\,m$

4. A missile is launched at 60° to the ground and strikes a target on a hill as shown below.



(a) If the initial speed of the missile was 100 ms^{-1} find:

(i) the time taken to reach the target

(ii) the height of the target above the launcher.

(a)
(i) Consider the horizontal motion of the projectile:

$u_H = u\cos\theta = 100\cos 60 = 50\,m\,s^{-1}$

$s_H = u_H \times t \qquad t = \frac{s_H}{u_H} = \frac{400}{50} = 8\,s$

(ii) Consider the vertical motion of the projectile:

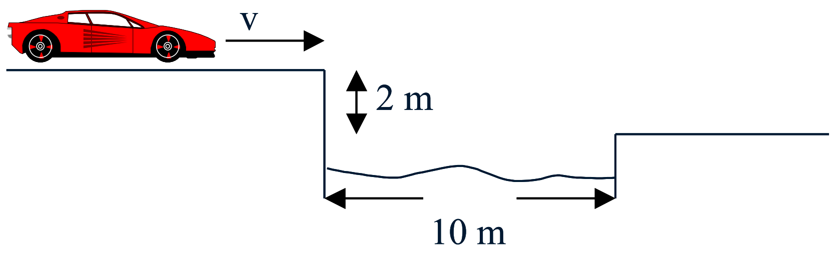
$u_V = u\sin\theta = 100\sin 60 = 86.6\,m\,s^{-1}$

$s_V = u_V t + \frac{1}{2}at^2$

$s_V = (86.6 \times 8) + \left(\frac{1}{2} \times -9.8 \times 8^2\right)$

$s_V = 692.8 - 313.6 = 379.2\,m$

5. A stunt driver hopes to jump across a canal of width 10 m. The drop to the other side is 2 m as shown.



(a) Calculate the horizontal speed required to make it to the other side.

(b) State any assumptions you have made.

(a)

$$s_V = u_V t + \tfrac{1}{2}at^2$$
$$2 = 0 + \left(\tfrac{1}{2} \times 9.8 \times t^2\right)$$
$$t^2 = \frac{2}{4.9} = 0.41$$
$$t = 0.64\,s$$

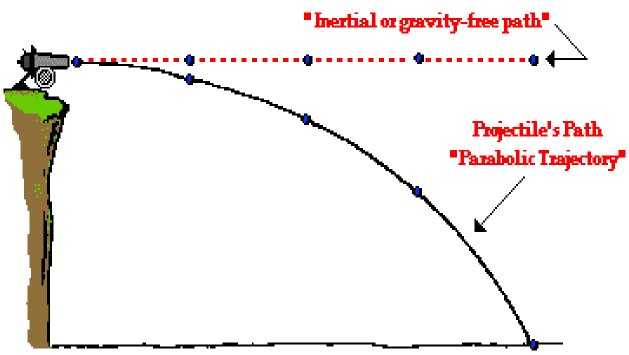
$$\overline{v_H} = \frac{s_H}{t} = \frac{10}{0.64} = 15.6\,m\,s^{-1}$$

(b) The calculations do not take into account the length of the car. In practice, the front of the car will accelerate downwards before the back.

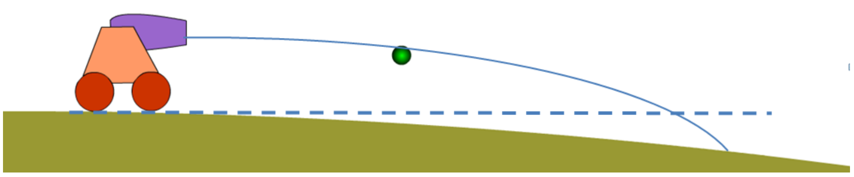
ORBITS AND NEWTON'S THOUGHT EXPERIMENT

Newton died in 1727, 230 years before the launch of Sputnik 1, the first man-made object to orbit Earth, in 1957. However, like all good Physicists, he did have a great imagination and conducted thought experiments.

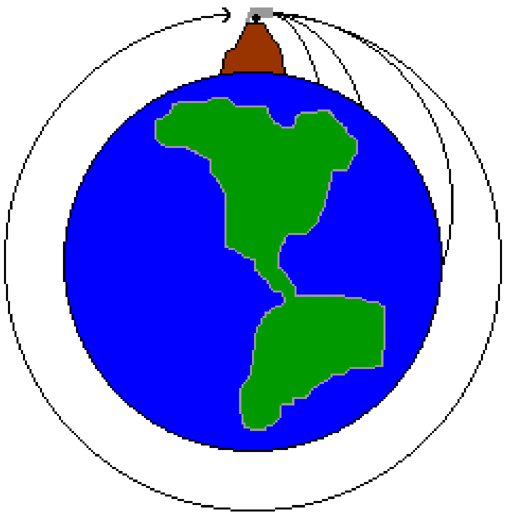
Newton considered the example of the cannon firing horizontally off a cliff. He knew that, as the Earth is approximately a sphere, the ground curves away from the projectile as it falls.



If we give the projectile a greater horizontal velocity, it will travel a greater distance before reaching the ground.



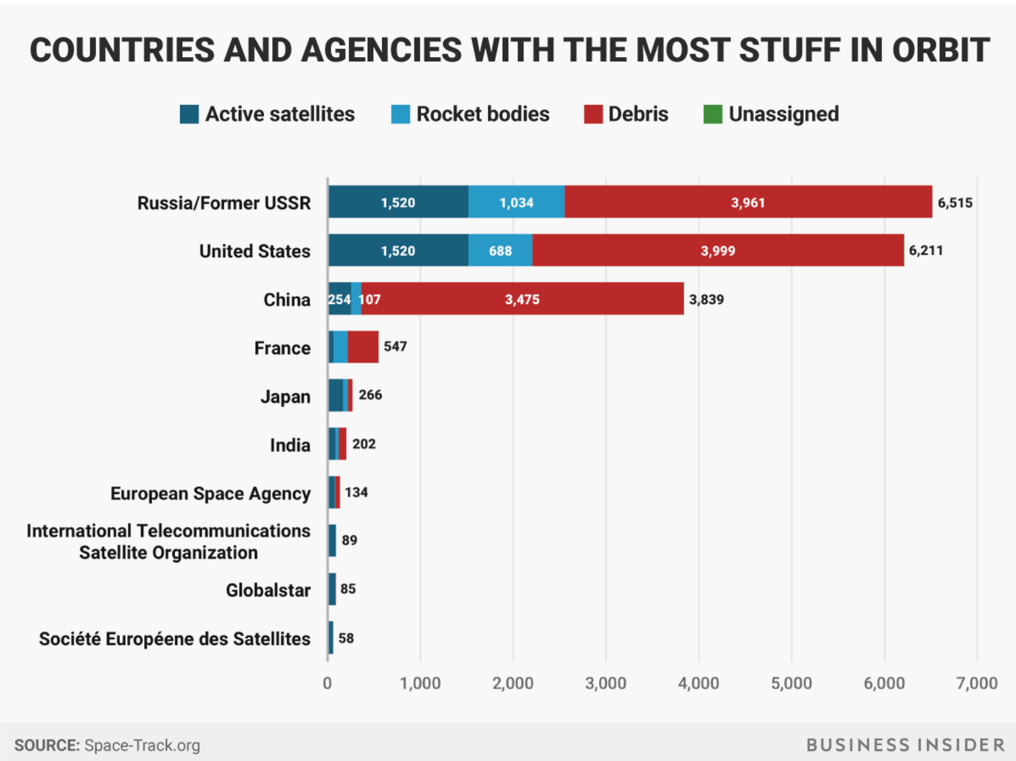
If that ground is also curving down and away from the projectile, the projectile would take even longer to land. (see the diagram, note the additional distance that the cannonball would travel due to the curvature of the Earth).

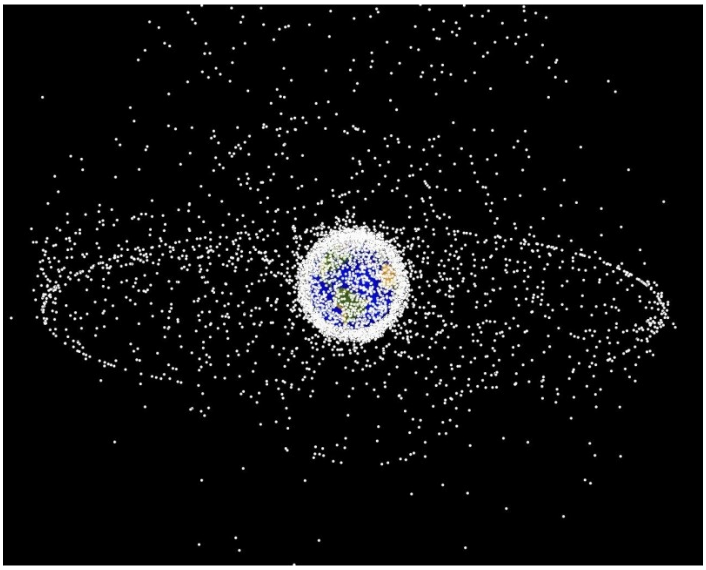


Newton theorized that there must be a horizontal launch velocity for a projectile which results in it following a course parallel to the curved surface of the Earth; causing it to go into orbit. Above this horizontal velocity the object would never return to Earth without an external force and would leave the Earth's *****, and is dependent on the planet's gravitational field.

To put a satellite into geostationary orbit the satellite would be travelling at approximately 3 kms^{-1}, if a rocket was launched vertically from Earth with a velocity of roughly 11 kms^{-1} it could escape the Earth's gravitational field.

As of 2019 a total of 8 378 objects have been launched into space. Currently, 4 994 are still in orbit – although 7 of them are in orbit around celestial bodies other than the Earth; meaning there are 4 987 satellites whizzing around above our heads every single day.. Radar is used to track more than 13,000 items that are larger than ten centimetres. This celestial clutter includes everything from the international space station (ISS) and the Hubble Space Telescope to defunct satellites, rocket stages, or nuts and bolts left behind by astronauts. And there are millions of smaller, harder-to-track objects such as flecks of paint and bits of plastic.





Note the geostationary and polar orbits that are rather busy and full!

Satellites in Low Earth Orbit (LEO), including the ISS, have a period of approximately 90 minutes, with the period depending on height above the Earth's surface.

Many communication satellites, for telecommunications and television, are in geostationary orbit. This orbit is at a greater radius, with a period of 24 hours and an altitude of almost 36,000 km. This allows the satellites to stay above the same point on the Earth's equator at all times and provide consistent communication across the globe.

Though high orbits have revolutionized global communications, they are prone to early failure of electronic components as they are not protected by the Earth's magnetic field and are exposed to very high levels of solar radiation and charge build-up. Satellites huge energies to achieve the required altitudes and powerful amplifiers to ensure successful transmission of data back to Earth.

To avoid these orbits many organisations choose to use a 'constellation' of satellites in LEO (low Earth orbit), placed between the atmosphere and the inner Van Allen Belt. These have their own issues, as gases from the upper atmosphere cause drag which can degrade the orbit.

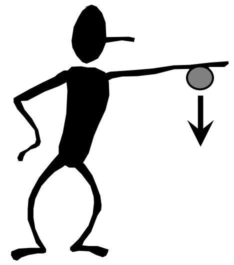
GRAVITY AND MASS

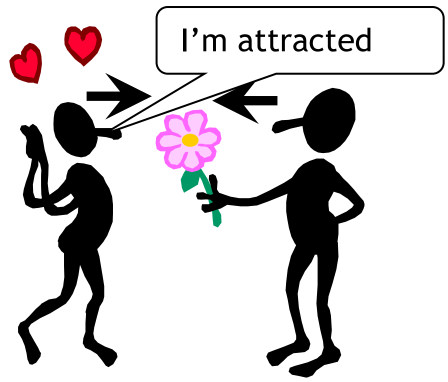
If there is one thing that causes confusion in physics then it is the distinction between mass and weight.

In Physics we must be very careful about our use of terms.

Do NOT use the term GRAVITY when you mean the "*force of gravity*". Try to think of gravity as a phenomenon rather than a force. If you wish to talk about the force of gravity on an object then you should use the term **weight** of the object or the **force of gravity.**

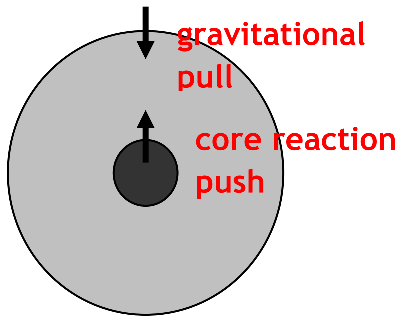
Mass is a measure of how much matter an object contains. This will only change if matter is added to or taken from the object.





The force of gravity is caused by **mass**, any object that has mass creates its own gravitational field. The magnitude of the field depends on the mass of the object and the distance from that mass. This field then exerts a force on any mass within it. You will have done an experiment to measure the Earth's gravitational field using a set of masses and a Newton balance. If you measured the acceleration due to gravity earlier in the unit, then you have effectively measured the gravitational field strength since the magnitude of these two quantities is the same.

The force of gravity permeates the entire universe; scientists believe that stars were formed by the gravitational attraction between hydrogen molecules in space. As mass accumulated the gravitational attraction on the gases increased so that the forces at the centre of the mass were big enough to cause the hydrogen molecules to fuse together. This process is nuclear fusion and generates the sun's energy. The energy radiating outwards from the centre of the sun counteracts the gravitational force trying to compress the sun inwards.



In time the hydrogen will be depleted, the reaction will stop and the sun will collapse under its own force of gravity, but this shouldn't worry you unless you expect to live for approximately four billion years.

It is believed that our solar system formed from the debris generated from the formation of the sun. This debris joined together to form planets, due to the gravitational attraction between the particles.

NEWTON'S UNIVERSAL LAW OF GRAVITATION

Newton formulated the laws of gravity, inventing calculus along the way to help him get his laws in a simple form. Remember, Newton didn't invent gravity; he produced the calculations that explained his observations.

His Law of Gravitation states that the gravitational attraction between two objects is directly proportional to the mass of each object and is inversely proportional to the square of their distance apart. Newton produced what is known as the Universal Law of Gravitation.

$$F = \frac{Gm_1m_2}{r^2}$$

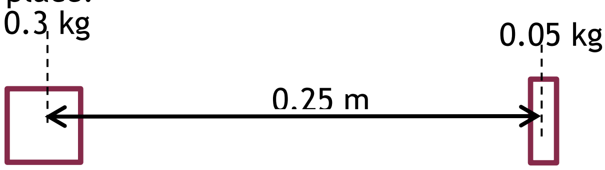
Where *G* is the universal constant of gravitation and has the value **6.67 x 10^{-11} m^{3}kg^{-1}s^{-2} or Nm^{2}kg^{-2}**

Gravitational force is **always attractive**, unlike electrostatic or magnetic forces.

Example: Consider a folder, of mass 0.3 kg and a pen, of mass 0.05 kg, sitting on a desk, 0.25 m apart. Calculate the magnitude of the gravitational force between the two masses.

Assume they can be approximated to point objects, where all their mass is concentrated in one place.

Solution:



m_1 = 0.3 kg
m_2 = 0.05 kg
r = 0.25 m
G = 6.67 x 10^{-11}m^3 kg^{-1} s^{-2}
F = ?

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{6.67\times10^{-11}\times0.3\times0.05}{0.25^2}$$

$$F = 1.6\times10^{-11}\,N$$

We do not notice the gravitational force between everyday objects because it is so small, in fact it is the weakest of the four fundamental forces of our universe. This is just as well, or you would have to fight against the force of gravity every time you walk past a large building! The force of gravity only becomes really apparent when *very* large masses are involved, e.g. planetary masses.

For example the force of attraction between two pupils of average mass [60kg] sitting 1 metre apart is 2.4×10^{-7}N. If there were no resistive forces and one pupil were able to move under the influence of this force towards the other, it would take 22 360s to cover the 1m distance.

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 60 \times 60}{1^2}$$

$$F = 2.4 \times 10^{-7} N$$

$$F = ma$$

$$2.4 \times 10^{-7} = 60 \times a$$

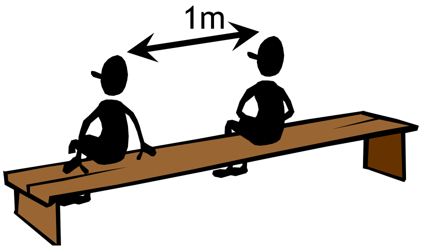
$$a = 4.0 \times 10^{-9} ms^{-1}$$

$$s = ut + \tfrac{1}{2}at^2$$

$$1 = 0 + \tfrac{1}{2} \times 4.0 \times 10^{-9} \times t^2$$

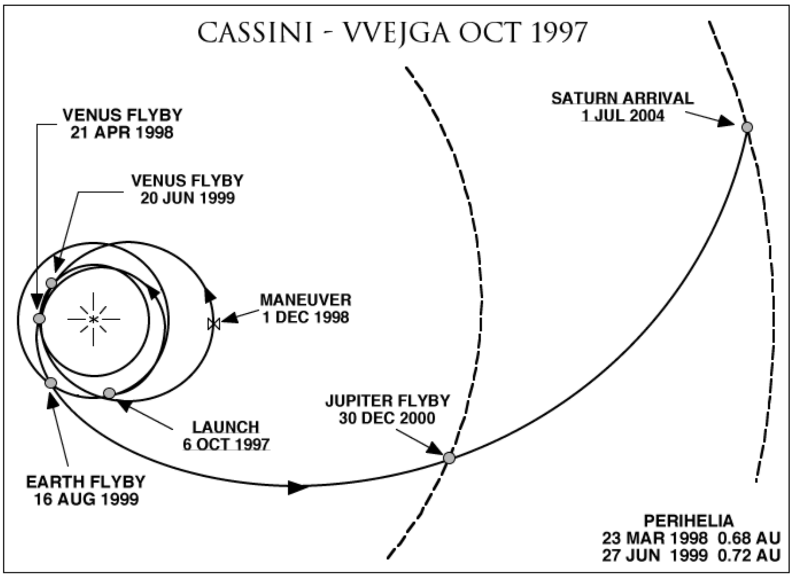
$$t^2 = \frac{2}{4.0 \times 10^{-9}} = 5.0 \times 10^8$$

$$t = 2.3 \times 10^4 s \quad \text{or 6 hours 12 min and 40s}$$



When this is applied to gas molecules or dust particles then the aggregation of larger and larger masses becomes easier to understand.

Another application of the gravitational force is the use made of the 'slingshot effect' by space agencies to get some 'free' energy to accelerate their spacecraft. Simply put they send the craft close to a planet, where it accelerates in the gravitational field of the planet. Here's the clever part, if the trajectory is correct the craft then speeds past the planet with the increased speed. Don't get it right and you still get a spectacular crash into the planet, which could be fun but a bit on the expensive side!



GRAVITATIONAL FIELDS

A gravitational field is the region around a mass where another mass experiences a force. The strength or intensity of the gravitational field, g, is defined as the force acting per unit mass placed at a point in the field.

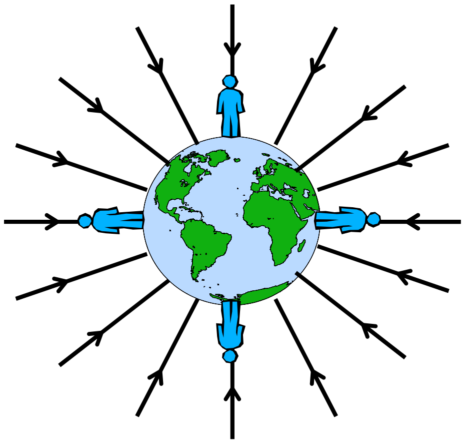
If a mass, m, experiences a gravitational force, F, then gravitational field strength, $g = \frac{F}{m}$ with units of N kg^{-1} (m s^{-2}):

$$g = \frac{F}{m} = \frac{W}{m}$$

Where *g*= gravitational field strength in Nkg^{-1} Gravitational field strength is the force on a 1kg mass placed in the field.

F or *W* is the weight in Newtons, *m* = mass being attracted in kilograms

We can visualise the direction of a field by drawing ***field lines***. For an isolated point mass the lines are always directed towards the mass, the field lines are ***radial***.



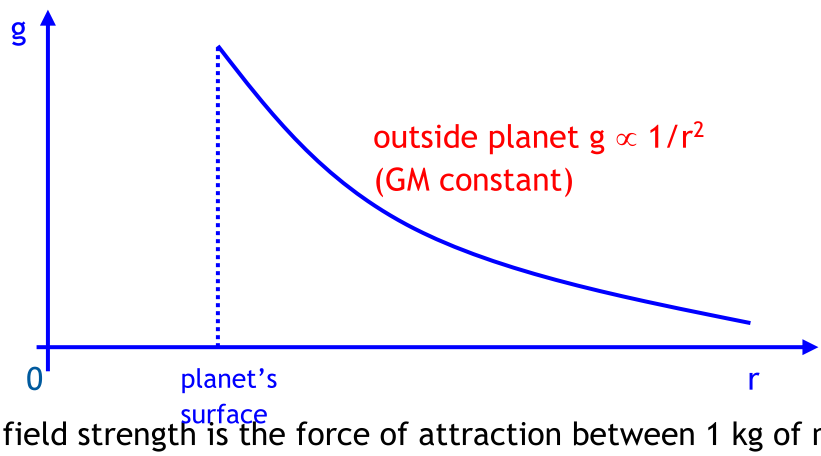
For a radial field:

$$F = \frac{GMm}{r^2}$$

$$\therefore \frac{F}{m} = \frac{GM}{r^2}$$

$$\therefore g = \frac{GM}{r^2}$$

As you get further away from the mass the strength of the field gets weaker. You can see from the diagram that the closer the field lines the stronger the field.. These field lines do not really exist (look out of the window!) but they do show a concept that is real. (This is also similar to magnetic field lines when you draw the direction that a compass will point around a magnet)



The gravitational field strength is the force of attraction between 1 kg of mass and the Earth. Remember the law of gravitation is a force of attraction between *two* masses, this means the 1 kg mass is attracting the Earth towards it, as well as the other way round.

Example 1: Show, using the universal law of gravitation, that the gravitational field strength on Earth is 9·8 N kg^{-1}.

$$F = \frac{Gm_1m_2}{r^2}$$

$$but$$

$$mg = \frac{Gm_1m_2}{r^2}$$

$$\cancel{m}g = \frac{Gm_1\cancel{m_2}}{r^2}$$

$$g = \frac{6.67\times10^{-11}\times 6\cdot 0\times 10^{24}}{(6\cdot 4\times 10^{6})^2}$$

$$g = 9.77Nkg^{-1}$$

Example 2:

(a) Calculate the gravitational field strength on the surface of the Moon.

(b) Calculate the gravitational force between the Moon and the Earth.

(a)

$$F = \frac{Gm_1m_2}{r^2}$$

$$but$$

$$mg = \frac{Gm_1m_2}{r^2}$$

$$\cancel{m}g = \frac{Gm_1\cancel{m_2}}{r^2}$$

$$g = \frac{6.67\times10^{-11}\times 7\cdot 3\times 10^{6}}{(1\cdot 7\times 10^{6})^2}$$

$$g = 1.68Nkg^{-1}$$

(b)

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{6.67\times10^{-11}\times 6\cdot 0\times 10^{24}\ \times 7\cdot 3\times 10^{6}}{(3.84\times 10^{8})^2}$$

$$F = 1.98\times10^{4}N$$

CHAPTER 6: SPECIAL RELATIVITY

These notes are produced with material from EducationScotland, Dick Orr, and Arfur Dogfrey and lots of reading from other sources. **Note most books transcribe the terms t and t' which appears to give a different equation.** It is the same equation but the terms are reversed. Always use the formula given in the SQA Relationships sheet.

Guides to Special Relativity:

https://www.dummies.com/education/science/physics/einsteins-special-relativity/

http://physicsforidiots.com/physics/relativity/

https://www.youtube.com/watch?v=ttZCKAMpcAo

https://www.pdfdrive.com/relativity-a-very-short-introduction-emilkirkegaarddk-e7286899.html

SUMMARY OF CONTENT

6		Special relativity	
	eq	$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$	$l' = l\sqrt{1-\frac{v^2}{c^2}}$
	a)	I know that the speed of light in a vacuum is the same for all observers.	
	b)	I know that measurements of space, time and distance for a moving observer are changed relative to those for a stationary observer, giving rise to time dilation and length contraction.	
	c)	I can use appropriate relationships to solve problems involving time dilation, length contraction and speed.	

SPECIAL RELATIVITY

Before we start this part of the course, please accept that the Newtonian Mechanics you have been taught so far is true – up to a point.

Useful Definitions and ideas

inertial reference frames	**Simply two objects that are moving at constant speed relative to one another.**
absolute reference frame	A unique, universal frame of reference from which everything could be defined or measured against. Einstein's theories prove no such reference frame exists.
the ether (aether)	Early theories suggested that electromagnetic waves (light) required a medium: (a space-filling substance or field) to travel through. This ether was believed to be an absolute reference frame. Modern theories have no requirement for this idea, and the Michelson-Morley experiment performed in 1887 provided no evidence for such a field.
Time dilation	**A difference in a time interval as measured by a stationary observer and a moving observer.**
Length contraction	**A difference in a length along the moving axis as measured by a stationary observer and a moving observer.**

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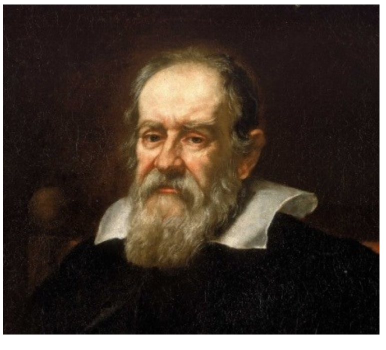

Galilean Invariance

Galileo was one of the first scientists to consider the idea of relativity.

He stated that the laws of Physics should be the same in all **inertial frames of reference**.

He first described this principle in 1632 using the example of a ship, travelling at **constant velocity**, without rocking, on a smooth sea; any **observer** doing experiments below the deck would not be able to tell whether the ship was **moving** or **stationary**.

http://www.biografiasyvidas.com/biografia/n/fotos/newton.jpg

Newtonian Relativity

Newton followed this up by expanding on Galileo's ideas.

He introduced the idea of **absolute** or **universal space time**.

He believed that it was the same time at all points in the **universe** as it was on **Earth**, not an unreasonable assumption.

According to Newton, **absolute time** exists independently of any perceiver and progresses at a consistent pace throughout the universe. **Absolute space**, in its own nature, without regard to anything external, remains always **similar** and **immovable**.

In other words, the **laws of Physics** are the same whether **moving at constant speed** or at **rest**.

Many people know that Einstein was renowned for his theories on Relativity but for some people that is far as their knowledge goes.

Einstein wrote two papers on relativity. His theory on Special Relativity was published in 1905 in a paper titled, "On the Electrodynamics of Moving Bodies" by Albert Einstein. **Special Relativity** deals with the concepts of **space and time as viewed by observers moving relative to one another with uniform velocities**. It is called “Special” as it deals with only one situation and that is one where objects are not accelerating, so are continuing at constant velocity. In 1916 Einstein published another paper, on General Relativity. **The Theory of General Relativity is the one which redefined the laws of gravity. The Theory of General Relativity also says that large objects cause outer space to bend in the same way a marble laid onto a large thin sheet of rubber would cause the rubber to bend**. The larger the object, the further space bends. Just like a bowling ball would make the rubber sheet bend much more than the marble would.

If you want more information refer to the other material in the Our Dynamic Universe Folder. Don’t give up too early, try to get to grips with one of the major works of the twentieth century. However, do be aware that the literature is not consistent on the choice of definition of terms so that you can find various forms of the equations (I know I found them) use intuition as your best form of defence!

NOTATION

t time interval measured in the **same** frame of reference as the event (e.g. the pulse of light, throwing up a ball, running a race)

t' time interval measured in a frame of reference moving relative to the one containing the event(e.g. on a train)

l length measured in the **same** frame of reference as the object (e.g. rod)

l' length measured in a frame of reference moving relative to the one containing the object (e.g. rod)

v relative velocity of the two frames of reference

(*Note*: Recall that no one frame of reference is any more ‘stationary’ or ‘moving’ than any other. There is no ‘absolute rest’.)

It is Einstein’s theory of Special Relativity that we need to introduce in this course.

SQA CfE Higher Physics	SQA CfE Advanced Higher Physics
SPECIAL RELATIVITY	**GENERAL RELATIVITY**
Uniform Motion Only	**Any motion -e.g. accelerate,curve etc**

Before we can go deeply into the idea of Special Relativity we must introduce the concept of Frames of Reference

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FRAMES OF REFERENCE

How we see things in the world is determined by our position or viewing point. We call this our ***frame of reference***. Relativity is all about observing events and measuring physical quantities, such as distance and time, from different reference frames. Frames of reference refer to any laboratory, vehicle, platform, spaceship, planet etc. and often have a relative velocity to another 'frame of reference'. **When the movement is in a *straight line with a constant velocity*** we call these ***inertial frames of reference***.

If you ask most people to explain what the term "motion" means, they will probably say something about movement, transport, travel, and other similar words that convey the idea of movement from one point to another. However, most people would omit to mention that all motion is **relative**. That is, motion can only be judged by comparing the position of the moving object with some other reference point or object.

In the language of physics, the reference against which the motion of an object is measured is called a "frame of reference". Thus, in the figure above, for example, the road is a frame of reference against which the motion of the car can be measured. Alternatively, the house could be a frame of reference against which the motion of the car is measured.

Thus;.

1. Car moves relative to the road

2. Person in car moves relative to the car

3. Car moves relative to the house

4. Person in the house moves relative to the house

An inertial frame of reference is one in which Newton's first law of motion holds, i.e you are travelling at constant velocity or are at rest.

Example. Imagine sitting in your stationary car on a flat road, with the gears in neutral. The car remains stationary, as there are no unbalanced forces acting on the car. Now put the car in gear and floor the accelerator. Forces from the engine act on the car and it picks up speed. Now if you were holding a cup of coffee at this point you are likely to spill this down your top. You will experience Newton's Laws of Motion.

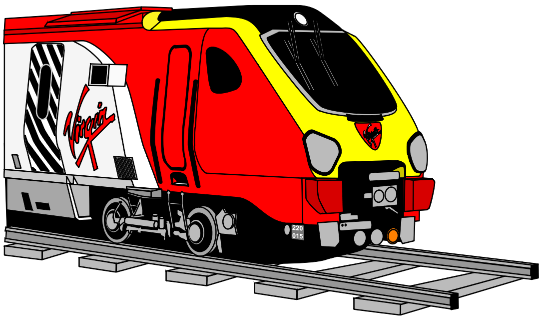
Remember this as a memory aid. If you'd spill your coffee you are not in an inertial reference frame. As the car continues to pick up speed, wind resistance increases on the car, and eventually the car will reach its maximum speed. At this point, the force from the engine is exactly balanced by the frictional and wind resistance forces, and hence the net force on the car is zero. The speed remains constant, at terminal velocity, in accordance with Newton's first law.

In this example, the road (or surface of the earth), against which the motion of the car is measured, is an inertial frame of reference. Frames of reference need not be stationary. It can be shown that if a stationary frame of reference is inertial, then it is also inertial if it moves at constant speed. Thus, in the figure above, the car is an inertial frame of reference whether it is stationary or moving relative to the road.

Here is another example, you probably feel that you are currently stationary, but remember that the Earth rotates on its axis once a day so at the equator a person would be moving at 1670 kmh^{-1}. The Earth is also moving with respect to the Sun, orbiting the Sun once a year requires the Earth to be moving at approximately 100 000 kmh^{-1}, with respect to the Sun. We are in the outer spiral of the Milky Way galaxy, so from an inertial reference frame in another galaxy we would observe someone on the Earth as travelling 800 000 kmh$^{-1.}$ Which speed is correct? They are all correct- it depends on your frame of reference

However, any accelerating frame of reference is not inertial. *Thus, the car in the figure above, if it is accelerating, is not an inertial frame of reference.* ***Remember that if you spill your coffee you are <u>not</u> in an inertial frame of reference.***

Note that without a frame of reference, it would not be possible to judge the state of motion of an object moving at constant speed. Newton believed that there was one overall, preferred, reference frame, called the "absolute" frame of reference, but Einstein disagreed.



Einstein asserted that there is no "preferred" inertial frame of reference against which motion should be measured. In other words, all inertial frames of reference are **equivalent**. It does not matter which inertial frame of reference is used to judge the motion of an object, in our example either the road or the house are acceptable frames of reference against which to judge the motion of the car.

Here is another example of the same event seen by three different observers, each in their own frame of reference:

Event 1: You are reading your Kindle on the train. The train is travelling at 60 mph.

Observer	Location	Observation
1	Passenger sitting next to you	You are stationary
2	Person standing on the platform	You are travelling towards them at 60 mph
3	Passenger on train travelling at 60 mph in opposite direction	You are travelling towards them at 120 mph.

This example works well as it only involves objects travelling at relatively low speeds. The comparison between reference frames does not work in the same way, however, if objects are moving close to the speed of light.

Event 2: You are reading your Kindle on an interstellar train. The train is travelling at 2×10^8 m s^{-1}.

Observer	Location	Observation
1	Passenger sitting next to you	You are stationary
2	Person standing on the platform	You are travelling towards them at 2×10^8 m s^{-1}
3	Passenger on train travelling at 2×10^8 m s^{-1} in opposite direction	*You are travelling towards them at 4×10^8 m s^{-1}* ✘

The observation made by observer three is impossible as an object cannot travel faster than the speed of light from **any** reference frame and it would certainly be impossible to *watch* something travel faster than light.

POSTULATES OF SPECIAL RELATIVITY

Einstein took this one stage further, and came up with his first postulate of special relativity, which generalises our discussion to ALL of the laws of physics:

Postulate 1:

All the laws of physics are the same in all inertial frames of reference.

We can give a simple example of this principle in action. Consider a person standing at the road side and a person sitting in a car moving at constant speed. This postulate states that both people must observe the same laws of physics for all phenomena. If the car is accelerating, then it does not constitute an inertial frame of reference, and the person on the ground and in the car need not observe the same laws of physics.

The second postulate is, in a sense, a corollary (consequence) of the first postulate:

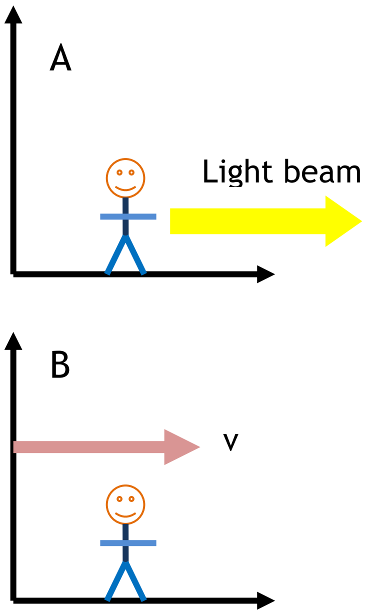
Postulate 2:

The speed of light (in a vacuum) is the same in all inertial frames of reference.

This postulate leads to a surprising difficulty when we consider what we expect to happen in everyday life. Suppose we are driving along at 50 mph, approaching a stationary person on the pavement. At the instant we pass the person, he throws a ball at 50 mph, relative to him and in the direction we and the car are travelling. In the frame of reference of the car, therefore, the ball is stationary.

The second postulate says that light does not behave in this way. Suppose that, instead of throwing a ball, the person on the pavement switches on a torch. The second postulate says that both we and him have to measure the same speed of light from the torch - no matter how fast we are travelling in the car.

Our consideration of throwing the ball suggests that if the speed of light is *c*, and our speed is *v* then the person on the pavement sees light travelling at speed *c*, but we should see it travelling at speed *c* - *v*. The second postulate says that this is **not** the case. The second postulate says that we see the light travelling at speed *c*.



The figure (left) illustrates this, but in the conventional notation used to represent frames of reference, in the context of relativity. In this example, frame A is the pavement, and frame B is the car.

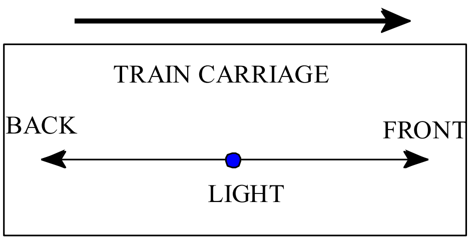
Person in frame A fires a light beam (e.g. switching on a torch) at the instant that frame B passes by. The second postulate implies that BOTH observers would measure the same speed for the light beam!

Einstein showed:

1. everyone sees the same speed for light (given the symbol *c*)
2. what happens when you try to go faster than c.

To understand this, our concepts of time, length, and mass must be changed. Consider two observers viewing the same event. One observer is stationary and one moving. If *c* is the same for both observers then they must experience different distances and times. Two events can be simultaneous in one frame of reference but when viewed from another frame of reference the events are not simultaneous. This is called the ***Relativity of Simultaneity***.

Imagine a train travelling with both its front and back doors open. A light is uncovered from under a dark casing, causing a beam of light to be emitted. The beam is emitted from the centre of a train carriage. A person on the train would see the beam of light arrive at the end walls of the train at the same time. But a person viewing the same event on the railway banking would see the beam of light exit from the back door before the front door because the train is moving forward.



These ideas seem very contrary to classical Newtonian Mechanics which you have dealt with so far.

Newtonian Mechanics states that:

1. the time interval between events is independent of the observer.

Newton	Einstein
d and t constant	c constant
c relative	d and t relative

2. the space interval is independent of the observer.

THE PRINCIPLES OF RELATIVITY

To summarise

Special relativity (which applies to observers/reference frames in relative motion with constant velocity) has two postulates:

1. **The laws of physics are the same for all observers in all parts of the universe.** ***All the laws of physics are the same in all inertial frames of reference*.**

2. **Light always travels at the same speed in a vacuum, 3.0×10^8 m s^{-1} (299,792,458 m s^{-1} to be more precise). (Light does slow down inside transparent material such as glass.)**

This means that no matter how fast you go, you can never catch up with a beam of light, since it always travels at $3{\cdot}0 \times 10^8$ m s^{-1}*relative to you*.

The best known experimental evidence that started the proof for the Theory of special relativity is the Michelson-Morley interferometer experiment. It actually failed to show the expected results, despite many people repeating the experiment. (*see later*).

Example: A "space car" is travelling through space at 90% of the speed of light (2.7×10^8 m s^{-1}) with its headlights on. The occupants of the car will see the beams of the headlights travel away from them at 3×10^8 m s^{-1}, so "common sense" or Newtonian Mechanics would say that the light from the headlights is travelling at 5.7×10^8 m s^{-1}. But it is not, it is travelling at 3×10^8 m s^{-1}.

An observer on Earth will also observe light from the headlamps travel at 3×10^8 m s^{-1}. The speed of light, c, is constant in and between all reference frames and for all observers.

These principles have strange consequences for the measurement of distance and time between reference frames.

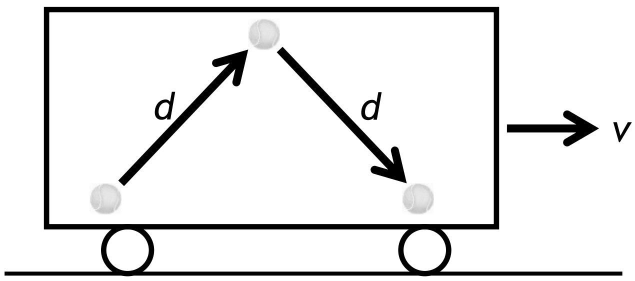
TIME DILATION

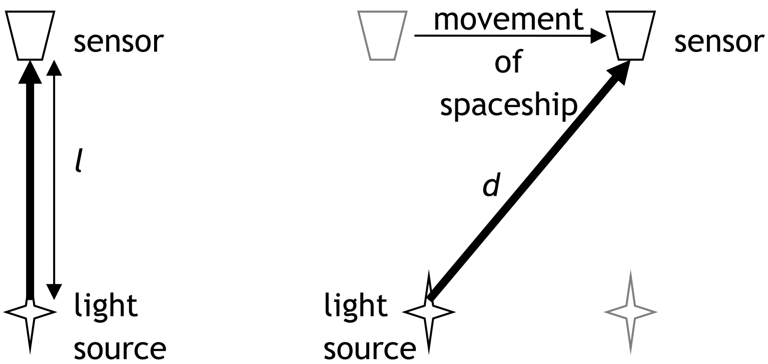
Time dilation is the increase in an observed time interval for an object moving relative to an observer, compared to that measured when they are in the stationary frame of reference.

A very simple thought experiment shows that one consequence of the speed of light being the same for all observers is that time experienced by all observers is not necessarily the same. There is no universal clock that we can all refer to – we can simply make measurements of time as we experience it.

Imagine you are on a spaceship travelling at constant speed (v) relative to a colleague on a space station. You are investigating a timing device based on the fact that the speed of light is constant. You fire a beam of light at a sensor and time how long it takes to arrive. From your point of view the light has less distance (l) to travel than from the point of view of your colleague on the space station (d). If you both observe the speed of light as the same then you cannot agree on when it arrives, i.e. both of you experience time in a different way.

Time is different for observers in different reference frames because the path they observe for a moving object is different.

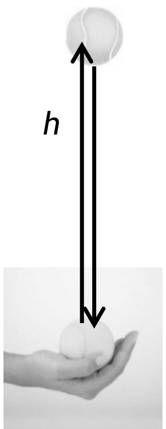




Your point of view, where the source and sensor are moving along together

Your stationary colleague's point of view, where the sensor moves after the light is fired

Example 1: How distance can be different for different observers



Event: Inside a moving train carriage, a tennis ball is thrown straight up and

caught in the same hand.

Observer 1: In Observer 1's reference frame they are stationary and the ball has gone straight up and down.

Observer 1 sees the ball travel a total distance of 2*h*.

The ball is travelling at a speed *s*.

The period of time for the ball to return to the observers hand is:

$$t = \frac{2h}{v_1}$$

Observer 2, standing on the platform watches the train go past at a speed, v_2, and sees the passenger throw the ball. However, to them, the passenger is also travelling horizontally, at speed *v*. This means that, to Observer 2, the tennis ball has travelled a horizontal distance, as well as a vertical one.

Observer 2 sees the ball travel a total distance of 2*d*.

The period of time for the ball to return to the observers hand is: $t' = \frac{2d}{v_1}$

For observer 2, the ball has travelled a greater distance, in the same time.

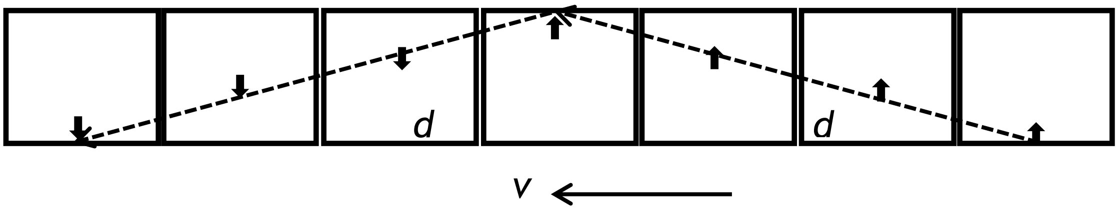
Example 2: How time can be different for different observers

Event: Observer 1 is in a spaceship travelling to the left at speed *s*. Inside the spaceship cabin, a pulsed laser beam is pointed vertically up at the ceiling and is reflected back down. The laser emits another pulse when the reflected pulse is detected by a photodiode.

Reference frame 1: Inside the cabin, the beam goes straight up, reflects off the ceiling and travels straight down.

Period of pulse: $t = \frac{2h}{c}$

Reference frame 2: Observer 2 on another, stationary ship.



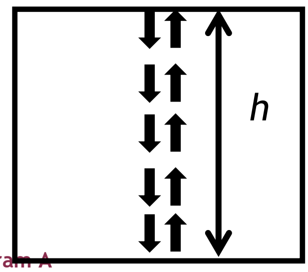
Period of pulse: $t' = \frac{2d}{c}$

The time for the experiment as observed by observer 2 (reference frame 2), t', is greater than the time, t, observed by observer 1 when moving with the photodiode. Neither observer would be aware of any difference until they met up and compared data.

For observer 2, the light has taken a greater time as the light has travelled a greater distance.

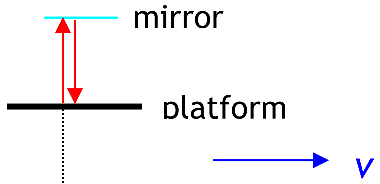
LET'S DO THE MATHS

Consider a person on a platform who shines a laser pulse upwards, reflecting the light off a mirror. The time interval for the pulse to travel up and down is t (no superscript). For travellers on board the train moving along the x-axis at high speed v,, the light travels as shown in diagram A.

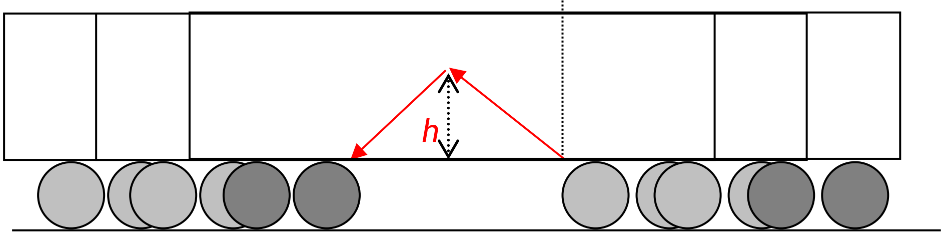


1 Diagram A

Person in same frame as 'event' measures a time t.



Total distance travelled bv pulse



Travellers in this different frame of reference observe the 'event' (eg out of the window of the train), which takes place in the platform frame of reference and measure a time t'.

2: Diagram B

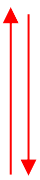
The time taken for the light to travel up and back, as measured by travellers in this frame, is t' (t dash or t priime).

In the time t' that it takes for the light to travel up and back down the train in this frame, the train has travelled a distance d.

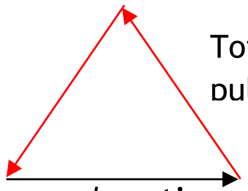
Both observers measure the *same* speed for the speed of light.

Train frame of reference

$2h = ct$



Platform frame of reference

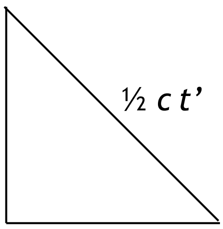


Total distance travelled by pulse= ct'

$d = vt'$

Horizontal distance travelled by train =

A right- angled triangle can be formed where the vertical side is the height, h, of the pulse (½ ct), the horizontal side is half of the distance, d, gone by the train (½ vt') and the hypotenuse is half the distance gone by the pulse as seen by the travellers on the train (½ct').



Applying Pythagoras to the triangle gives:

$$\left(\tfrac{1}{2}ct'\right)^2 = \left(\tfrac{1}{2}ct\right)^2 + \left(\tfrac{1}{2}vt'\right)^2$$

$$(ct')^2 = (ct)^2 + (vt')^2$$

$$(c^2 - v^2)\,t'^2 = c^2 t^2$$

$$\left(1-\frac{v^2}{c^2}\right) t'^2 = t^2$$

$$t'\sqrt{1-(v/c)^2} = t \qquad (1)$$

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (2)$$

Where

t time interval measured in the **same** frame of reference as the event (e.g. the pulse of light, throwing up a ball, running a race).

t' time interval measured in a frame of reference moving relative to the one containing the event(e.g. on a train).

v relative velocity of the two frames of reference.

WHAT ASSUMPTIONS HAVE WE MADE?

(i) The two frames are moving relative to each other along the *x*-axis, i.e. the train passes the platform. There is no bending or circular motion involved.

(ii) We require two travellers on the train since the start and finish places are separate. This is fine since two clocks can be synchronised in the same frame of reference.

ADDITIONAL NOTES

One famous situation leading from this theory is the thought experiment regarding the twin paradox. (which is not currently possible):

At the age of 25 you leave your twin behind on Earth to go on a space mission. You are in a spaceship travelling at 90% of the speed of light (this is why it is a thought experiment) and you go on a journey for 20 years as *you* measure it. When you get back you will find that 46 years have elapsed on Earth. Your clock will have run slower than those on Earth, but both your clock **and** the one on Earth are correct. You are 45, but your twin is 71, not just in terms of what the clock says but biologically as well. This is "real" time to both of you (even though it seems different).

You can read about this paradox in more detail by asking your Physics teacher for the additional note or reading it for yourself from Russell Stannard's excellent book "Relativity- a very short introduction" Oxford. (2008) ISBN 978-0-19-923622-0

At velocities which we are more familiar with, a retired airline pilot who may have spent 35,000 hours travelling at, say, 180 m s^{-1} whilst in the air will be 23 μs younger than if they had stayed on the ground. Scott Kelly, a twin, who used to be 6 minutes younger than his older twin is now 6 minutes and 5 milliseconds younger, due to his 520 days on the ISS orbiting at 28,160 kmh^{-1} (17,500 mph). Even your calculators are unable to show this and you'd need to prove it using something like Microsoft Excel.

TIME DILATION FORMULA AND THE LORENTZ TRANSFORMATION

The formula linking these can be shown to be:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Note this is often written as:

$$t' = t\gamma$$

where γ is known as the Lorentz Factor. It is often used in the study of special relativity and is given by:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Example:

A rocket is travelling at a constant 2.7×10^8 m s^{-1} compared to an observer on Earth. The pilot measures the journey as taking 240 minutes. (*Note this is an impossible scenario with current technology*)

How long did the journey take when measured from Earth?

Solution: **Note as time dilation is about relative times etc there is no requirement to change the units of time to seconds. Whatever unit you use will be the same unit for your unknown time.**

t = 240 minutes

$v = 2.7 \times 10^8$ m s^{-1}

$c = 3 \times 10^8$ m s^{-1}

t' = ?

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \frac{240}{\sqrt{1 - \frac{(2\cdot7 \times 10^8)^2}{(3 \times 10^8)^2}}} = 551\text{minutes}$$

An observer on Earth would measure the journey as taking 551 minutes, i.e. 551 minutes would have passed from their point of reference.

ANSWERING LORENZ FACTOR QUESTIONS

1) Make sure you know what the terms mean.

t time interval measured in the **same** frame of reference as the event (e.g. the pulse of light, throwing up a ball, running a race). *Referred to in books as "proper time"*

t' time interval measured in a frame of reference moving relative to the one containing the event (e.g. on a train)

l length measured in the **same** frame of reference as the object (e.g. rod) *Referred to in books as "proper length"*

l' length measured in a frame of reference moving relative to the one containing the object (e.g. rod).

v relative velocity of the two frames of reference.

(*Note*: Recall that no one frame of reference is any more 'stationary' or 'moving' than any other. There is no 'absolute rest'.

2) Think through the question and try to decide what you would expect to happen.

3) **Remember that** $\frac{v^2}{c^2}$ is the same as $\left(\frac{v}{c}\right)^2$

4) Note that the question can give the value of *v* in terms as a portion of *c*. e.g. the speed in the above example could have been given as 90% *c* or 0.9 times *c* or 9/10 of the speed of light. So using the question previously given we would calculate it as shown.

$$t' = \frac{240}{\sqrt{1 - 0 \cdot 9^2}}$$

$$t' = \frac{240}{\sqrt{1 - 0 \cdot 81}}$$

$$t' = 551 \text{ minutes}$$

5) You might find it easier to square both sides which would give you the following equation $(t')^2 = \dfrac{(t)^2}{1 - \dfrac{v^2}{c^2}}$

6) The ratio of *t* to *t'* can be given and again this would be calculated in the following way

$$\sqrt{1-\frac{v^2}{c^2}} = Ratio(t:t')$$

$$1-\frac{v^2}{c^2} = (Ratio(t:t'))^2$$

7) The ratio of *t'* to *t* can be given and again this would be calculated in the following way

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = Ratio(t':t)$$

$$\frac{1}{1-\frac{v^2}{c^2}} = (Ratio(t':t))^2$$

8) Don't separate v^2 from c^2 until after you have dealt with the 1.

9) Remember that it is $1-\frac{v^2}{c^2}$ so if $\frac{v^2}{c^2}$ is taken to the other side it becomes $+\frac{v^2}{c^2}$.

10) $1-\frac{v^2}{c^2}$ must give a value less than 1.

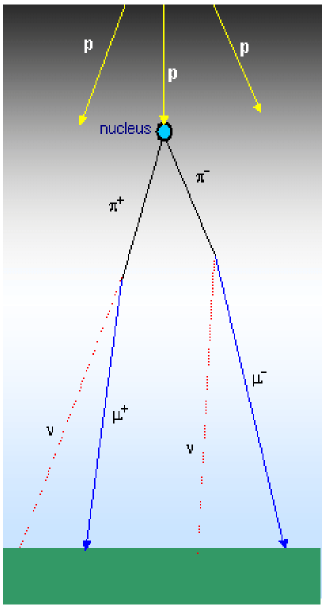
11) It is not necessary to put times into seconds, whatever the given units of time your calculated time quantity will have the same units as the equation works out ratios.

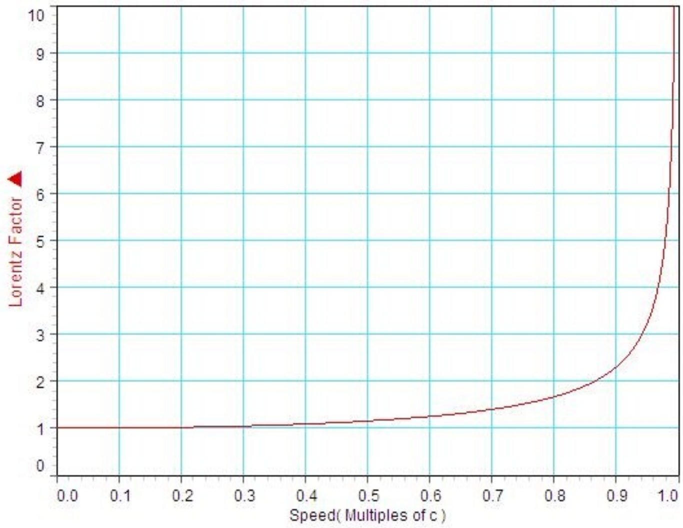
12) As we will see later the length transformation equation looks similar but is written in terms of t'=tγ compared with the time transformation written as l'=l/γ

13) Some people recommend working out the $\sqrt{1-\frac{v^2}{c^2}}$ bit first and then using t'=tγ or l'=l/γ, but remember that $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

RELATIVISTIC TIME DIFFERENCES IN EVERYDAY LIFE?

A graph of the Lorentz factor versus speed (measured as a multiple of the speed of light) is shown below.





We can see that for small speeds (i.e. less than 0.1 times the speed of light) the Lorentz factor is approximately 1 and relativistic effects are negligibly small. Even 0.1 times the speed of light is 30,000,000 m s^{-1} or 108,000,000 km h^{-1} or about 67,500,000 mph – a tremendously fast speed compared to everyday life.

However, the speed of satellites is fast enough that even these small changes will add up over time and seriously affect the synchronisation of global positioning systems (GPS) and television satellites with users on the Earth. They have to be specially programmed to allow for the effects of special relativity (and also general relativity, which is not covered here). Very precise measurements of these small changes in time have been performed on fast-flying aircraft and agree with predicted results within experimental error.

Further evidence in support of special relativity comes from the field of particle physics and the detection of particles called muons at the surface of the Earth. Muons are created in the upper atmosphere by cosmic rays (high-energy protons from space).

The half-life of a muon is 2.2 μs and so moving at 0.994 c they would only expect to travel about 660 m before half of them decayed. Muons formed at, say 12000 m would take 40 μs or about 20 half-lives to reach the ground. This would mean that only $1/2^{20}$ of the original number would be detected. The fact that the proportion reaching the ground is much higher than this means that the muons are existing for longer.

At 0.994c the formula for time dilation gives the half-life for the muons to be 20 μs. This means that at 0.994c the proportion reaching the ground should be 0.25, indicating the number of muons per second detected at the ground is much greater than expected. This is because the 'muon clock' runs slowly compared to the observer on Earth and the muon reaches the ground.

http://www.scivee.tv/node/2415

Example: Using the figures above, show, by calculation, why time dilation is necessary to explain the observation of muons at the surface of the Earth.

Solution:

t = 2.2 μs = 2.2×10^{-6} s

v = $0.994 \times 3.00 \times 10^{8}$ = 2.982×10^{8} m s^{-1}

d = ?

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \frac{2.2 \times 10^{-6}}{\sqrt{1 - (0.994)^2}}$$

$$t' = 20 \mu\, s$$

$$d = vt$$
$$d = 2.982 \times 10^{8} \times 2.2 \times 10^{-6}$$
$$d = 656 m$$

$$d' = vt'$$
$$d' = 2.982 \times 10^{8} \times 20 \times 10^{-6}$$
$$d' = 6.0 \times 10^{3} m = 6000 m$$

In the reference frame of an observer on Earth the half-life of the muon is recorded as 20 μs during which it travels 6000 m. From this perspective, a period of only two half-lives is needed for the muons to reach the earth from 12000 m altitude.

It is useful to mention the significance of the term $\sqrt{1-(v/c)^2}$.

(The reciprocal $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ is known as the gamma factor, or Lorenz Transformation.)

This term occurs in relativity equations and its size determines when relativity effects will be observed. At everyday speeds it is almost unity (one).

You should be able to *interpret* the final equation, stating what each of the symbols represent, and what the equation means in terms of the time interval for each observer.

Note: In our thought experiment above with the pulse emitted on the platform the laser pulse starts and finishes at the *same* place on the platform. Thus equation (1) is used to calculate the time interval *t'* registered in a frame of reference, e.g. the train, for an event which takes place in a *different* frame of reference from the 'event'.

For example if $v = 0.4c$ then $(v/c)^2 = 0.16$ and $\sqrt{1-(v/c)^2}$ = $\sqrt{1-0.16}$ = 0.917.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{0.917} = 1.091$$

Let us use this value of $v = 0.4c$ in our thought experiment with a laser pulse time of 8.0 ns.

$$t' = t\gamma$$
$$t' = 8.0 \times 1.091$$
$$t' = 8.7ns$$

Thus, if t = 8.0 ns, we can calculate t', giving t' = 8.7 ns. A longer time interval is 'measured' by travellers on the train. This effect, known as *time dilation* (dilation = expanding), is a direct result of the postulate that the speed of light is measured to be the same by all observers.

Time dilation leads to observers being unable to agree about simultaneous events. Two events may appear to be simultaneous to one observer, but may *not* be simultaneous for others.

ANOTHER EXAMPLE AND SOME EFFECTS

Consider a space ship passing Earth at a velocity of 0.5c. It emits a pulse (or on Earth we observe 'ticks' of their clock) of duration ΔT = 2.0 ns. We on Earth can 'measure' the duration t' we observe. Note that we are *not* in the same frame of reference as the 'event' so our time interval is t' not t. The duration of the event, in the frame of the event on the space ship, is t.

Using equation

$$t' = \frac{t}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$$

$$t' = \frac{2.0\times10^{-9}}{\sqrt{1-0.5^2}}$$
$$t' = \frac{2.0\times10^{-9}}{\sqrt{1-0.25}}$$
$$t' = 2.3n\ s$$

$$t' = \frac{2.0\times10^{-9}}{\sqrt{1-0.5^2}}$$
$$t' = \frac{2.0\times10^{-9}}{0.87}$$
$$t' = 2.3n\ s$$

gives us an observed time interval of t × 1 /0.87 = 2.3 ns.

We consider their clock is running slow, time is passing more quickly for us so they could end up 'younger'! Although time dilation can give rise to interesting discussions on time travel (into the future), the explanation of the twin paradox requires more consideration since any acceleration may involve general relativity. and returning from a long fast space trip would require a 'change' frames of reference, and hence a knowledge that the observer is moving.

When $\sqrt{1-(v/c)^2}$ is almost unity no effect is noticeable. It is useful to calculate this term (or the reciprocal) for various speeds, for example:

- a supersonic plane 422 m s^{-1} (900 mph);
- × 10^8 m s^{-1};
- 0.3 × 10^8 m s^{-1} (10%c);
- 1.0 × 10^8 m s^{-1};
- 2.0 × 10^8 m s^{-1},

- 2.8×10^8 m s^{-1}
- and 99% *c*.

If you do these calculations you will clearly see that effects in everyday life are not noticeable. We need ***v* > 10%*c* for any noticeable effects**.

LENGTH CONTRACTION

https://www.youtube.com/watch?v=FPzGAksFCbs

Length contraction is the shortening of the measured length of an object moving relative to the observer's frame.

Another implication of Einstein's Special theory of Relativity is the shortening of length when an object is moving. Consider the muons discussed above. Their large speed means they experience a longer half-life due to time dilation. An equivalent way of thinking about this is that the muons experience the height of the atmosphere as smaller (or contracted) by the same proportion as the time has increased (or dilated). A similar formula for time dilation can be derived. Note that ***the contraction only takes place in the direction that the object is travelling***:

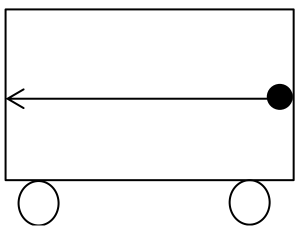
$$l' = l\sqrt{1-\frac{v^2}{c^2}}$$

where

l proper length or rest length refers to the length of an object in the object's rest frame.so it is the.length measured in the <u>**same**</u> frame of reference as the object

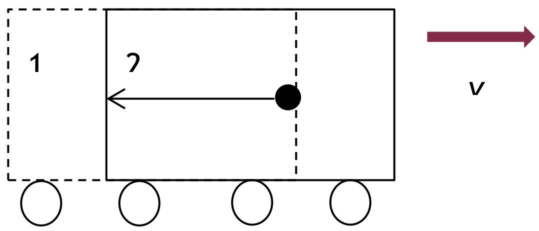
l' length measured in a frame of reference moving relative to the one containing the object.

THOUGHT EXPERIMENT 2



The observer on board the carriage and to them the carriage appears stationary relative to them.

For the observer on the train the ball goes from one end of the train to the other and back



The carriage is moving to the right with a velocity *v*.

To the observer on the platform the ball travels to the end of the carriage, but it has moved forwards, so the total distance travelled is less.

Length contraction is only observed when objects travel at a speed close to the speed of light and in a direction parallel to the direction in which the observed body is travelling.

l = length measured on the platform by the observer

l' = length measured on the train by the observer

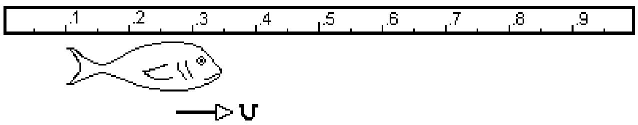
v = speed the object is moving at (ms^{-1})

c = speed of light (ms^{-1})

HOW DO YOU MEASURE A FISH?

Suppose an oceanographer wished to measure the length of a fish that swims by his metre stick. He would note, where the head was and *at the same moment* he would notice where the tail was. The fish would then be said to be the same length as the distance between the two marks on the metre stick. If the oceanographer were to notice where the head was, wait a moment and then look where the tail was the fact that the fish was moving through the water would give a shorter measurement. Let's see that in pictures:

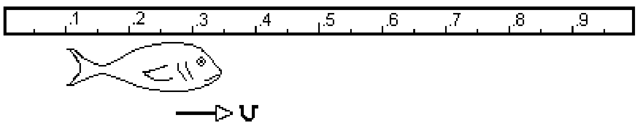
Picture 1:Look at both ends simultaneously:
head point: 0.35 metres
tail point: 0.1 metres
Length = 0.35 metres - 0.1 metres
=0.25 metres



Now, the "wrong" way

Picutre 2: Look at the head first:

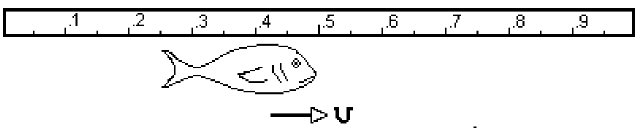
head point: 0.35 metres.



(wait a moment)

Picture 3: Now look at the tail:

tail point: 0.25 metres.



length = 0.35 metres - 0.25 metres = 0.1 metres.

For two objects to be the same length their endpoints must coincide **SIMULTANEOUSLY**. The measuring process for a moving object includes the concept of simultaneity. But, as we will see events deemed simultaneous in one inertial frame are not always simultaneous in another inertial frame. Thus the length of an object will be different when measured in different inertial frames. Since all these inertial frames are equally valid (postulate 1) each of their measurements are as correct as any other's.

For more details on Length Contraction read the additional notes in Special Relativity Paradoxes.

LENGTH DILATION / TUTORIAL 3

1. Write down the relationship involving the proper length l and contracted length l' of a moving object observed in two different frames of reference moving at a speed, v, relative to one another (where the proper length is the length measured by an observer at rest with respect to the object and the contracted length is the length measured by another observer moving at a speed, v, relative to the object).

2. In the table shown, use the relativity equation for length contraction to calculate the value of each missing quantity (a) to (f) for an object moving at a constant speed relative to the Earth.

Contracted length	*Proper length*	*Speed of object* / m s^{-1}
(a)	5·00 m	$1{\cdot}00 \times 10^{8}$
(b)	15.0 m	$2{\cdot}00 \times 10^{8}$
0·15 km	(c)	$2{\cdot}25 \times 10^{8}$
150 mm	(d)	$1{\cdot}04 \times 10^{8}$
30 m	35 m	(e)
10 m	11 m	(f)

3. A rocket has a length of 20 m when at rest on the Earth. An observer, at rest on the Earth, watches the rocket as it passes at a constant speed of $1{\cdot}8 \times 10^{8}$ ms^{-1}. Calculate the length of the rocket as measured by the observer.

4. A pi meson is moving at 0·90 c relative to a magnet. The magnet has a length of 2·00 m when at rest to the Earth. Calculate the length of the magnet in the reference frame of the pi meson.

5. In the year 2050 a spacecraft flies over a base station on the Earth. The spacecraft has a speed of 0·8 c. The length of the moving spacecraft is measured as 160 m by a person on the Earth. The spacecraft later lands and the same person measures the length of the now stationary spacecraft.

Calculate the length of the stationary spacecraft.

6. A rocket is travelling at 0·50 c relative to a space station.
Astronauts on the rocket measure the length of the space station to be 0·80 km. Calculate the length of the space station according to a technician on the space station.

7. A metre stick has a length of 1·00 m when at rest on the Earth. When in motion relative to an observer on the Earth the same metre stick has a length of 0·50 m. Calculate the speed, in ms^{-1}, of the metre stick.

8. A spaceship has a length of 220 m when measured at rest on the Earth. The spaceship moves away from the Earth at a constant speed and an observer, on the Earth, now measures its length to be 150 m.

Calculate the speed of the spaceship in ms^{-1}.

9. The length of a rocket is measured when at rest and also when moving at a constant speed by an observer at rest relative to the rocket. The observed length is 99·0 % of its length when at rest. Calculate the speed of the rocket.

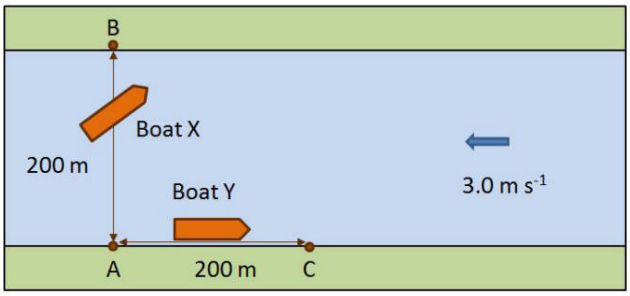
THE MICHELSON-MORLEY EXPERIMENT

The prevalent belief in the nineteenth century was that light is a wave, carried by a subtle medium, the *aether*, which is at rest in the universe. The sun is at rest, in the centre of the universe, and the earth moves through the aether and around the sun at about 30 kms^{-1}.

The Michelson-Morley (M&M) experiment was designed to verify this belief. If light is sent back and forth on earth, in the direction of the earth's movement, then the round trip should take longer than it would if there were no aether. It should also take a little longer, but not as much as in the parallel direction, for light moving back and forth in the direction perpendicular to the earth's movement. The M&M experiment used two identical rods perpendicular to each other along which the light moved back and forth.

A BOAT RACE: A MODEL FOR THE MICHELSON MORELY EXPERIMENT

This can be modelled by thinking about two boats travelling the same distance but one crossing a river (the ether) and one going parallel to the ether (the river)



Here the river represents the ether, and the boats represent the two beams of light going off perpendicular to each other. Boat Y has to move against the river tide on the way out, but is assisted by the tide on the return part of the journey. Boat X will always arrive first.

Both boats, X and Y have a speed of 5 ms^{-1}. Boat X has to cross the river from A to B and back to A, while Boat Y has to travel from A to C and back again.

Questions

(a) Calculate the velocity of X relative to the river bed (Use a vector diagram).

(b) Calculate the time for X to travel from A to B to A.

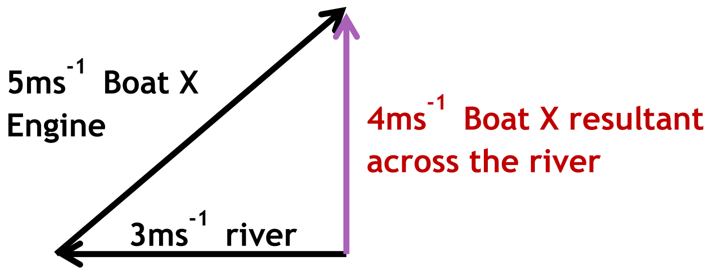
(c) Calculate the velocity of Y relative to the river bed going from A to C and the time.

(d) Calculate the velocity of the boat Y going from C to A and its time.

(e) State which boat wins and by what time interval.

Answers

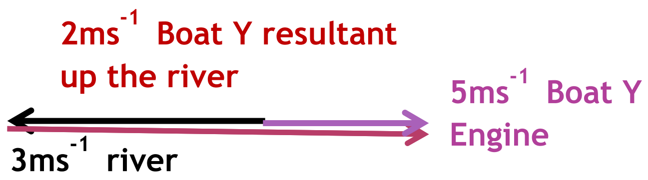
(a) Velocity is 4 m s^{-1} (since it's a 3, 4, 5 triangle).



(b) What is the time for X to travel from A to B to A?

$$time = \frac{distance}{speed} = \frac{400}{4} = 100\,s$$

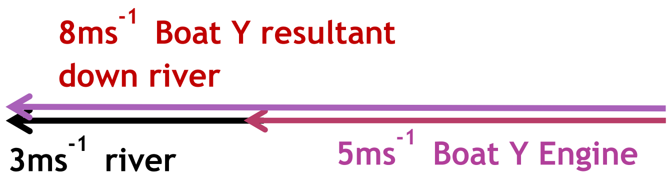
(c) What is the velocity of Y relative to the river bed going from A to C and the time?



$$by\ vector\ addition \qquad Velocity = 5 - 3 = 2\,m\,s^{-1}$$

$$time = \frac{distance}{speed} = \frac{200}{2} = 100\,s$$

(d) What is the velocity of the boat Y going from C to A and its time?



$$by\ vector\ addition \qquad Velocity = 5 + 3 = 8\,m\,s^{-1}$$

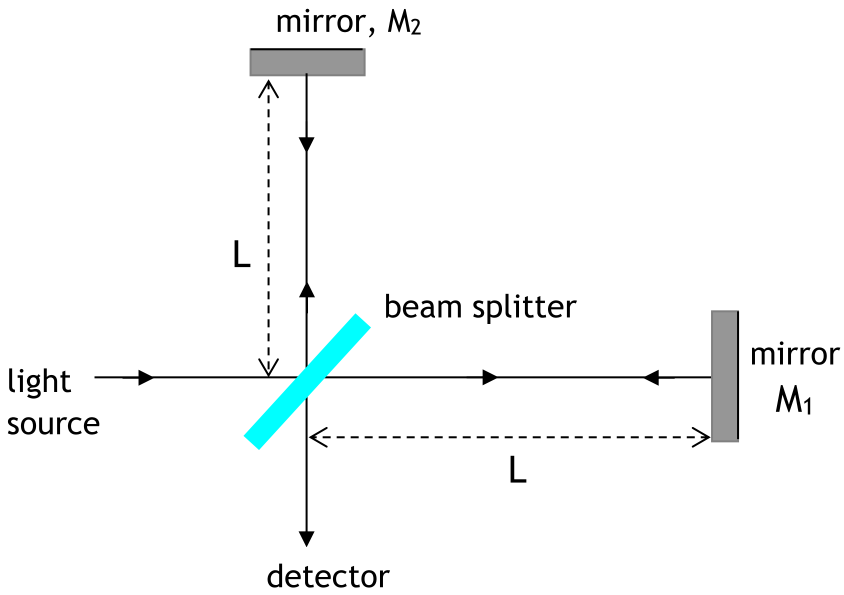
$$time = \frac{distance}{speed} = \frac{200}{8} = 25\,s$$

(a) Which boat wins and by how much? Boat X wins by 25 s

To see how this fits with the Michelson Morely Experiment check out the link below. https://tinyurl.com/MichelsonMorely

BACK TO THE MICHELSON MORELY EXPERIMENT

The apparatus consists of a light source, a beam splitter (a half-silvered glass plate) and two mirrors, M_1 and M_2, each of which is equidistant from the beam splitter. The beam splitter is at 45° to the incident beam and the return beams pass to the detector, a telescope.



Sometimes one light beam may be travelling in the same direction as the aether and the other beam at right angles to the aether. They will have slightly different speeds. The detector should show interference between the two beams travelling perpendicular to each other. The aim was to measure the speed of the Earth relative to the aether.

No discernible interference fringes were found, despite repeating the experiment at different times in the Earth's orbit and with different orientation. The world's physicists used these results to add on a few qualities to the ether. They assumed that the movement through the ether "compressed" the apparatus in the direction of the motion so that the "no movement" reading always resulted. On the prevailing theory and experimental accuracy, the small destructive interference should have been observed.

This experiment, first performed in 1881, repeated in 1887, and often thereafter, could find no indication of a difference. But the belief could not be shaken, and Hendrik Lorentz, in 1895 and again in 1904, thought the explanation could possibly be that the rod placed in the direction of the earth's movement might contract due to the earth's movement, just enough to make the round trip time equal to the case when there is no movement. The equation he developed is known as the Lorentz transformation (LT).

This null result had great importance since it could not be explained.

It was against this background of theories that were fundamentally untestable that Einstein and others stated that any experimental science such as physics must have its fundamental entities defined in terms of some experiment. To avoid this was to invent ghosts. This is the basis of Operational Definition. An operational definition is simply a way of defining a physical quantity or quality in terms of an operation performed. Instead of saying length is "the amount of distance between two points" a modern physicist would give a procedure (an *operation*) for measuring the distance between two points. ("lay metre sticks end to end and count how many" or "send a light signal from point A to point B and reflect it back to A again; then multiply the elapsed time for the signal by ½ c.") The result of such a measurement would be the **definition** of the length. Physics is a way of predicting and explaining observations encountered under certain circumstances. All Physics should be based on observation and experiment. Any concept that could not, in principle, be connected with experiment was declared to be not physics but metaphysics.

http://www.youtube.com/watch?v=AKhvqO5UBsA
http://www.youtube.com/watch?v=SUf8zA-LwUk&feature=related

To summarise

1. Defining t, t', l and l' as shown below, equations for time dilation and length contraction can be derived. The equations (1) and (2) are provided on the SQA data sheet.

t and *l*time interval ('event') or length of object under discussion in a frame of reference.

t' and *l'* time interval or length of object 'measured' by travellers in a *different* frame of reference.

v relative velocity of the two frames of reference.

$$t' = \frac{t}{\sqrt{1-(v/c)^2}} \qquad (1)$$

$$l' = l\sqrt{1-(v/c)^2} \qquad (2)$$

2. No object can travel faster than the speed of light.

3. Relativistic effects are negligible when relative velocity is less than10% of the speed of light.

4. Experimental verification is provided by observing the life time of fast-moving muons.

5. There is not a separate conservation of mass, but a combined conservation of mass and energy. A greater energy than expected is required to increase the speed of an object as its speed approaches the speed of light.

http://www.scivee.tv/node/2415 http://hyperphysics.phy-astr.gsu.edu

CHAPTER 7: THE EXPANDING UNIVERSE

SUMMARY OF CONTENT

8.	The expanding Universe
eq	$f_o = f_s\left(\frac{v}{v \pm v_s}\right)$ $\quad f_{observed} = f_{source}\frac{v}{[v+v_{source}]}$ $\quad v = H_o d$ $\quad z = \frac{v}{c}$ $z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$
a)	I know that the Doppler effect causes shifts in wavelengths of sound and light.
b)	I can use appropriate relationship to solve problems involving the observed frequency, source frequency, source speed and wave speed.
c)	I know that the light from objects moving away from us is shifted to longer wavelengths (redshift).
d)	I know that the redshift of a galaxy is the change in wavelength divided by the emitted wavelength. For slowly moving galaxies, redshift is the ratio of the recessional velocity of the galaxy to the velocity of light..
e)	I can use appropriate relationships to solve problems involving redshift, observed wavelength, emitted wavelength, and recessional velocity
f)	I can use appropriate relationship to solve problems involving the Hubble constant, the recessional velocity of a galaxy and its distance from us.
g)	I know that Hubble's law allows us to estimate the age of the Universe.
h)	I know that measurements of the velocities of galaxies and their distance from us lead to the theory of the expanding Universe.
i)	I know that the mass of a galaxy can be estimated by the orbital speed of stars within it.
j)	I know that evidence supporting the existence of dark matter comes from estimations of the mass of galaxies.
k)	I know that evidence supporting the existence of dark energy comes from the accelerating rate of expansion of the Universe.
l)	I know that the temperature of stellar objects is related to the distribution of emitted radiation over a wide range of wavelengths.

🏆	m)	I know that the peak wavelength of this distribution is shorter for hotter objects than for cooler objects.
🏆	n)	I know that hotter objects emit more radiation per unit surface area per unit time than cooler objects.
🏆	o)	I know of evidence supporting the big bang theory and subsequent expansion of the Universe: cosmic microwave background radiation, the abundance of the elements hydrogen and helium, the darkness of the sky (Olbers' paradox) and the large number of galaxies showing redshift rather than blueshift.

DISTANCES IN SPACE

When dealing with Space we know that very large distances are involved. On Earth we assume the light arrives instantly as the speed is so great and we don't have massive distances: even if we consider a light beam reflecting off a geostationary satellite this would only take about 0.24 s to do the round trip from earth to the satellite and back. When dealing with astronomical distances we must take account of the vastness of space. Light, even travelling at the universal speed limit of 3×10^{8} ms^{-1} takes 8 mins to reach us from the sun and the light from the next nearest star takes 4.3 years to reach us.

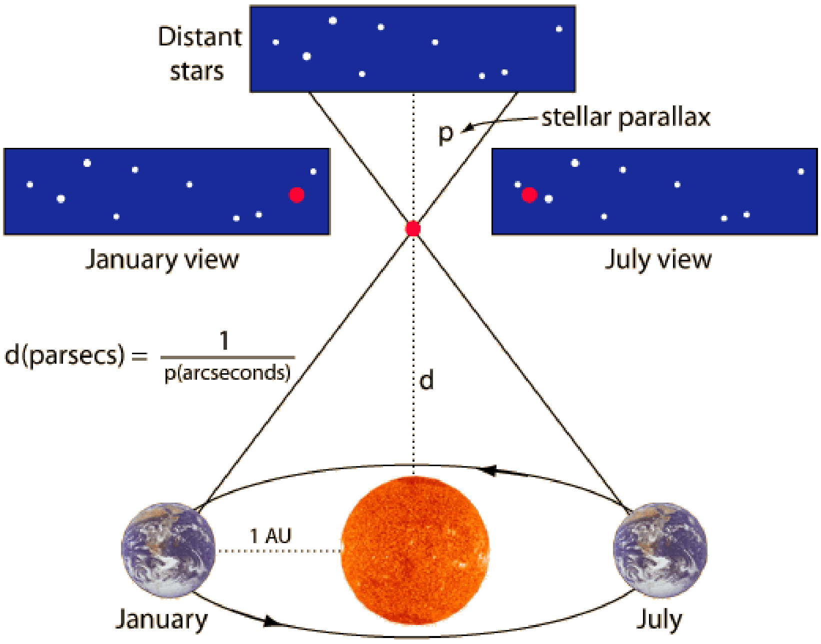
Astronomers use many different units to describe distances in space. You met the light year- the distance light travels in one year at National 5. Astronomers also use the Astronomical Unit (AU) and the parsec (pc).

ASTRONOMICAL UNIT

Astronomical Unit: a unit of measurement equal to 149.6 million kilometres, the mean distance from the centre of the earth to the centre of the sun.

PARSEC

The parsec is derived from two words, parallax and arcseconds. When you've measured angles you've probably measured in degrees and know there are 360 degrees in a circle. This can be broken down into degrees minutes and seconds of arc, and one second of arc is 1/3600 degrees. Parallax is the apparent change in location of any nearby star when the star is viewed against the background of more distant stars. So in the diagram below we can look at the position of the same star at six month intervals and note its change in position compared to more distant stars.



http://hyperphysics.phy-astr.gsu.edu/hbase/Astro/para.html

THE DOPPLER EFFECT

Definition:

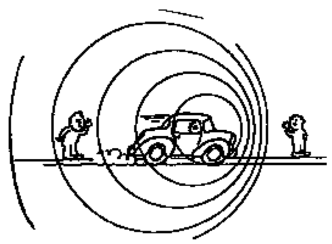
The Doppler Effect is the apparent change in frequency of a wave when the source and observer are moving relative to each other.

The Doppler Effect is produced with all wave motions, including electromagnetic waves.

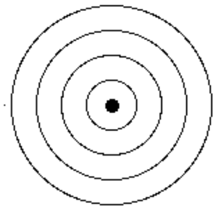
The Doppler Effect is applied in many different disciplines:

- The Doppler Effect is used by meteorologists to track storms.
- The military uses Doppler Effect to be able to determine the speed of their submarines.
- An echocardiogram uses the Doppler Effect to measure the velocity of blood flow and cardiac tissue and is one of the most widely used diagnostic tests in cardiology. For example it is used to measure the speed of blood flow in veins to check for deep vein thrombosis [DVT].

In this course we will concentrate on a wave source moving at constant speed relative to a stationary observer.



You have already experienced the Doppler Effect many times. You may have noticed a change in pitch as a car comes first towards you then passes and goes away from you. The most noticeable is when a police car, ambulance or fire engine passes you. You hear the pitch of their siren increase as it moves towards you and then decrease as it moves away. Another memorable example is the sound of a very fast moving vehicle, such as a motorbike passing you (or passing a microphone on the television), the sound of the engine rises in frequency as it approaches and falls in frequency as it moves away.

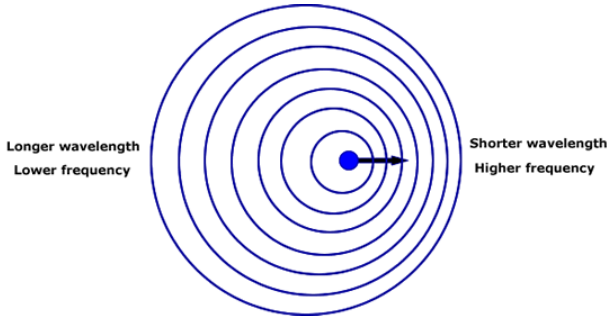


When the source moves towards you, more waves reach you per second and the frequency heard is increased. If the source moves away from you, fewer waves reach you each second and the frequency heard decreases.

Why do we hear a change of frequency?

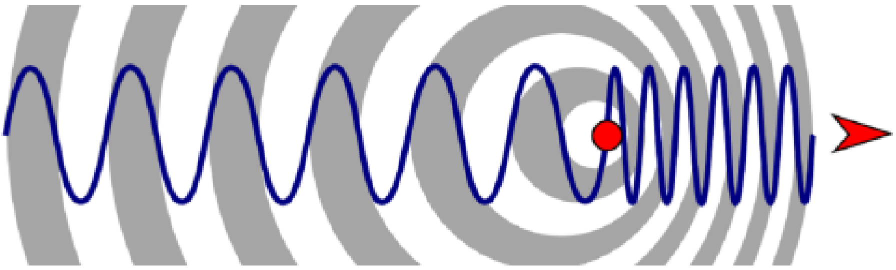
A stationary sound source produces sound waves at a constant frequency f, and the wavefronts propagate symmetrically out from the source at a constant speed, which is the speed of sound in the medium. The distance between wave-fronts is the wavelength. All observers will hear the same frequency, which will be equal to the actual frequency of the source:

$$f_{source} = f_{Observed}$$



where f_{source} is the frequency produced by the source and $f_{observed}$ is the observed frequency.

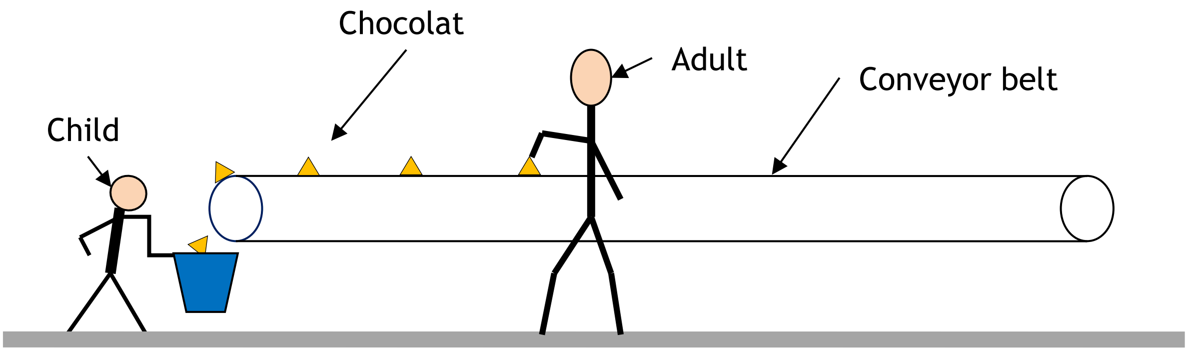
The sound source now moves to the right with a speed v_s. The wavefronts are produced with the same frequency as before. **Therefore the period of each wave is the same as before.** However, in the time taken for the production of each new wave the source has moved some distance to the right. This means that the wavefronts on the left arrive further apart and the wavefronts on the right arrive closer together. This leads to the spreading out and bunching up of waves you can see above and hence the change in frequency.



https://www.youtube.com/watch?v=h4OnBYrbCjY

ANALOGY: SWEETS ON A CONVEYOR BELT

Imagine a long conveyor belt running at a steady speed. An adult standing about halfway along deposits sweets onto the belt at a regular rate, say one sweet per second (frequency).



Chocolates on conveyor belt

A child standing at the end of the conveyor belt collects the sweets in a bucket as they fall off the end. As long as they are both standing still, the child will be collecting the sweets at the same rate (frequency) as they are being deposited by the adult.

If the adult walks steadily away from the child, still depositing the sweets at the same rate, the child now receives the sweets at a lower rate (frequency). The sweets will be further spaced out on the conveyor belt (longer 'wavelength').

Conversely if the adult walks towards the child, the child will receive the sweets at a higher rate (frequency) and they will be spaced closer together (shorter 'wavelength').

EQUATIONS

There are equations that we can use to calculate the apparent change in frequency and we can derive these equations for the Doppler effect. For a *moving source* there is an equation for the source moving towards the observer and another for the source moving away from the observer.

Similarly for a moving observer there are two equations, one for moving away from and one for moving towards the source, but these last two are not covered in this course.

For this course, the only equations required are for a stationary observer with the source moving away and moving towards the observer.

DERIVING THE DOPPLER EQUATION

Derivation: Stationary observer and source moving away

Beware in these derivations the frequency of the source is the actual frequency generated by the object producing the sound waves. The speed of sound would have a value of 340 ms^{-1}. The sound is generated from a moving object which is moving with speed v_{source} The speed of the source is the speed at which the vehicle is moving, not the speed of the sound. This generates an observed frequency and wavelength in a stationary observer. Other derivations use different symbols, you must define your terms.

These are the symbols that the SQA use to define the terms. Most other books use the same symbols but use them to represent different terms so the equation can look different.

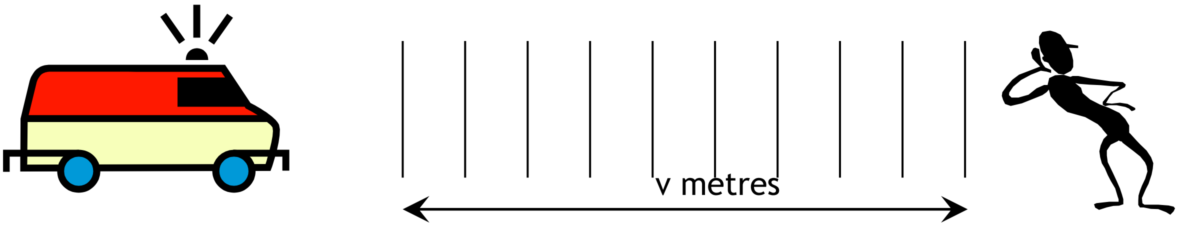
speed of sound = v ***speed of source = v_s***

frequency source = f_s ***observed frequency = f_o***

We can consider how this effect occurs. A source produces a sound of frequency f_s.

The frequency of the source will remain constant, it is the observed frequency that changes.

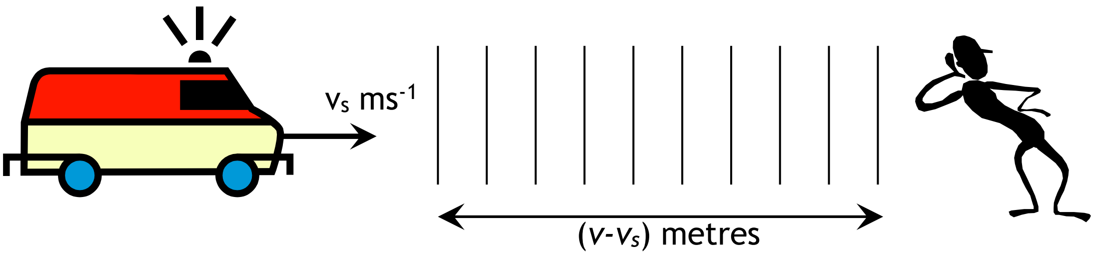
Movement towards observer: The waves will 'bunch up'



Stationary: in 1 second there will be f_s waves produced. The sound will travel a distance of *v* metres in *1 second*. This means there are f_s waves in a distance of *v* metres.

Moving at speed v_s: In the same way as above, in 1 second f_s waves will cover a **distance** of **(*v* - v_s)** metres.

As $v = \frac{d}{t}$ then numerically in one second the value of *v* is equal to the value of *d*



The observed wavelength of the waves will be

$$\lambda_{observed} = \frac{v}{f}$$

$$\lambda_{observed} = \frac{(v - v_s)}{f_s}$$

$$f_{observed} = \frac{v}{\lambda_{rest}}$$

$$f_o = \frac{v}{\frac{(v - v_s)}{f_s}}$$

$$f_o = f_s \frac{v}{(v - v_s)}$$

This gives an <u>observed</u> frequency that is **higher/greater** than that of the source.

If the source is moving away from the observer then the frequency observed will be **lower/smaller** than that of the source.

$$f_o = f_s \frac{v}{(v + v_s)}$$

Frequency increases as they come towards you and frequency decreases as they move away.

Remember the correct formula from ADD when the object moves AWAY

and TAKE AWAY (subtract) when the object comes TOWARDS

We can also see from these equations that if the magnitude of v_{source} is very small compared to v there is little effect on $f_{observed}$. This is why for the effect to be noticeable, v_{source} should be reasonably large in comparison with the speed of the waves (v).

https://www.youtube.com/watch?v=h4OnBYrbCjY (What does Motion do to waves?)

https://www.youtube.com/watch?v=LIvVzJ6KZpk (Fun Science- a bit silly but he does film along a canal!)

Providing objects emitting or reflecting light are travelling at speeds less than 10%c then we can use this Doppler equation to find out fast objects are moving.

For a stationary observer with a light source moving **towards** them, the relationship between the original frequency, f_s, of the source and the observed frequency is:

$$f_o = f_s\left(\frac{c}{c - v_s}\right)$$

For a stationary observer with a light source moving **away** from them, the relationship between the original frequency, f_s, of the source and the observed frequency is:

$$f_o = f_s\left(\frac{c}{c + v_s}\right)$$

This second scenario is exactly what is observed when we look at the light from distant stars, galaxies and supernovae. These relationships also allow us to calculate the speed at which an exoplanet is orbiting its parent star, or the velocity of stars orbiting a galactic core (called the recessional velocity, *v*), which has lead us to theorise the existence of dark matter. We will deal with this further later in the chapter.

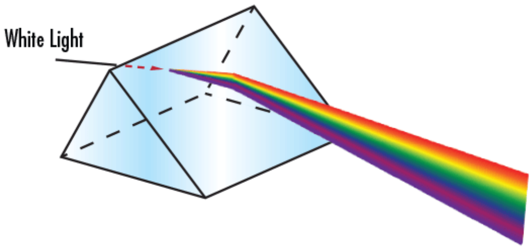
http://www.youtube.com/watch?v=Y5KaeCZ_AaY (as stated by Sheldon Cooper)

RECESSIONAL VELOCITY

Recessional velocity is the rate at which an astronomical object is moving away, typically from Earth. It can be measured by shifts in spectral lines or estimated by general reddening of a galactic spectra.

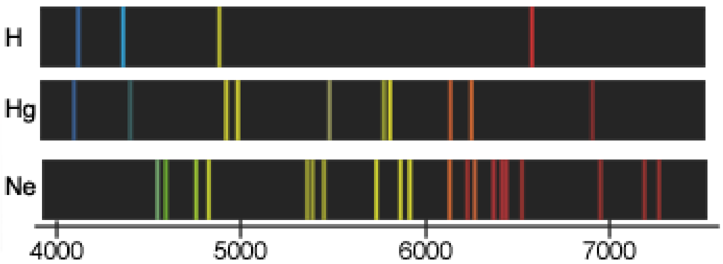
BIG BANG THEORY

LINE SPECTRA



Light coming from the sun appears white to us, but passing this light through a prism spreads the light into its constituent colours, the colours of the rainbow or a spectrum. The prism simply refracts the light; short wavelengths are refracted by a greater amount than longer wavelengths.

Astronomers during the nineteenth century discovered that when sunlight is passed through a prism dark lines appear at specific wavelengths. The dark lines were determined to be caused by the absorption of certain wavelengths of light by atoms lying between the light source and the observer. There is a unique "fingerprint" of absorption lines for each element.   Similarly, the spectrum of light from a heated, glowing gas of a given element gives off bright emission lines of the same wavelengths as the absorption lines for that element. Spectral analysis, identification of absorption and emission spectra, of sunlight, starlight, and light reflected from planet, asteroid, and comet surfaces can be used to identify elements found in the atmospheres and on the surfaces of those bodies. The light emitted by a star is made up of the line spectra emitted by the different elements present in that star. Each of these line spectra is an identifying signature for an element and these spectra are constant throughout the universe. You will learn a lot more about spectra in the Particles and Waves unit of this course.

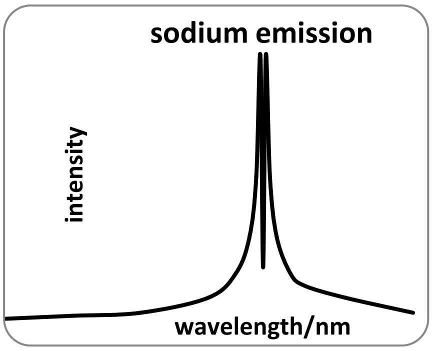


Each element has its own characteristic line emission spectrum and the corresponding absorption spectrum.

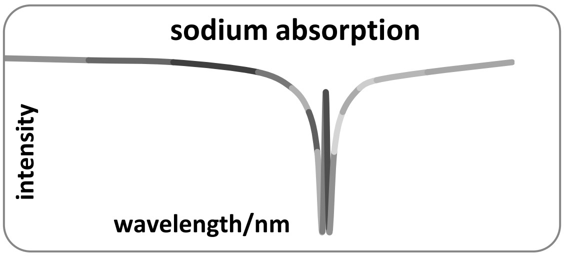
These can be indicated on a diagram as a coloured strip or as intensity versus wavelength graph.

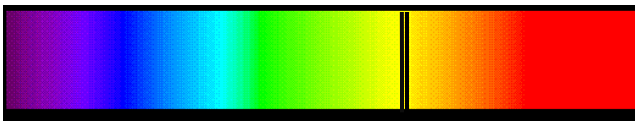
Scientists have used line spectra to discover new elements. The element helium was discovered by studying line spectra emitted by the Sun.

Sodium emission spectrum, produced when Sodium is heated.



Sodium absorption spectrum, produced when white light is passed through sodium gas.



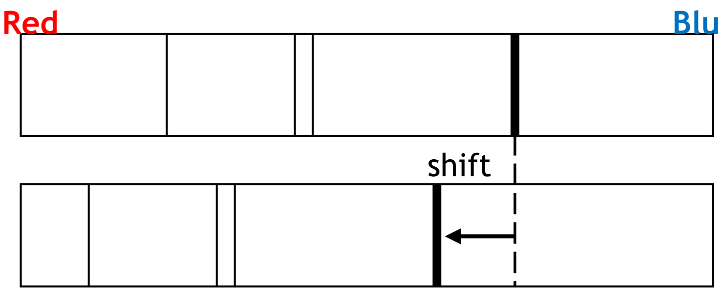


Notice that the same frequencies are seen as the coloured lines in the top diagram and the black lines in the bottom diagram. Changes in the frequency is also observed as a change in the wavelength (remember that v = fλ) This can be observed in both stars and galaxies.

REDSHIFT, Z

The astronomer Edwin Hubble noticed in the 1920's that the light from some distant galaxies was shifted towards the red end of the spectrum.

Hubble examined the spectral lines from various elements and found that each galaxy was shifted towards the red by a specific amount. He interpreted the frequency shift of extra-galactic starlight to mean that distant galaxies are moving away from us, causing the Doppler Effect to be observed. The bigger the magnitude of the shift the faster the galaxy was moving.



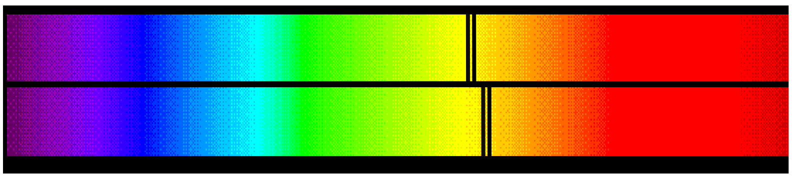
spectrum of element on Earth

spectrum of element from galaxy

Redshift is an example of the Doppler Effect. **Redshift is the term given to the change in frequency of the light emitted by stars, as observed from Earth, due to the stars moving away from us.**

Normal

Redshifte



Redshift

Redshift, z, of a galaxy is defined as the change in wavelength divided by the original wavelength, and given the symbol z.

So, redshift

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

Where *z = redshift,* λ_{obs} *= observed wavelength*

λ_{rest} *= wavelength of the source*

Note: Redshift is a <u>dimensionless</u> quantity since it is a ratio of two lengths.

Example

An astronomer observes the spectrum of light from a star. The spectrum contains the emission lines for hydrogen. The astronomer compares this spectrum with the spectrum from a hydrogen lamp. The line, which has a wavelength of 656 nm, from the lamp is found to be shifted to 663 nm in the spectrum from the star. Calculate the redshift for this light.

z = redshift (NB it is a ratio so has no units)

$\boldsymbol{\lambda_o = 663\ nm}$

$\boldsymbol{\lambda = 656\ nm}$

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

$$\boldsymbol{z = \frac{\lambda_o - \lambda}{\lambda}}$$

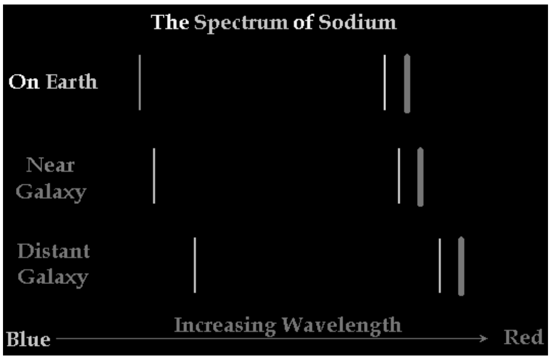
$$\boldsymbol{z = \frac{663n - 656n}{656n}}$$

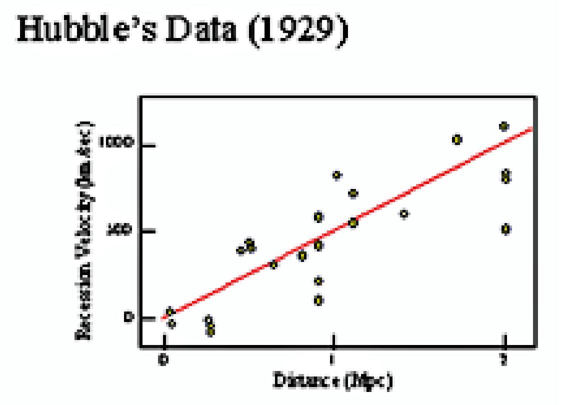
<u>z=0.0107</u>

NB Make sure you quote to the correct sig fig

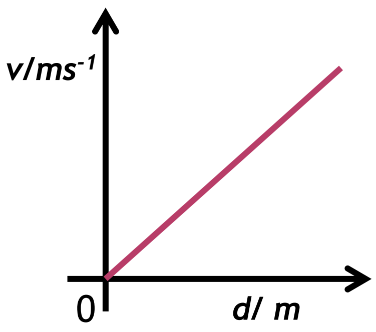
HUBBLE'S LAW

Galaxies are moving away from the Earth and each other in all directions, which suggest that the universe is expanding; so in the past galaxies were closer to each other. By working back in time it is possible to calculate a time where all the galaxies were in fact at the same point in space. This allows the age of the universe to be calculated.





The graph plots the data collected by Hubble giving the relationship between the velocity of a galaxy (recessional velocity) and its distance from us.



$$\boldsymbol{gradient = \frac{v}{d} = Hubble\ contant}$$

From the equation for

$$v = \frac{d}{t}$$

$$rearranging \Rightarrow \frac{v}{d} = \frac{1}{t}$$

The gradient of this graphs is called the Hubble Constant and has units of s^{-1}.

If we take the inverse of the Hubble constant we should be able to predict the age of the Universe.

HUBBLE'S CONSTANT

The value of the Hubble constant is not known accurately, as the exact gradient of the line of best fit is subject to much debate. However, as more accurate measurements are made, the range of possible values has reduced. It is currently thought to lie between 50 - 80 kms^{-1} Mpc^{-1}, with the most recent data putting it at 70.4 ± 1.4 kms^{-1} Mpc^{-1}. The units for the Hubble constant are always in units of time^{-1}

1Mpc = 3·26 x 10^{6} light years = 3·1 x 10^{22}m

Question

Calculate 70.4 kms^{-1} Mpc^{-1} in terms of SI units and state what these units would be.

For this course the Hubble Constant is given as

Hubble's Constant Ho 2·3 x 10^{-18} s^{-1}

This would give a predicted age of the universe as:

$$t = \frac{1}{2.3 \times 10^{-18}} = 4.35 \times 10^{17}\ s$$
$$= \frac{4.35 \times 10^{17}}{3600 \times 24 \times 365}\ years$$
$$= 13.78 \times 10^{9}\ years$$

13.8 billion years old

MOE ON REDSHIFT

Hubble noticed that further galaxies are redshifted more than closer galaxies indicating the further away a galaxy is the faster it is travelling we can calculate the value for the recessional velocity of galaxies from this data.

For small velocities (less than 1/10th c, the speed of light), cosmological redshift is related to recession velocity (v) through:

$$z = \frac{v_{galaxy}}{c}$$

Where z= redshift

v= recessional velocity

c= speed of light

So for the previous example on page 76, with a redshift of 0.0107, we can calculate the recessional velocity of the galaxy from which the star light was observed.

$$z = \frac{v}{c}$$

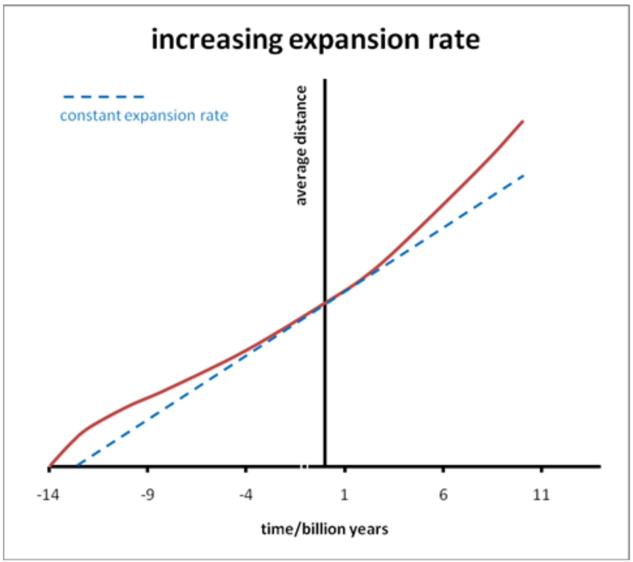
$$0.0107 = \frac{v}{3.0 \times 10^{8}}$$

$$v = 3.2 \times 10^{6}\ ms^{-1}$$

THE EXPANDING UNIVERSE

From Hubble's graph of speed versus distance, we can obtain an estimate of how long it took for a galaxy to reach its current position. Assuming they have been moving away from us at a constant speed, the time taken for a particular galaxy to reach its current position can be found by dividing the distance by the speed.

Since Hubble's time, there have been other major breakthroughs in astronomy. All of these support the findings of Hubble, and allow the age of the universe to be calculated even more accurately.



The force of gravity acts between all matter in the universe. Matter clumps together due to the force of gravity, such as the contraction of hydrogen gas to create new stars, the grouping of stars to create galaxies and the grouping of galaxies to create local groups and superclusters.

Hubble's Law and subsequent observations, show that the rate of expansion of the universe is ***increasing***. There must be a force acting against the force of gravity, pushing matter apart. This force is causing a significant acceleration and so it is much greater in magnitude than the force of gravity. As yet, astronomers and cosmologists have not been able to determine a source of energy capable of producing this force. For lack of a better term it is, for now, simply referred to as **dark energy**.

TASK

1. Complete the practical where you make your own mini universe and work out your own Hubble constant.

https://www.mrsphysics.co.uk/higher/the-expanding-universe/

http://www.schoolsobservatory.org.uk/astro/cosmos/uniball

2. Check out the example exam question below and answer part b:

Units of Hubble's constant H_0:

Example: Based on SQA 2016

An observatory collects information from a distant star which indicates that a redshift has taken place. The wavelength of the light from the source is 656 nm, while the wavelength of the observed light is 676 nm.

(a) Calculate the recessional velocity of the star.

z = redshift (NB it is a ratio so has no units) $\lambda_o = 676\ nm$ $\lambda = 656\ nm$	
$z = \frac{\lambda_o - \lambda}{\lambda}$	
$z = \frac{676n - 656n}{656n}$	**z=0.0305**

(b) If the recessional velocity of a distant galaxy is 1.2×10^{7} ms^{-1}, show that the approximate distance to this galaxy is 5.2×10^{24} m.

THE EXPANDING UNIVERSE

Scientists have gathered a lot of evidence and information about the universe. They have used their observations to develop a theory called the Big Bang. The theory states that originally all the matter in the universe was concentrated into a single incredibly tiny point, called a singularity. This began to enlarge rapidly in a hot explosion, and it is still expanding today. This explosion is called the Big Bang, and happened about 13.77 billion years ago (that's 13,800,000,000 years using the scientific definition of 1 billion = 1,000 million).

The universe started with a sudden appearance of energy which consequently became matter and is now everything around us. There were two theories regarding the universe:

The Steady State Universe: where the universe had always been and would always continue to be in existence.

The Created Universe: where at some time in the past the universe was created.

Ironically the term 'Big Bang' was coined by Fred Hoyle a British astronomer who was the leading supporter of the Steady State theory and who was vehemently opposed to the idea of the Big Bang and expanding universe..

Scientists have also discovered that the Universe expanding at an increasing rate, i.e. the acceleration is increasing. This was the conclusion of astronomers in 1998 when observing distant supernovae. Their discovery was a great shock to the scientific community and was awarded the Nobel Prize in Physics in 2011.

REDSHIFT (SEE PREVIOUS NOTES) / EVIDENCE FROM THE EXPANDING UNIVERSE

From the expanding universe idea we can extrapolate our figures back (we consider running time backwards) and come to the conclusion that all matter in the Universe was all collected at a single point called the initial singularity.

COSMIC MICROWAVE BACKGROUND RADIATION

Cosmic Microwave Background Radiation (CMBR) observations support the hot Big Bang theory that the universe is expanding out from a single point, as Hubble postulated.

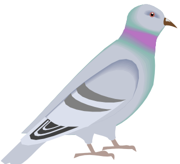
During the very early stages of the universe, say when it was one millionth its present size, the temperature would have been around 3,000,000 K. If an electron became bound with a nucleus, high energy radiation would immediately strip it off. The universe was in a plasma state.

The universe expanded and cooled. As space expanded, the wavelength of photons became longer, so each photon had less energy.

Three physicists, had produced a paper in 1948 that if the Big Bang had actually taken place then there would be a residual background EM radiation, in the microwave region, in every direction in the sky representing a temperature of around 2.7 K. This value for this equivalent temperature was arrived at by considering how the light produced at the Big Bang would have changed as the universe expanded.

The discovery of this background radiation was another example of scientists finding something they weren't looking for.

Arnold Penzias and Robert Wilson were working with a special radio telescope [shown in the picture] experimenting with satellite communication. They were getting a residual signal that seemed to come from outside the galaxy. At first they though it was actually due to pigeon droppings from the pigeons that roosted in the horn. Finally they realised that they had found the echo of the Big Bang.

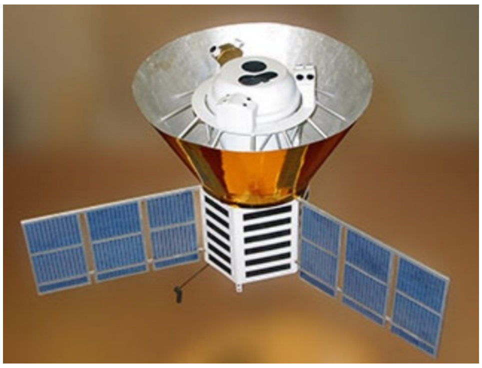


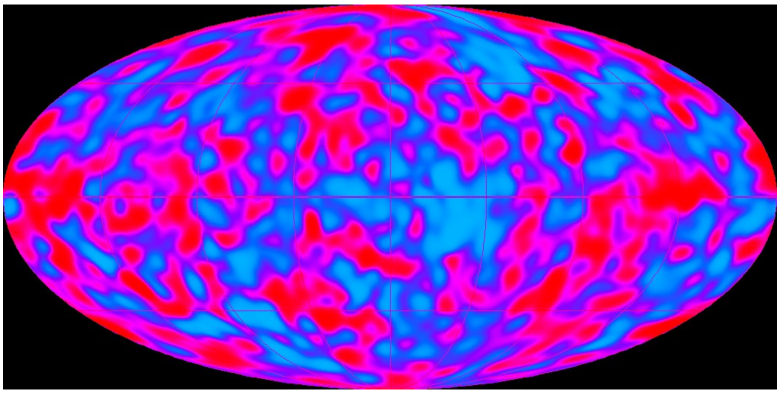
As one of their colleagues commented,

"Thus, they looked for dung but found gold, which is just opposite of the experience of most of us."

He may not have actually said the word dung but I'm not going to write in the actual word as I find it too rude!

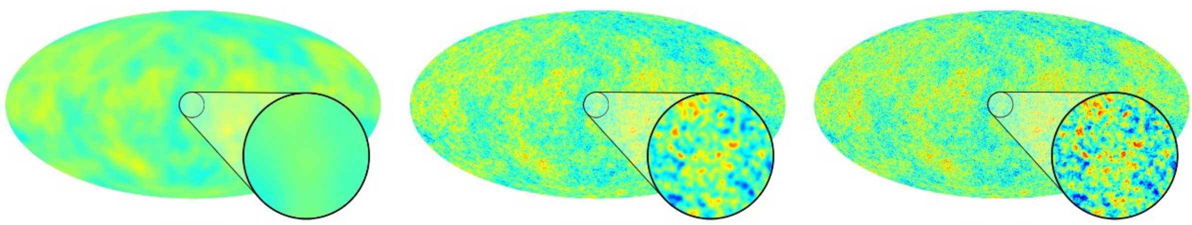
In 1989 a satellite was launched to study the background radiation, it was called the Cosmic Background Explorer [COBE].





In 1992 it was announced that COBE had managed to **measure fluctuations in the background radiation**. This was further evidence to support the Big Bang theory.

An image of the fluctuations is shown.



The three images above show the sky at the resolutions of COBE (launched 1990), WMAP (launched 2001) and Planck (launches 2009). All the images have the same colour scale.

Image credit: Chris North Cardiff University.

BLACK BODY RADIATION

Stars are black bodies in that they absorb all the radiation. The existence of black-body (thermal) radiation, at a temperature of about 3000 K, filling the universe at the time of recombination (when electrons combined with nuclei to form atoms), is consistent with a universe which has expanded and cooled from a much hotter and denser original state at much earlier times – as the Big Bang theory predicts.

THE HELIUM PROBLEM

Other evidence to support the Big Bang theory includes the relative abundances of hydrogen and helium in the universe. Scientists predicted that there should be a significantly greater proportion of hydrogen in the universe. The next most abundant should be helium. The most abundant elements in the universe are hydrogen (70–75%) and helium (25-30%). However, the abundance of helium in the universe cannot be explained by the formation inside stars as most of it remains locked up in their cores. However, it can be accounted for by being formed during the dense phase of the early universe, i.e. shortly after the Big Bang.

The elements present in the universe can be determined by spectroscopy, which you will study later in Particles and Waves.

The latest proportions are given in the table shown. These observations conform to the predicted proportions.

Element	relative abundance
Hydrogen	10 000
Helium	1 000
Oxygen	6
Carbon	1
All others	1

OLBERS' PARADOX

WHY IS THE NIGHT SKY DARK?

This question can be traced back to around 1576 and Thomas Digges, but it was first stated formally by the Prussian astronomer Heinrich Olbers in 1823, hence the name. It was commonly assumed, prior to the expansion of the universe being demonstrated by Hubble in 1929 that the universe was:

1. infinite
2. eternal
3. static.

If this was true, no matter which direction you looked, your line of sight would eventually intersect with a star. The entire sky would be virtually as bright as the Sun!

Olbers' own explanation – that invisible interstellar dust absorbed the light – would make the intensity of starlight decrease exponentially with distance.

It can be shown that the amount of dust needed to do this would be so great as to block out the Sun! Also the radiation would heat up the dust so much that it would start to glow, becoming visible in the infrared region.

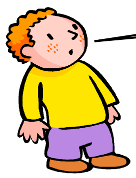
One solution to the paradox comes from considering the finite time light takes to reach us. Consider a galaxy very distant from our own. We only become aware of its existence when light from it reaches us. So if the universe is not infinitely old, galaxies must exist that are so distant that their light has not reached us yet. If the universe is not infinitely old, that implies that it was created some time in the past, which is consistent with the Big Bang theory.

Olber's paradox asks the question, "why is the sky dark at night?"

This is not as obvious as you first might imagine.

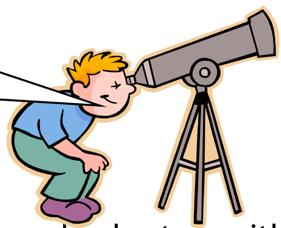
Tam says:

The sun has set at night



Wullie replies:

What about all the stars?
They're suns as well



The Big Bang theory gives a finite age to the universe, and only stars within the observable universe can be seen. So the main reason why the night sky is dark is due to the finite age of the galaxies, not the expansion of the universe. **This is consistent with the hot Big Bang model, but not with a steady-state universe.**

A SUMMARY OF SOME OF THE EVIDENCE

Evidence for big bang	Interpretation
The light from other galaxies is red-shifted.	The other galaxies are moving away from us.
The further away the galaxy, the more its light is red-shifted.	The most likely explanation is that the whole universe is expanding. This supports the theory that the start of the universe could have been from a single explosion.

Cosmic Microwave Background	The relatively uniform background radiation is the remains of energy created just after the Big Bang.
Proportions of H and He	The abundance of light elements is what we would expect from calculations about the formation of matter, called "Big Bang nucleosynthesis"

FURTHER QUESTIONS ON THE UNIVERSE

WHERE IN THE UNIVERSE WAS THE BIG BANG?

Everywhere! It's a bit like asking 'Where in your body were you born?'

WHAT DOES THE UNIVERSE EXPAND INTO?

The basic problem lies with the nature of spacetime. In order for the universe to expand into some larger space, that large space would need to exist in another universe. After all, what does 'space' even mean outside of the universe?

If you try to imagine yourself watching our universe from the outside, doesn't that mean you are in a universe surrounding ours? The key is to think of the universe as a collection of information rather than a collection of objects. Think of it more like the flash memory in an MP3 player. When you record more tracks of music, the memory chip itself doesn't become any larger.

For more information read the document

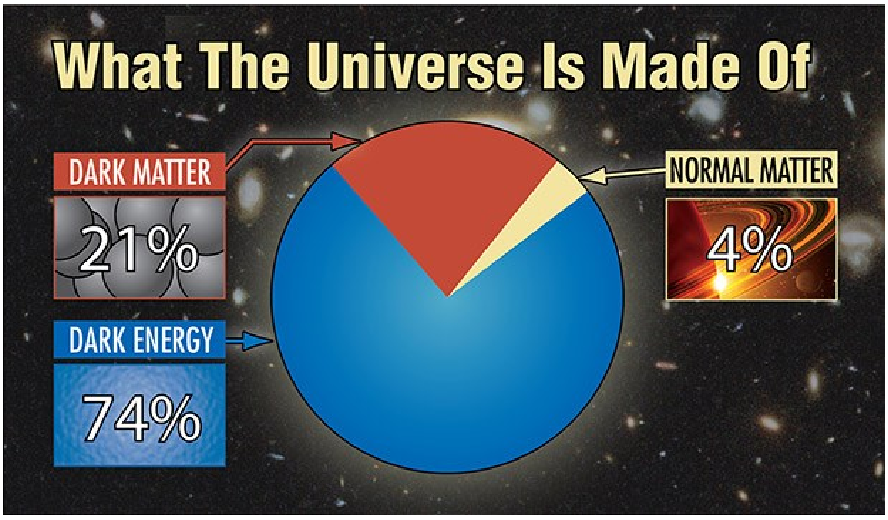
The Expanding Universe and Big Bang Theory Teacher's Notes by Nathan Benson

https://wp.me/s8d1Qu-odu

EINSTEIN'S GREATEST BLUNDER THE COSMOLOGICAL CONSTANT

http://www.jb.man.ac.uk/~jpl/cosmo/blunder.html

DARK ENERGY VS. DARK MATTER



Our universe may contain 100 billion galaxies, each with billions of stars, great clouds of gas and dust, planets and moons. The stars produce an abundance of energy, from radio waves to X-rays, which streak across the universe at the speed of light.

Yet everything that we can see only accounts for only about 4 % of the **total mass and energy** in the universe.

About one-quarter of the universe consists of dark matter, which releases no detectable energy, but which exerts a gravitational pull on all the visible matter in the universe.

What dark matter consists of is uncertain at present, though it is believed not to be the type of matter with which we are familiar – electrons, neutrons, and protons.

While dark energy repels, dark matter attracts. And **dark matter's influence shows up even in individual galaxies, while dark energy acts only on the scale of the entire universe.**

Remember Dark Energy REpels , Dark MAtter Attracts

Are dark matter and dark energy related? No one knows. The leading theory says that dark matter consists of a type of subatomic particle that has not yet been detected, although upcoming experiments with the world's most powerful particle accelerator may reveal its presence. Dark energy may have its own particle, although there is little evidence of one.

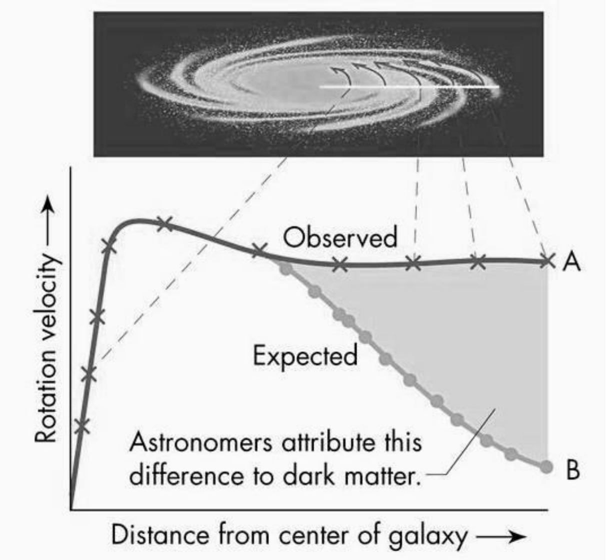
Instead, dark matter and dark energy appear to be competing forces in our universe. The only things they seem to have in common is that both were forged in the Big Bang, and both remain mysterious. While dark matter pulls matter inward, dark energy pushes it outward. Also, dark energy shows itself only on the largest cosmic scale, dark matter exerts its influence on individual galaxies as well as the universe at large.

WHAT DO WE OBSERVE?

The stars are travelling too fast.

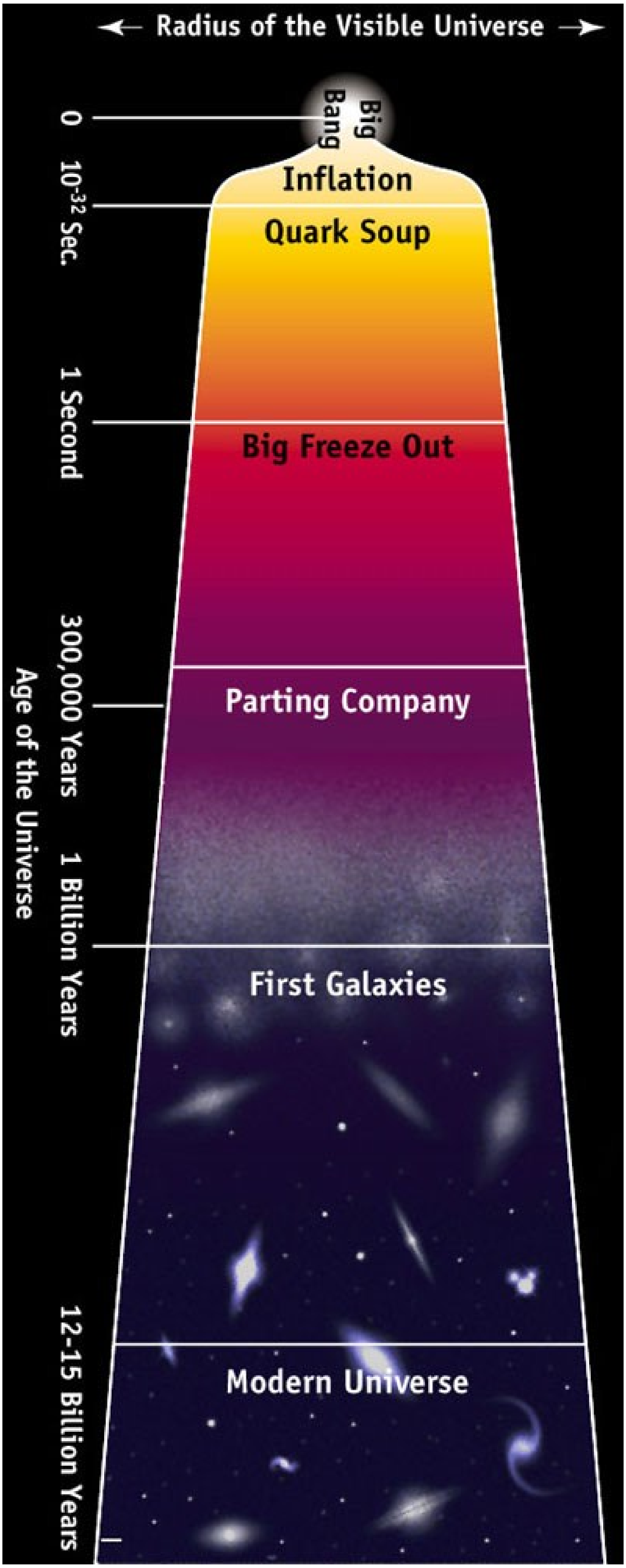
As you can see from the graph above, the velocity of stars does not drop off as expected, at greater orbital radii. At these high velocities the observed mass of the galaxy should not be enough to hold on to many of its stars and we should see them fly off into intergalactic space.

The only logical conclusion that astronomers have to explain this consistent observation is that there must be a significant amount of mass that we cannot see. Hence the name: **dark matter**.



THE BIG BANG THEOR

yhttp://archive.ncsa.illinois.edu/Cyberia/Cosmos/Images/CosmicTimeline_gr.jpg
http://www.schoolsobservatory.org.uk/astro/cosmos/bb_history


Radius of the Visible Universe
Big Bang
Inflation
Quark Soup
Big Freeze Out
Parting Company
First Galaxies
Modern Universe
0
10^-32 Sec.
1 Second
300,000 Years
1 Billion Years
12-15 Billion Years
Age of the Universe

TEMPERATURE OF STARS -

A simple background to Black Body Radiation can give an understanding of stars, and allows you to access questions on the exam papers.

DETERMINING THE TEMPERATURE OF DISTANCE STARS & GALAXIES.

From everyday experience we know that substances glow (incandesce) when heated to very high temperatures. The colour of light emitted depends only on temperature.

When an object is heated it does not initially glow, but radiates large amounts of energy as infrared radiation. We can feel this if we place our hand near, but not touching, a hot object.

As an object becomes hotter it starts to glow a dull red, followed by bright red, then orange, yellow and finally white (white hot). At extremely high temperatures it becomes a bright blue-white colour.

The observant amongst you may realize that these are the colours in the visible spectrum.

The temperature of an object determines the frequency of light it emits. This idea has been with us for a long time; Jožef Stefan proposed in 1879 that the power irradiated from an object was proportional to its temperature in Kelvin to the fourth power.

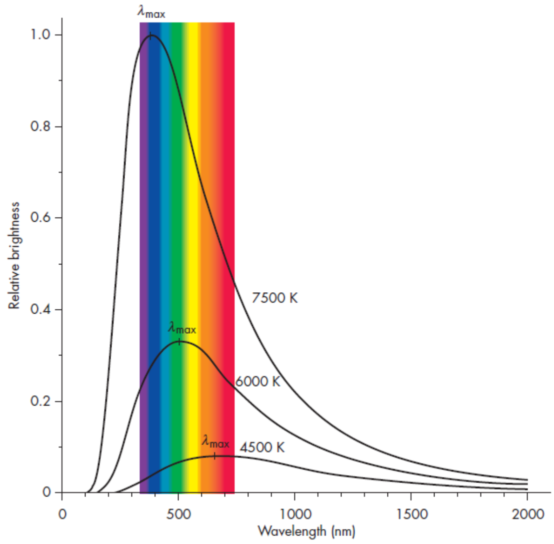


Figure 2 Courtesy of John D Fix Astronomy Journey to the Cosmic Frontier 5th Ed.

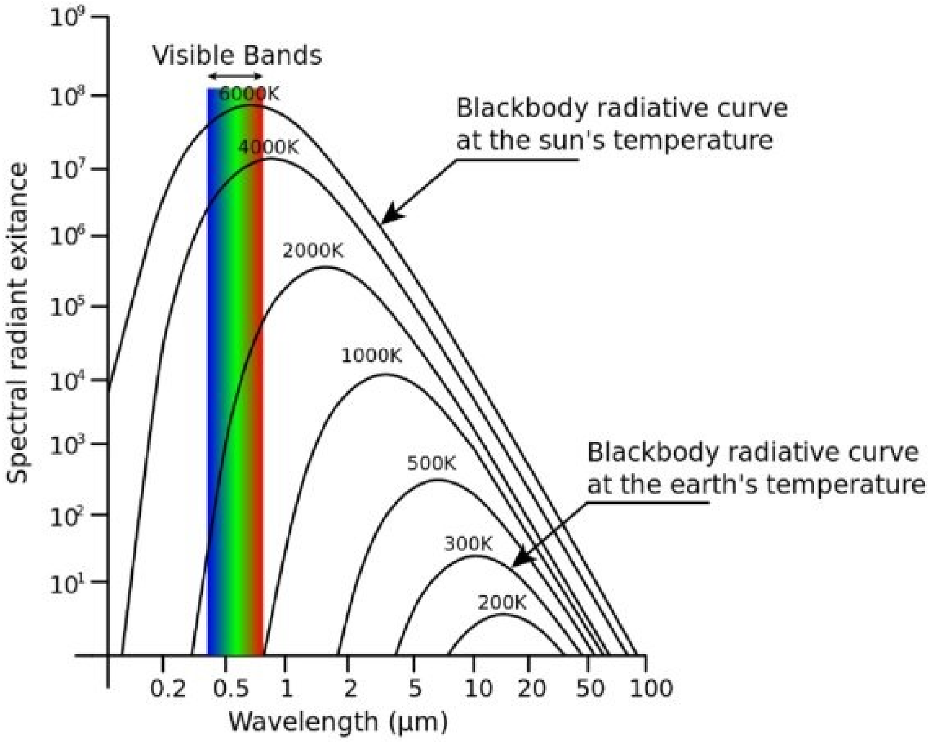
$$P = \sigma T^4$$

Where σ is Stefan's (also referred to as the Stefan- Boltzmann) constant and

$$\sigma = 5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}.$$

What this means is that by examining the spectrum of a distant star, its temperature can effectively be determined.

A "black-body" is an ideal object whose temperature is the same at every point and constant in time. It is useful for study because such an object produces a spectrum which is dependent only on the temperature. The spectrum of black-body radiation is continuous from zero energy at zero wavelength, rising to a maximum at some wavelength determined by Wien's law (AH Physics), and then falling off to zero at infinite wavelength. The hotter the black body, the more energy is produced at EVERY wavelength. Black-bodies are useful models because many stars radiate approximately as black-body.



The black-body spectrum has three main features:

1. **The basic shape is more or less the same (apart from a scaling factor) at all temperatures; it is like a skewed bell curve, falling off gently on the long wavelength side of the peak, and much more sharply on the shorter wavelength side.**
2. **As the temperature of the object is increased, the peak of the intensity spectrum shifts towards the shorter wavelengths.**
3. **As the temperature of the object is increased, the intensity increases for all wavelengths.**

The properties of the spectrum are characterised by a single parameter, temperature, hence it is sometimes referred to as a thermal radiation spectrum.

When the temperature is raised the peak moves towards the short wavelength end (blue), which gives the visible effect of changing from red to orange to yellow to white to blue-white, in that order.

BANG?

The Big Bang was not like a conventional explosion in which matter is thrown apart from a single point into a pre-existing empty space.

Most cosmologists think space, time and matter originated at the Big Bang and before that there was no space, time or matter in the sense we think of them today.

Rather than thinking of galaxies as flying away from each other through space, it is much better to think of them as being essentially at rest in an expanding space.

EXPLANATIONS FOR USEFUL TERMS:

NB many of the terms here are from theories that have been discredited, but it is to help you understand some of the language used in cosmology. These explanations will not form definitions that can be used in your exam, for that you will need to go through the notes.

BLACK BODY RADIATION

A "black-body" is an ideal object whose temperature is the same at every point and constant in time. It is useful for study because such an object produces a spectrum which is dependent only on the temperature. The spectrum of black-body radiation is continuous from zero energy at zero wavelength, rising to a maximum at some wavelength determined by Wien's law, and then falling off to zero at infinite wavelength. The hotter the black body, the more energy is produced at EVERY wavelength. Black-bodies are useful models because many stars radiate approximately as black-body.

HELIOCENTRIC UNIVERSE

This is a model of the universe that puts the sun at its centre.

HUBBLE EXPANSION

Used to describe the expansion of the universe based on the redshifted light of distant galaxies. The relationship between recession velocity and distance is described by Hubble's Law, where $recessional\ velocity\ of\ an\ object = Hubble\ constant \times distance\ of\ object\ from\ Earth$

$$v = H_0 \times d$$

INFINITE

We talk about the universe being infinite in relation to steady state theory, we mean several things. First, steady state theory predicts the universe to be **infinitely big**, without limits. Secondly, steady state theory says the universe is **infinitely old**, as old as time itself.

ISOTROPIC

An isotropic universe has no preferred direction. It acts the same in every direction. For example, the redshift of distant galaxy clusters looks the same from our location as it does from another distant cluster in the universe.

NOT EXPANDING

A universe that is not expanding is called static. According to steady state theory, the universe just is. It cannot be expanding because steady state theorists think this is the way the universe has always looked. A universe that is expanding would have looked very different at a time billions of years ago. There is no evidence for a universe that is not expanding.

PRIMORDIAL SOUP

This is the name for the time in the evolution of the universe where the universe was too hot to form ordinary matter. The universe was opaque and glowed due to photons that were being continuously scattered by electrons.

REDSHIFT

When a radiating object moves away from us, we observe a *redshift* in its light, or the light waves it emits are getting longer (shifting to the red part of the spectrum).

STEADY STATE THEORY

Theory of the creation of the universe that says the universe has been and always will be like it is today. It assumes that the universe is uniform, infinite, and not expanding.
For more about steady state theory go to:
http://cfpa.berkeley.edu/Education/IUP/Big_Bang_Primer.html.

UNIFORM

When we talk about a homogeneous universe, we are making the assumption that the universe is *uniform*, or has the same makeup throughout. So, we figure that the matter density of our local region, or lets say the amount of galaxies, stars, gas and dust per a certain volume is pretty much the same anywhere in the universe.

VISIBLE SPECTRUM

This is the part of the Electromagnetic Spectrum that contains the light we can see. The colours in the *visible Spectrum* from longest wavelength to shortest are: red, orange, yellow, green, blue, and violet.

A closed universe: if the mean density exceeds a critical value then the universe will expand to a finite size and then begin to collapse back in on itself, slowly at first, and then at an ever-increasing rate until all the galaxies collide and the universe ends in a Big Crunch.

An open universe: if the mean density is less than the critical value, gravity will slow the rate of expansion towards a steady value and the expansion will continue forever. If it is assumed that effectively all matter in the universe is luminous, then the mean density turns out to be little more than 1% of the critical value. If that was the case the universe would certainly be ‘open’ and destined to expand forever.

A flat universe: if the mean density equals the critical value, the universe will be able to expand forever with the recession velocities tending towards zero.

SPACE EXPANDING

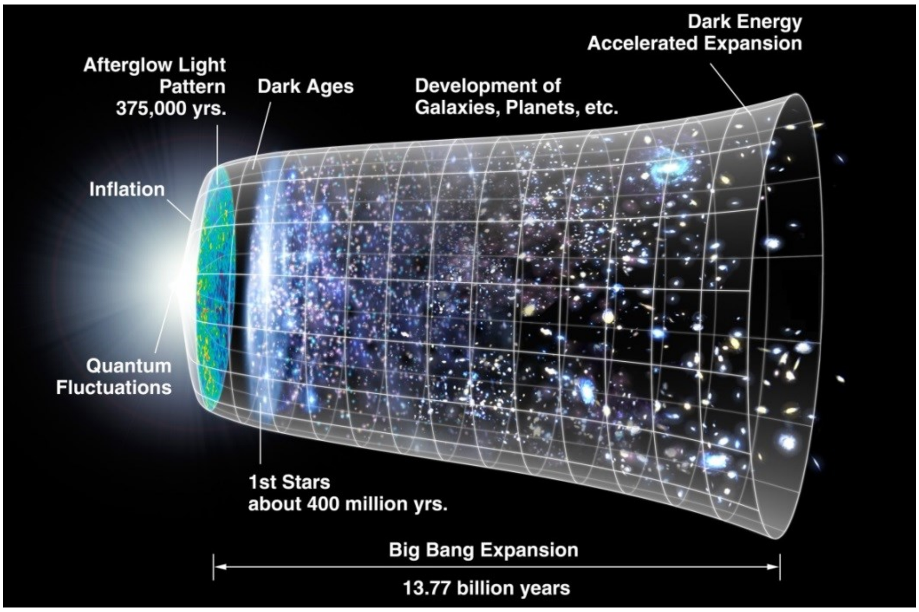
It is generally considered that a better way to think of redshift is that it is due to space itself expanding. As space expands, the waves become stretched, ie their wavelength increases.

In the time it takes light to travel from one galaxy to another, space has expanded so the distance between the galaxies and the wavelength of light have both been stretched by the same factor.

EXPANSION NOT EXPLOSION

Although we talk about the Big Bang, it is important to emphasise that the universe, ie space, is expanding. There are a number of characteristics that indicate it is an expansion and not the result of an explosion.

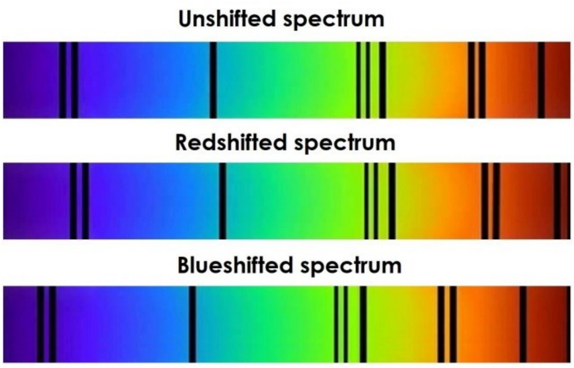
Explosion	Expansion
Different bits fly off at different speeds	Expansion explains the large-scale symmetry we see in the distribution of galaxies
Fast parts overtake slow parts	Expanding space explains the redshifts and the Hubble law
Difficult to imagine a suitable mechanism to produce the range of velocities from 100 kms^{-1} to almost the speed of light	Expansion also explains redshifts and the Hubble law even if we are not at the centre of the universe
Seems likely velocity would be related to some physical property, eg if given the same energy, less massive galaxies would be moving faster	Balloon analogy – every galaxy moves away from every other as the space expands
If this was the case a definite correlation between mass and velocity would be expected - this is not observed	No galaxy is located at the centre
Hubble's Law works well even if we only plot data for galaxies of similar mass	Not only are we not at the centre of the universe, it doesn't even need to have a centre
Faster galaxies would leave slower ones behind, resulting in those near the centre (start) being more closely packed than those on the periphery (finish), like runners in a marathon, but this is not observed	

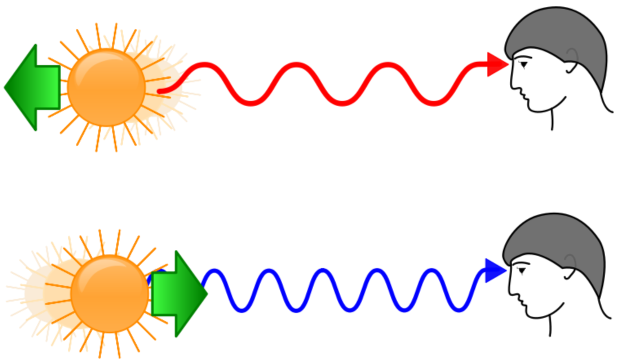


3By NASA/WMAP Science Team - Original version: NASA; modified by Cherkash, Public Domain, https://commons.wikimedia.org/w/index.php?curid=11885244

BLUESHIFT.

Note: If there is a decrease in wavelength, ie the line spectrum has moved towards the blue end of the spectrum, this makes z negative, which means the body is moving towards us. This is referred to as a blueshift. This occurs with individual stars as they orbit other stars in the Universe, but not for the galaxies, which are always moving away from us.





The End of OUR DYNAMIC UNIVERSE